

# Dynamic Field Theory

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# Discrete “neurons”

- or activation variables: how do they arise?  
How do they sample sensory/motor spaces...
- no evidence that neural discreteness matters  
for behavior

# Continuity in space

- hypothesis: behavior is embedded in continua
  - the space of possible behaviors, e.g. space of movements, percepts, timing structures
  - neuronal substrate is continuous (maps, broad tuning)
- ( $\Rightarrow$  need to understand how categorical behavior may emerge from such continua)

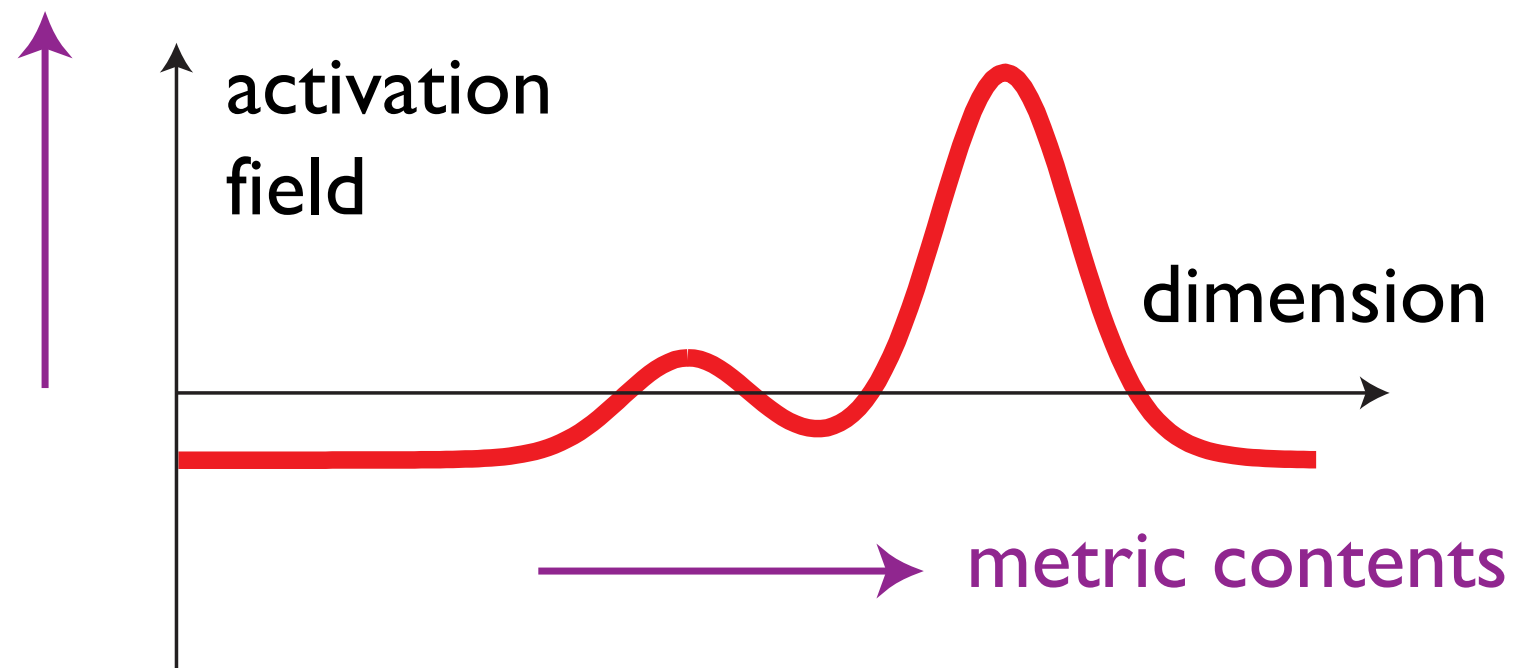
# Dynamical Field Theory: space

- in DFT, continuous spaces are dimension over which activation fields are defined
- homologous to sensory surfaces, e.g., visual or auditory space (retinal, allocentric, ...)
- homologous to motor surfaces, e.g., saccadic end-points or direction of movement of the end-effector in outer space
- feature spaces, e.g., localized visual orientations, color, impedance, ...
- abstract spaces, e.g., ordinal space, along which serial order is represented

# Dynamical Field Theory: space

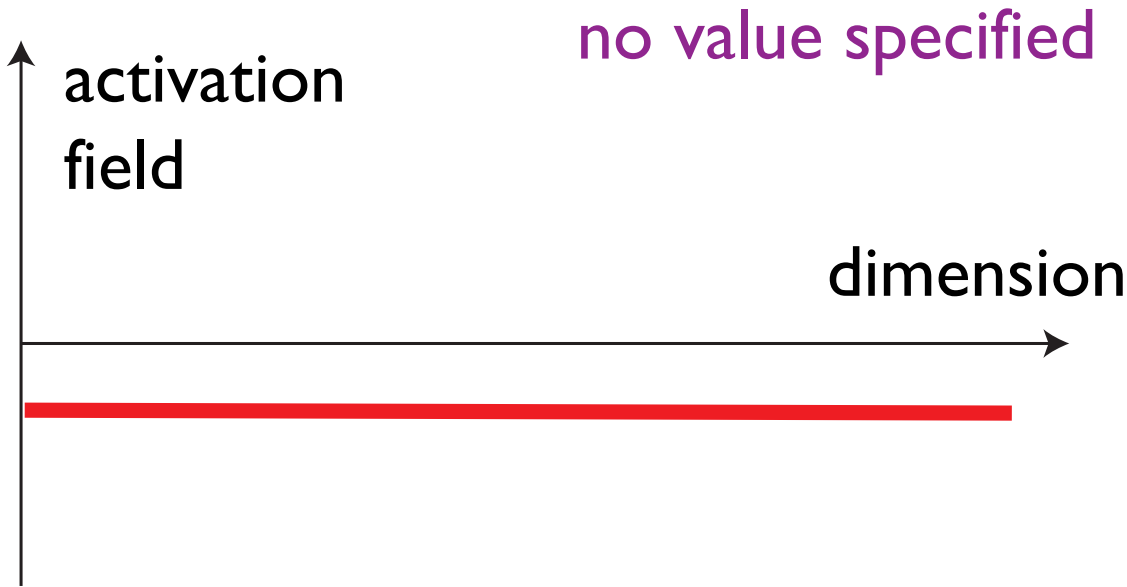
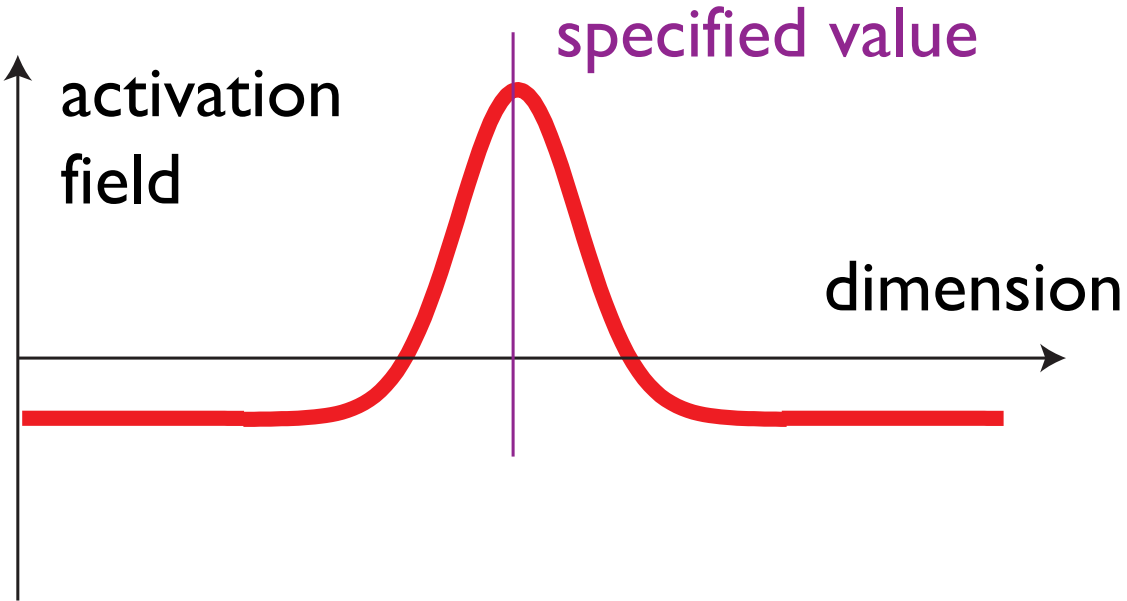
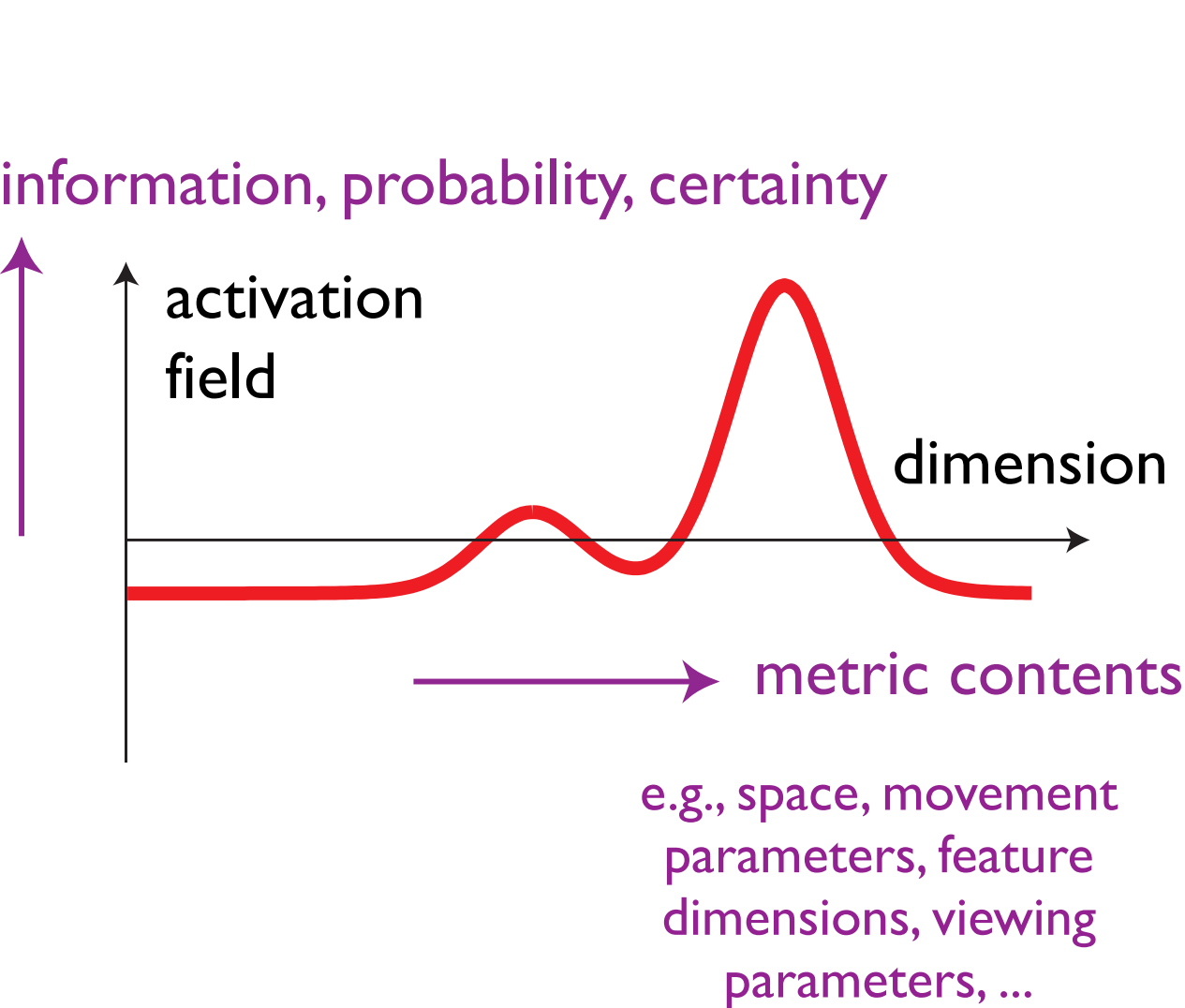
- fields: continuous activation variables defined over continuous spaces

information, probability, certainty



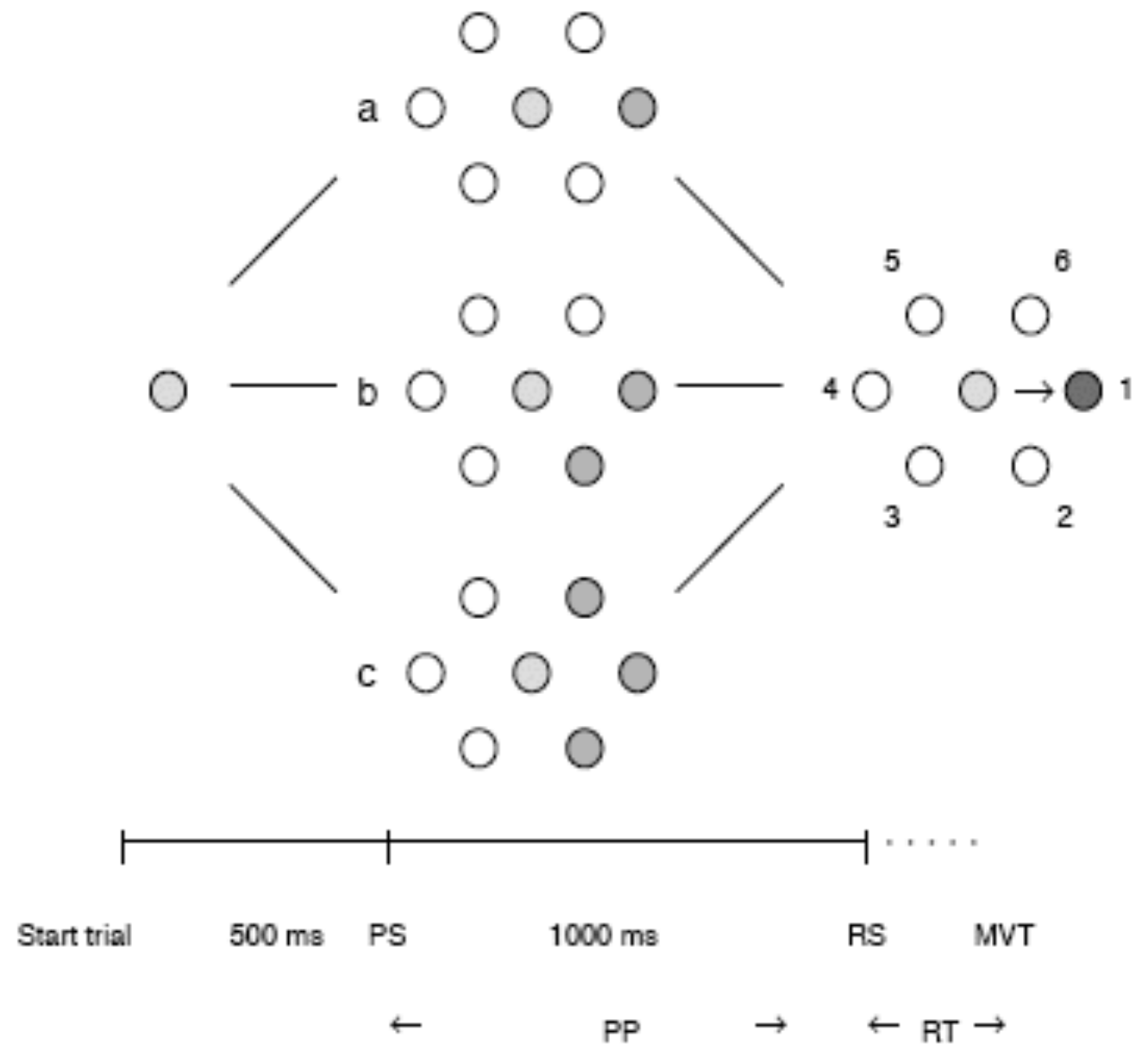
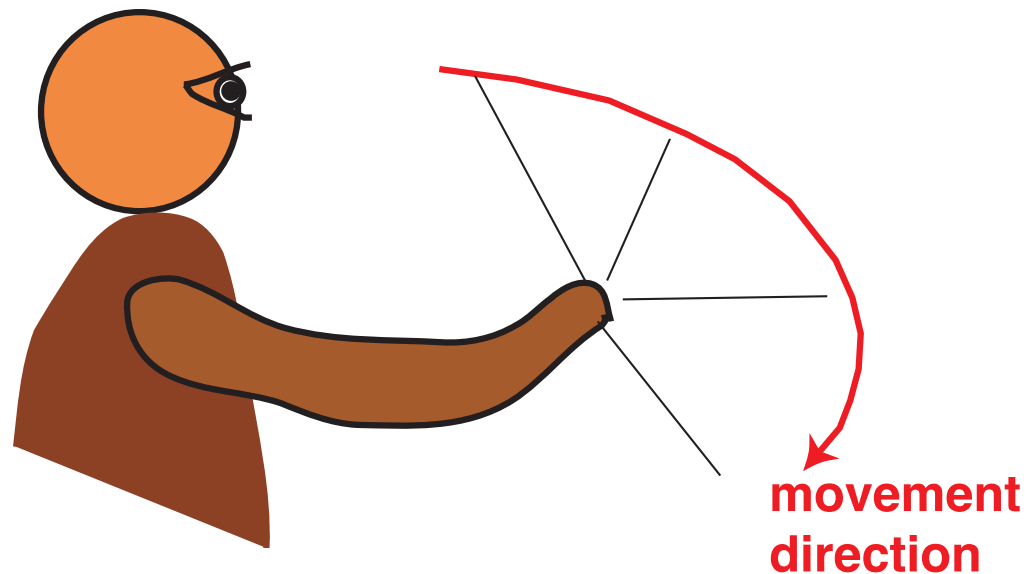
e.g., retinal space, movement parameters, feature dimensions, viewing parameters, ...

# activation fields

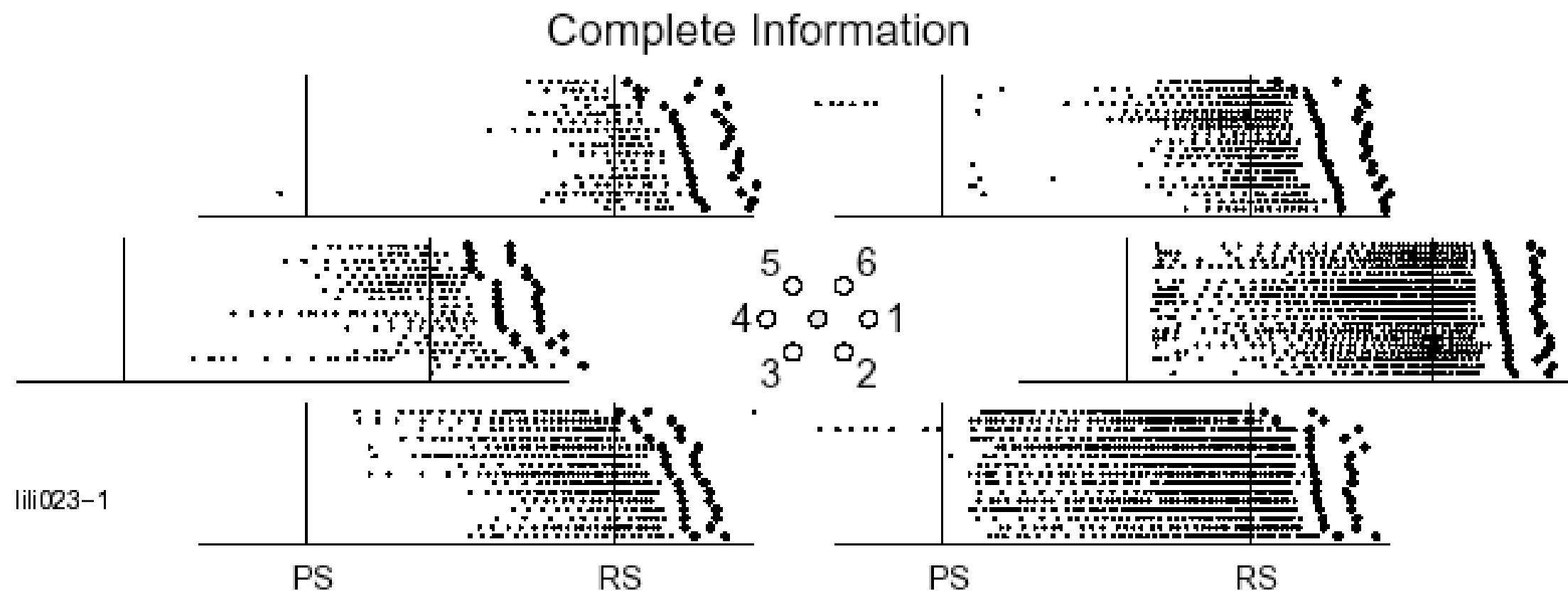


# Neurophysiological grounding of DFT

## example: movement planning



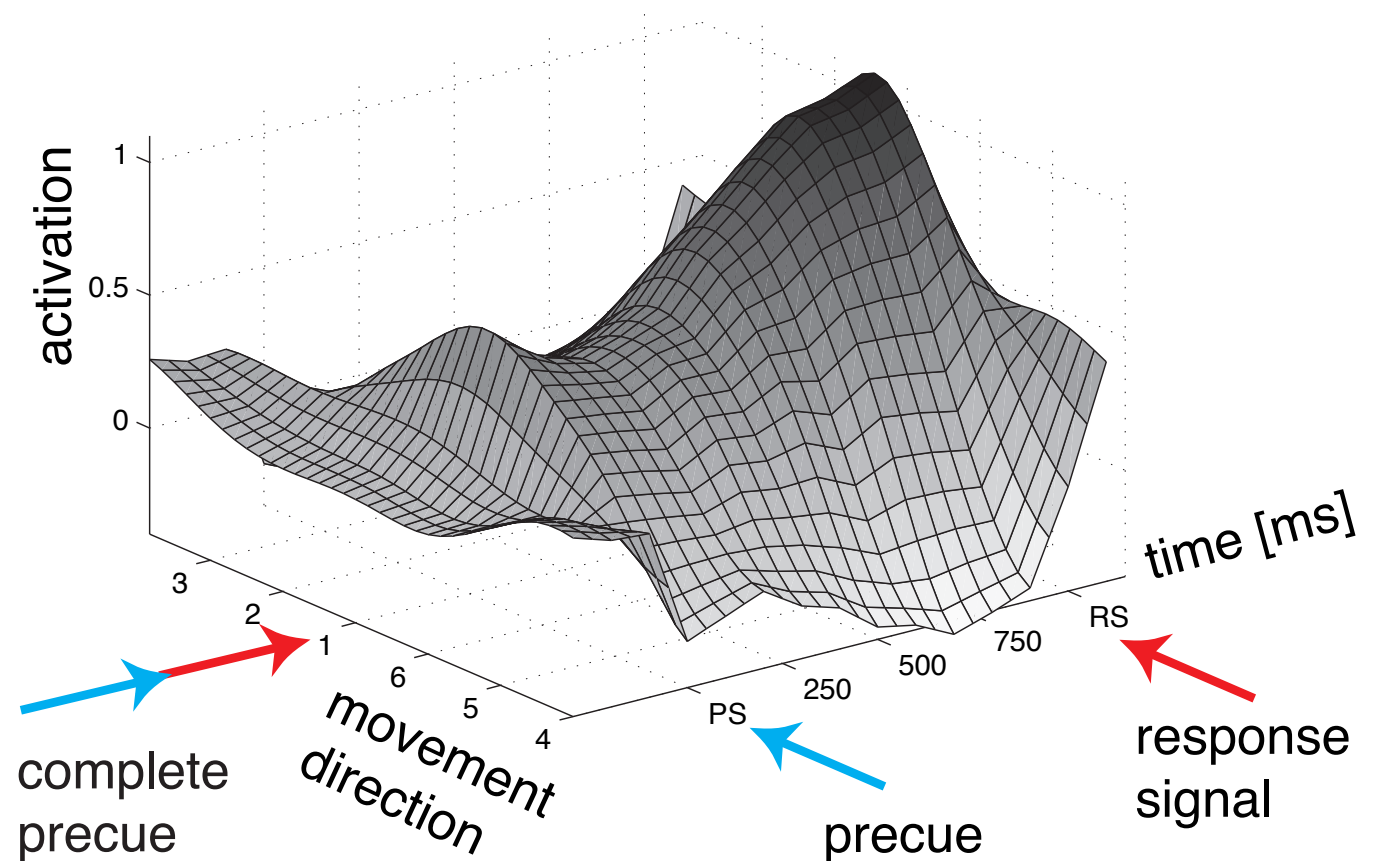
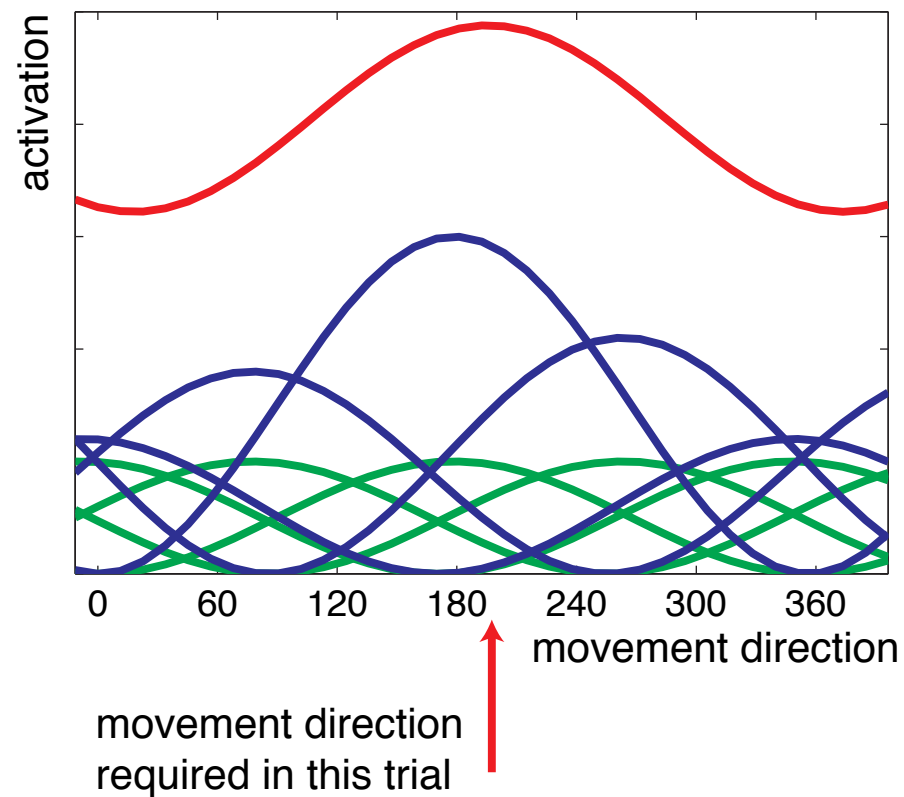
- tuning of cells in motor and premotor cortex to direction of end-effector movement path





# Distribution of Population Activation (DPA)

Distribution of population activation =  
 $\sum_{\text{neurons}} \text{tuning curve} * \text{current firing rate}$

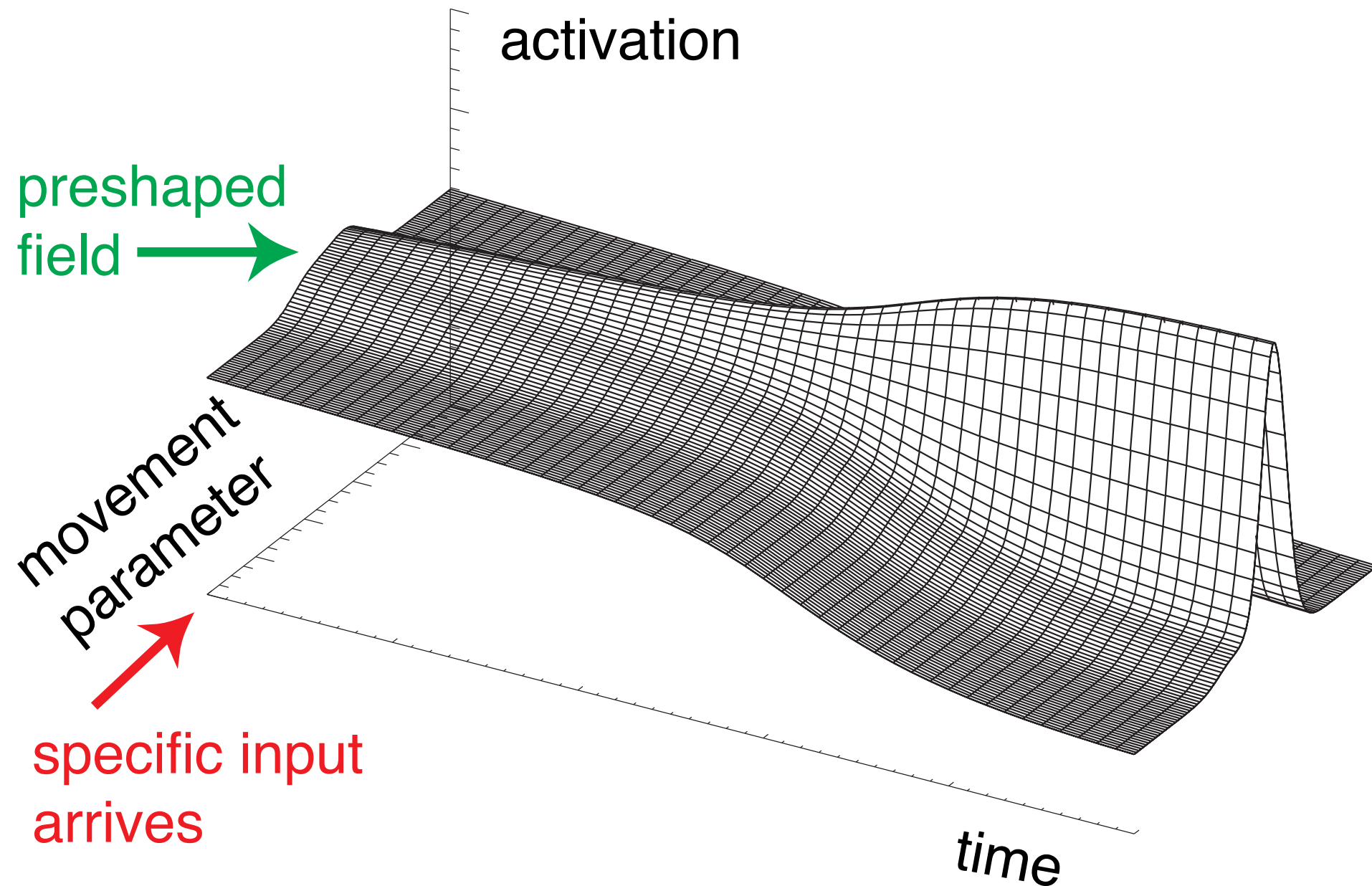


[Bastian, Riehle, Schöner, 2003]

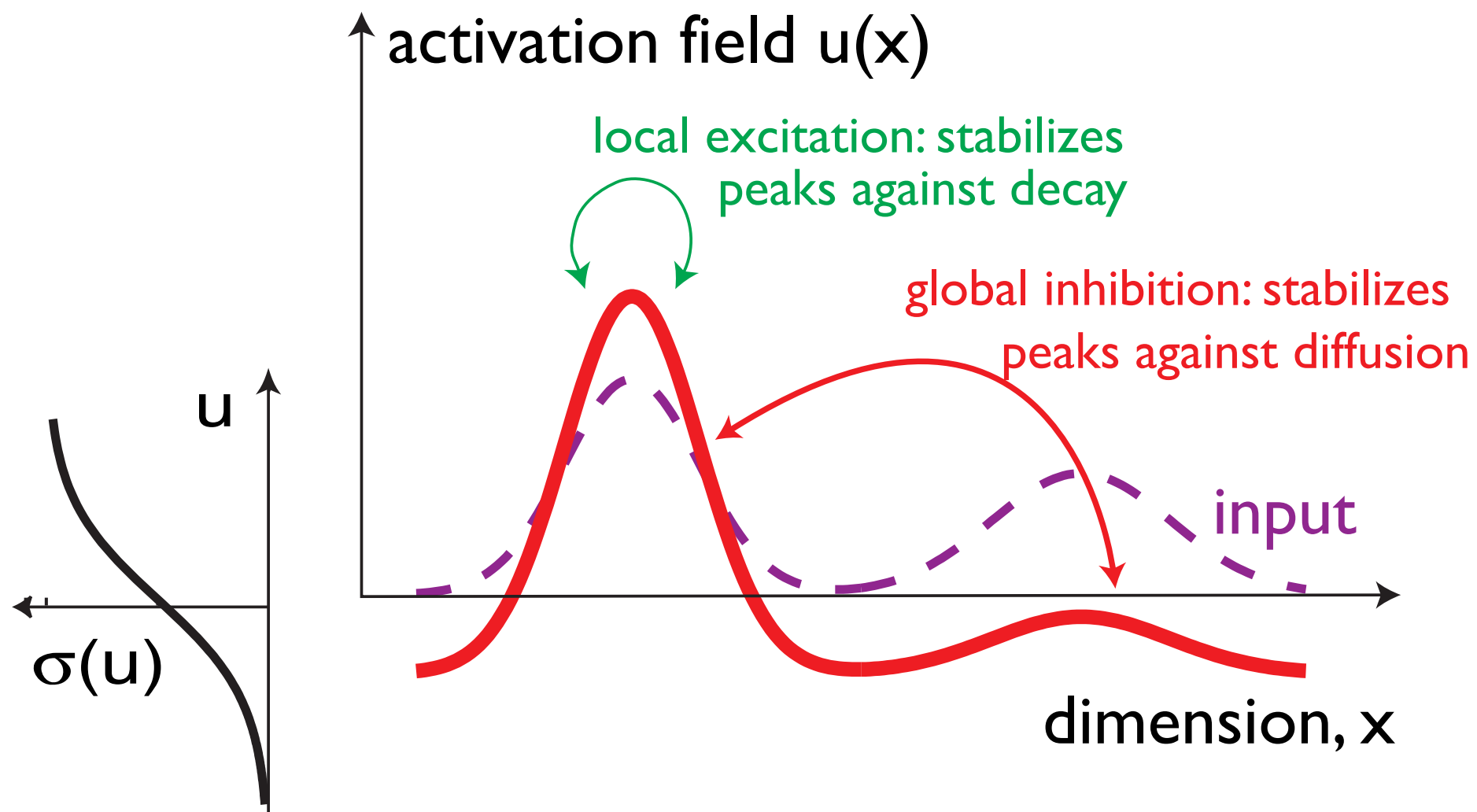
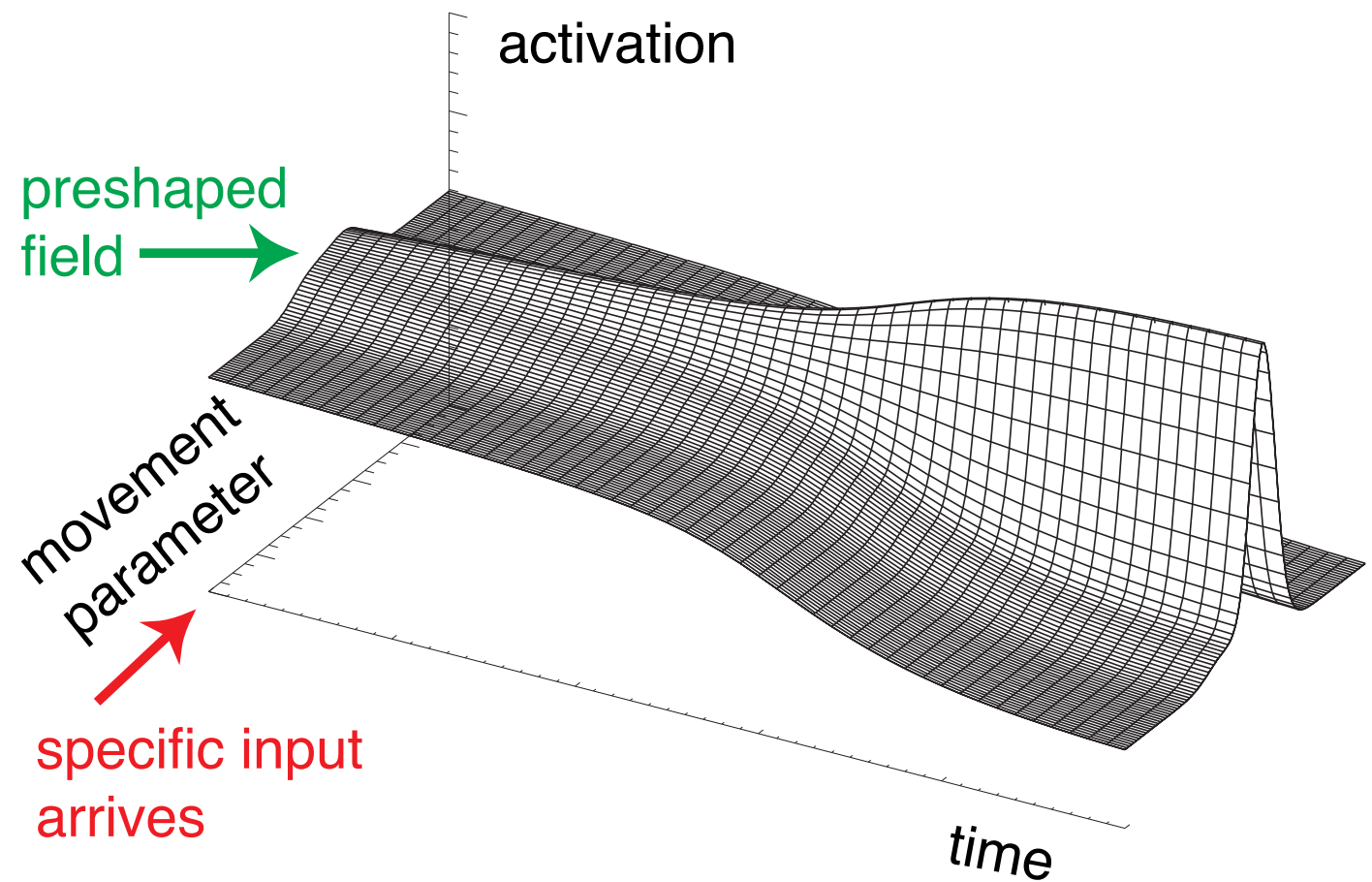
# Distributions of Population Activation are abstract

- neurons are not **localized** within DPA!
- cortical neurons really are sensitive to many dimensions
  - motor: arm configuration, force direction
  - visual: many feature dimensions such as spatial frequency, orientation, direction...
- => DPA is a **projection** from that high-dimensional space onto a single dimension

# evolution of activation fields in time: neuronal dynamics



the dynamics such  
activation fields is  
structured so that  
localized peaks  
emerge as attractor  
solutions



# mathematical formalization

Amari equation

$$\tau \dot{u}(x, t) = -u(x, t) + h + S(x, t) + \int w(x - x') \sigma(u(x', t)) dx'$$

where

- time scale is  $\tau$
- resting level is  $h < 0$
- input is  $S(x, t)$
- interaction kernel is

$$w(x - x') = w_i + w_e \exp \left[ -\frac{(x - x')^2}{2\sigma_i^2} \right]$$

- sigmoidal nonlinearity is

$$\sigma(u) = \frac{1}{1 + \exp[-\beta(u - u_0)]}$$

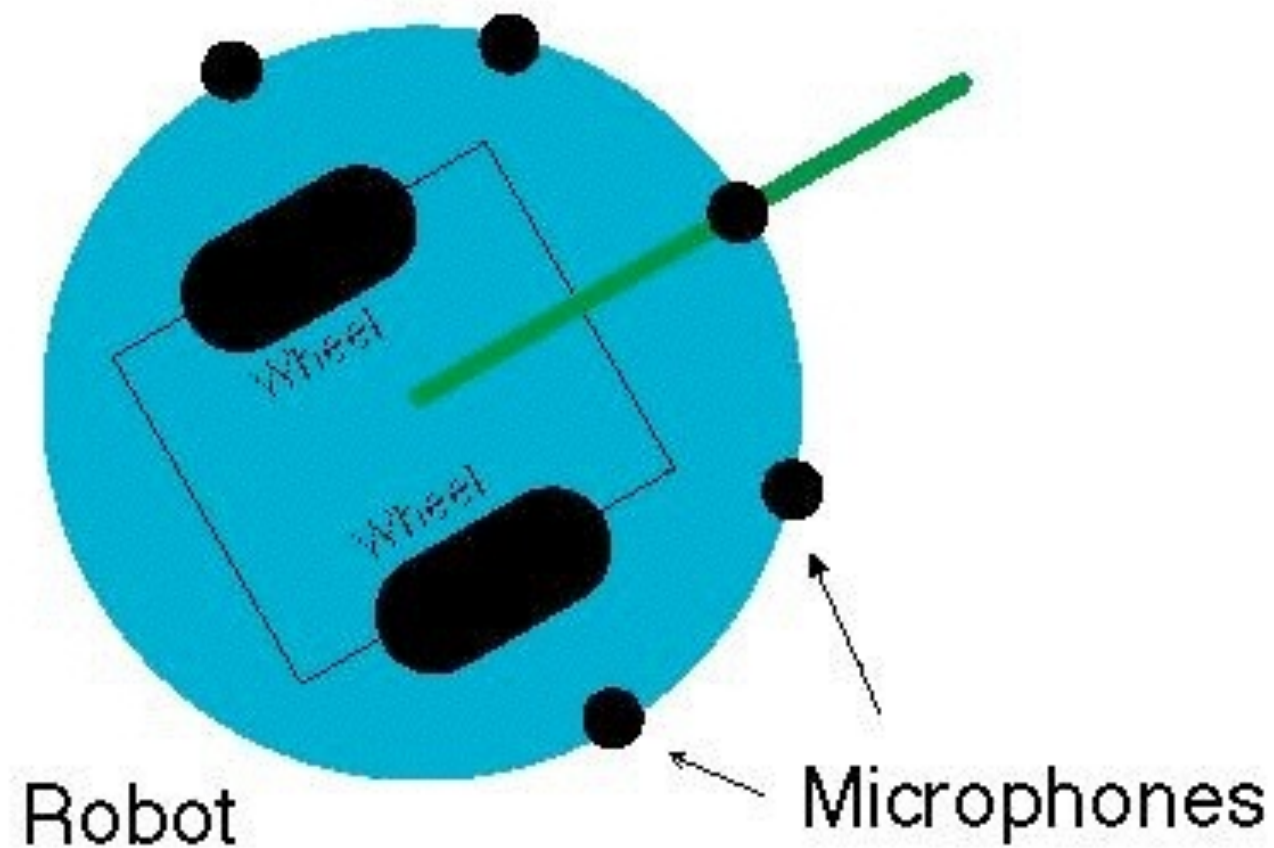
=> simulations

# solutions and instabilities

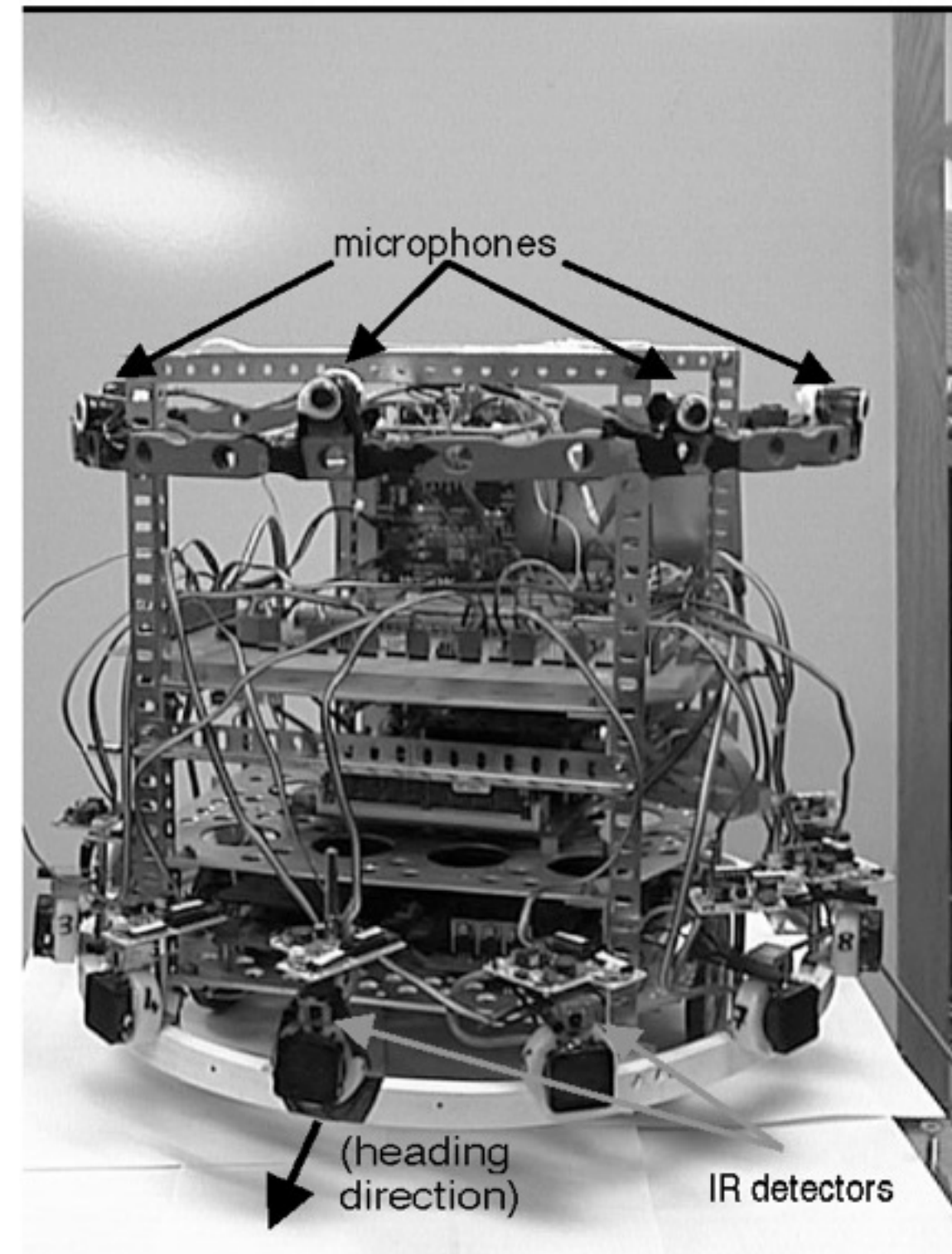
- input driven solution (sub-threshold) vs. self-stabilized solution (peak, supra-threshold)
- detection instability
- reverse detection instability
- selection
- selection instability
- memory instability
- detection instability from boost



# Illustration: linking to sensors



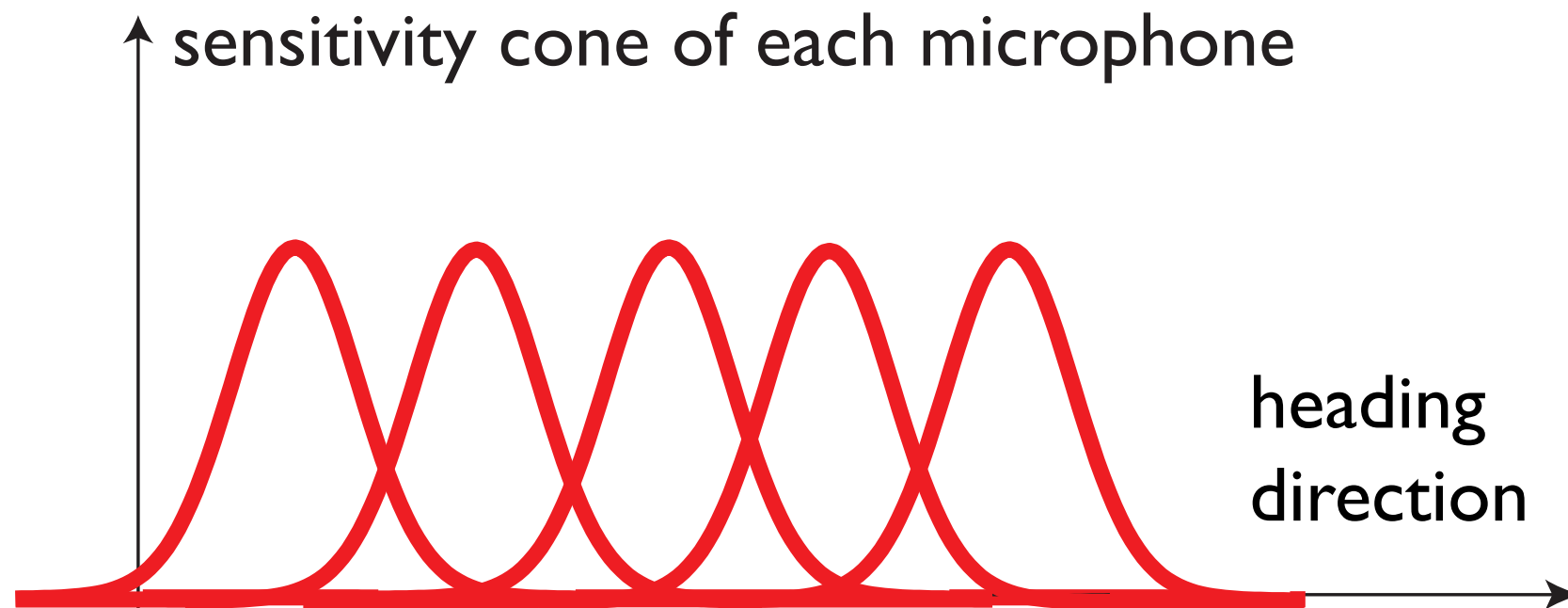
[from Bicho, Mallet, Schöner, Int J Rob Res, 2000]



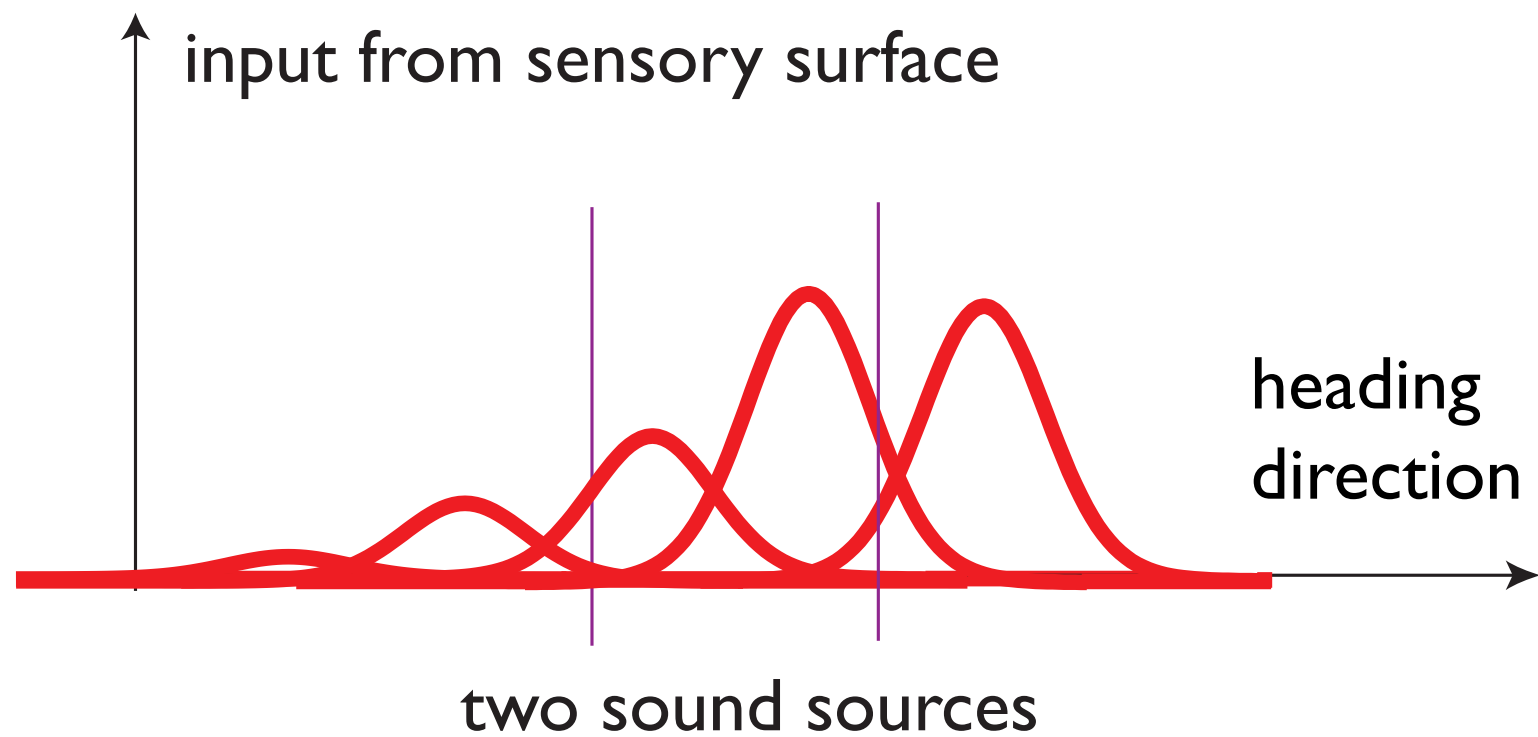
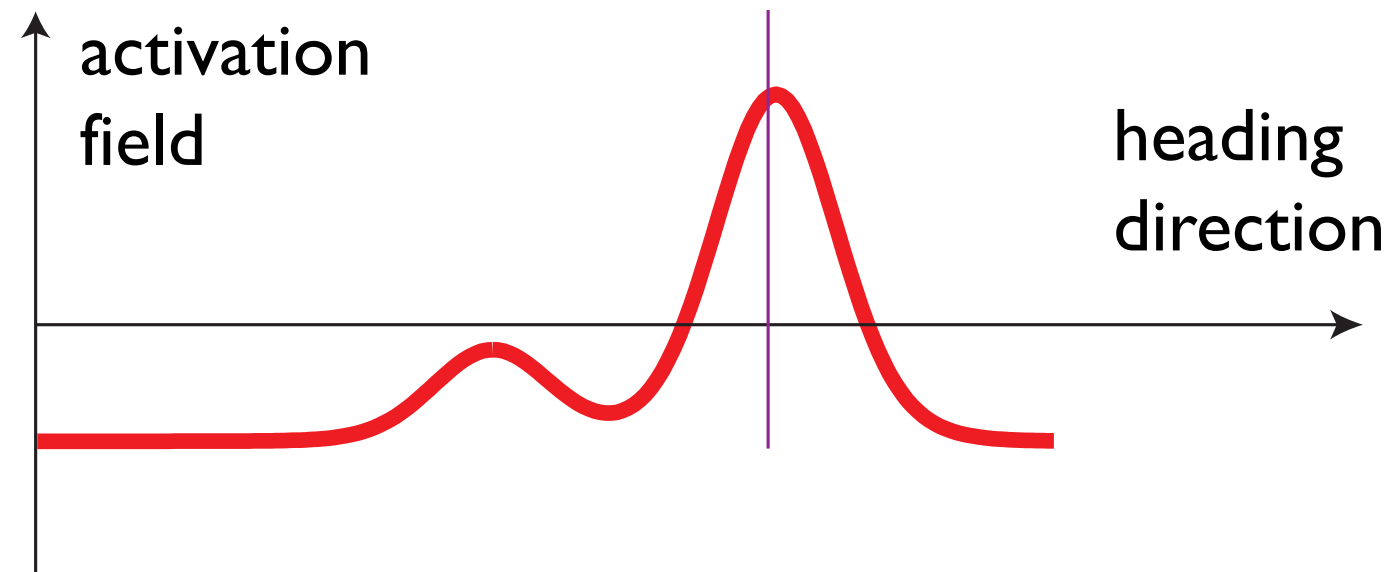


# Sensory surface

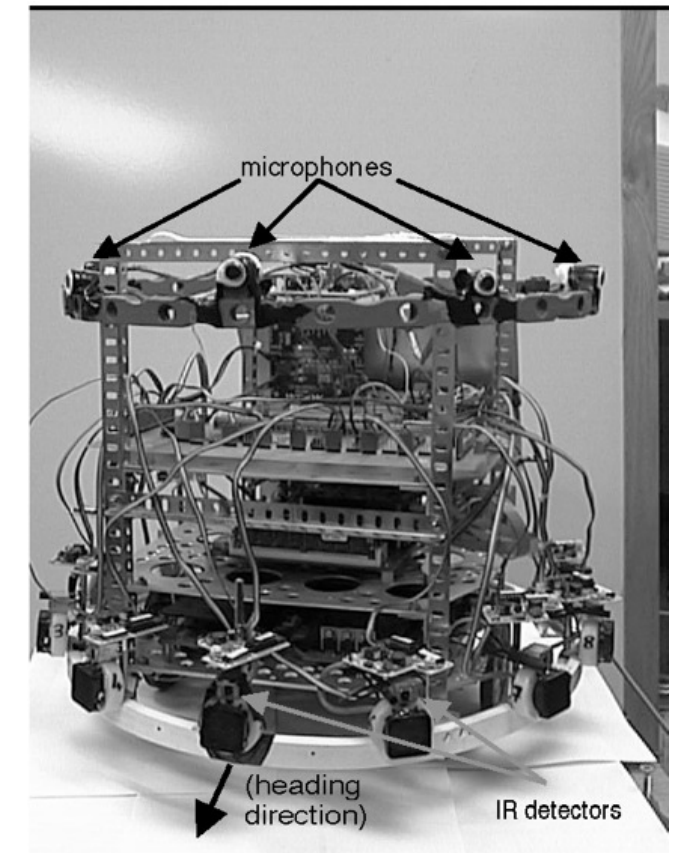
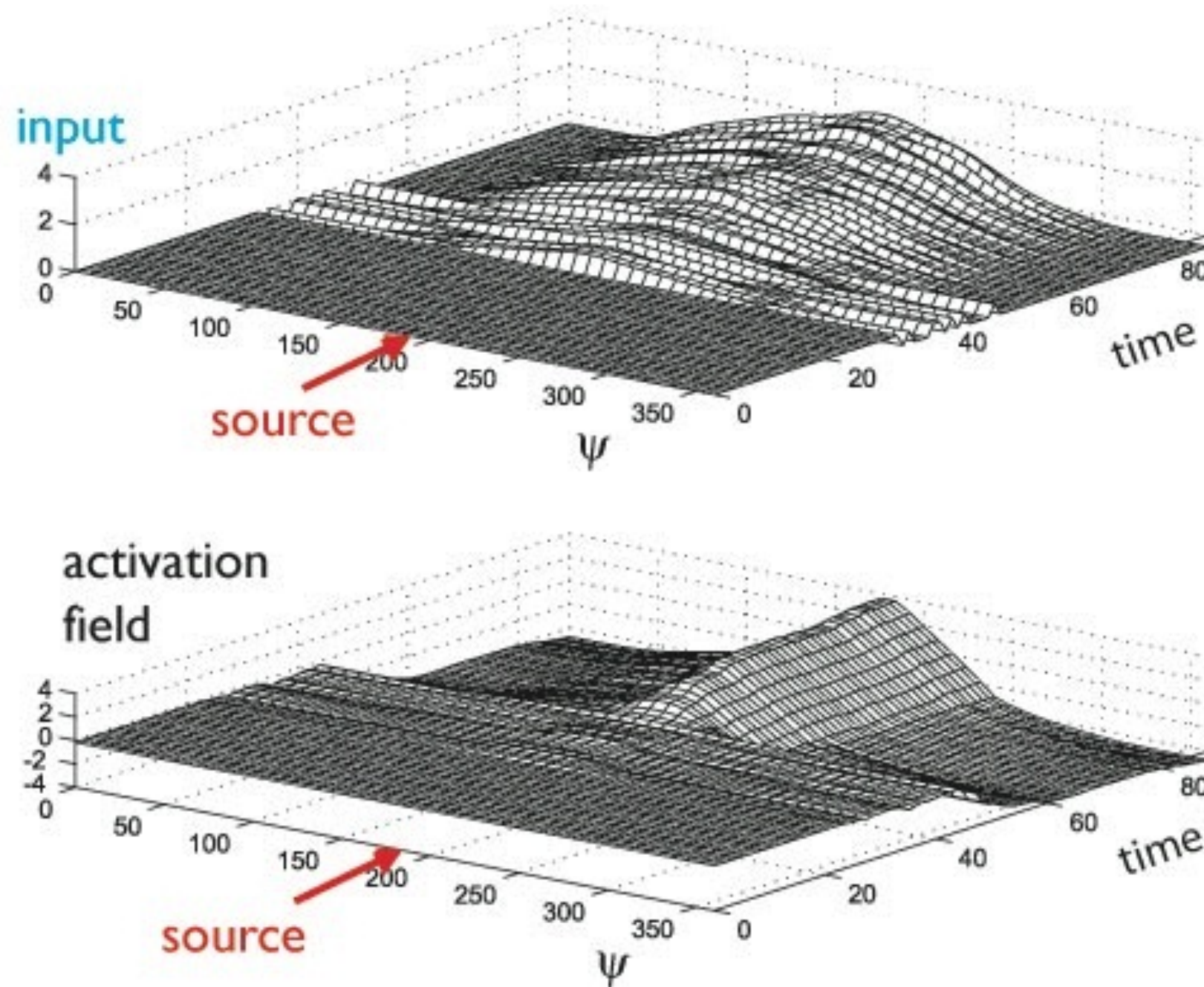
- each microphone samples heading direction



- each microphone provides input to the field



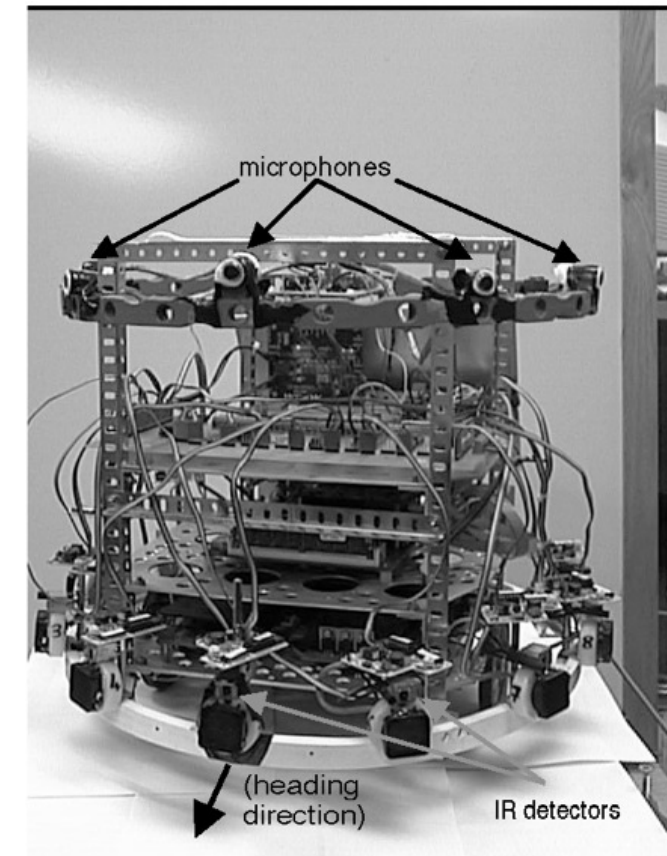
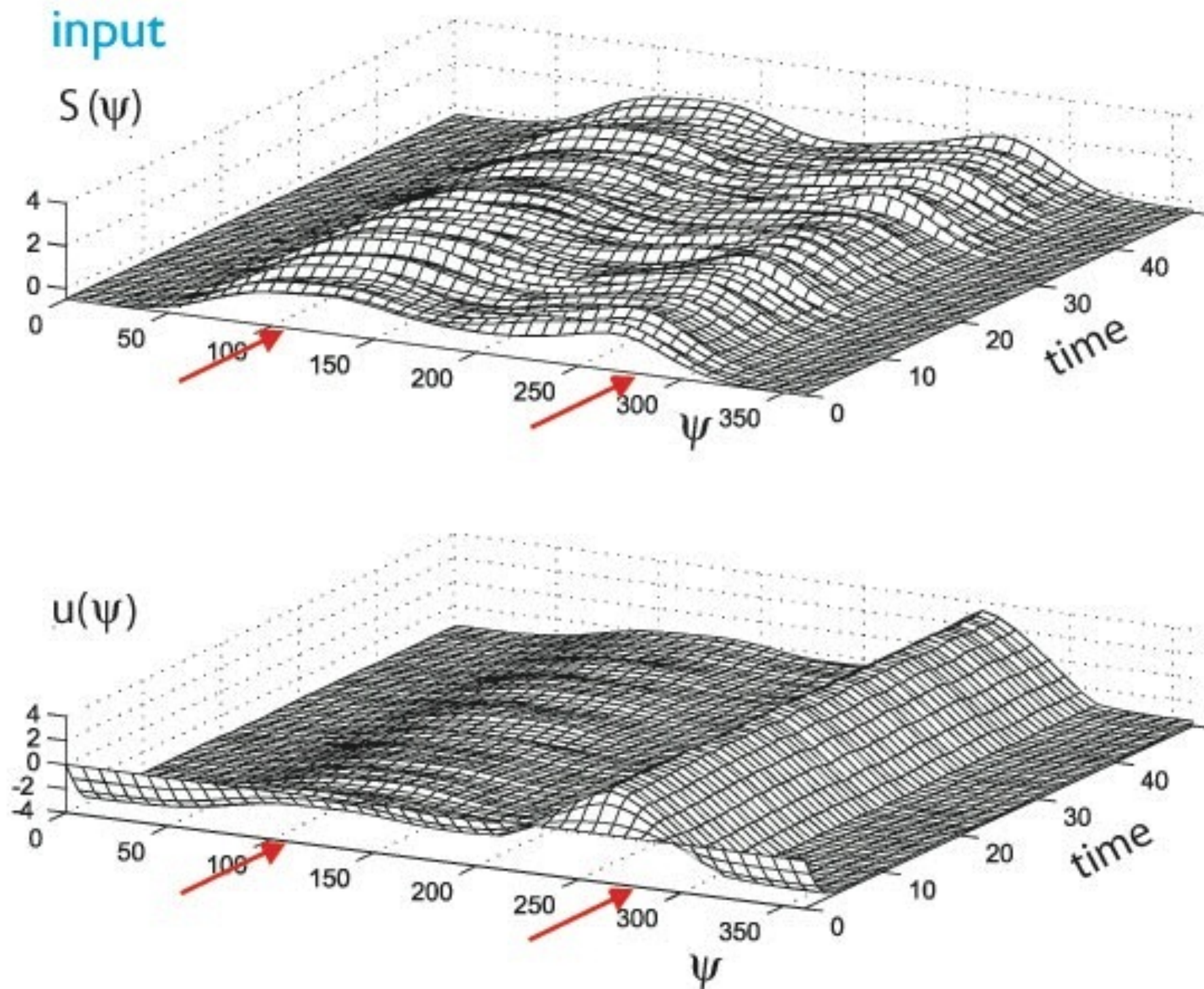
# Detection instability induced by increasing intensity of sound source



[from Bicho, Mallet, Schöner: Int. J. Rob. Res., 2000]

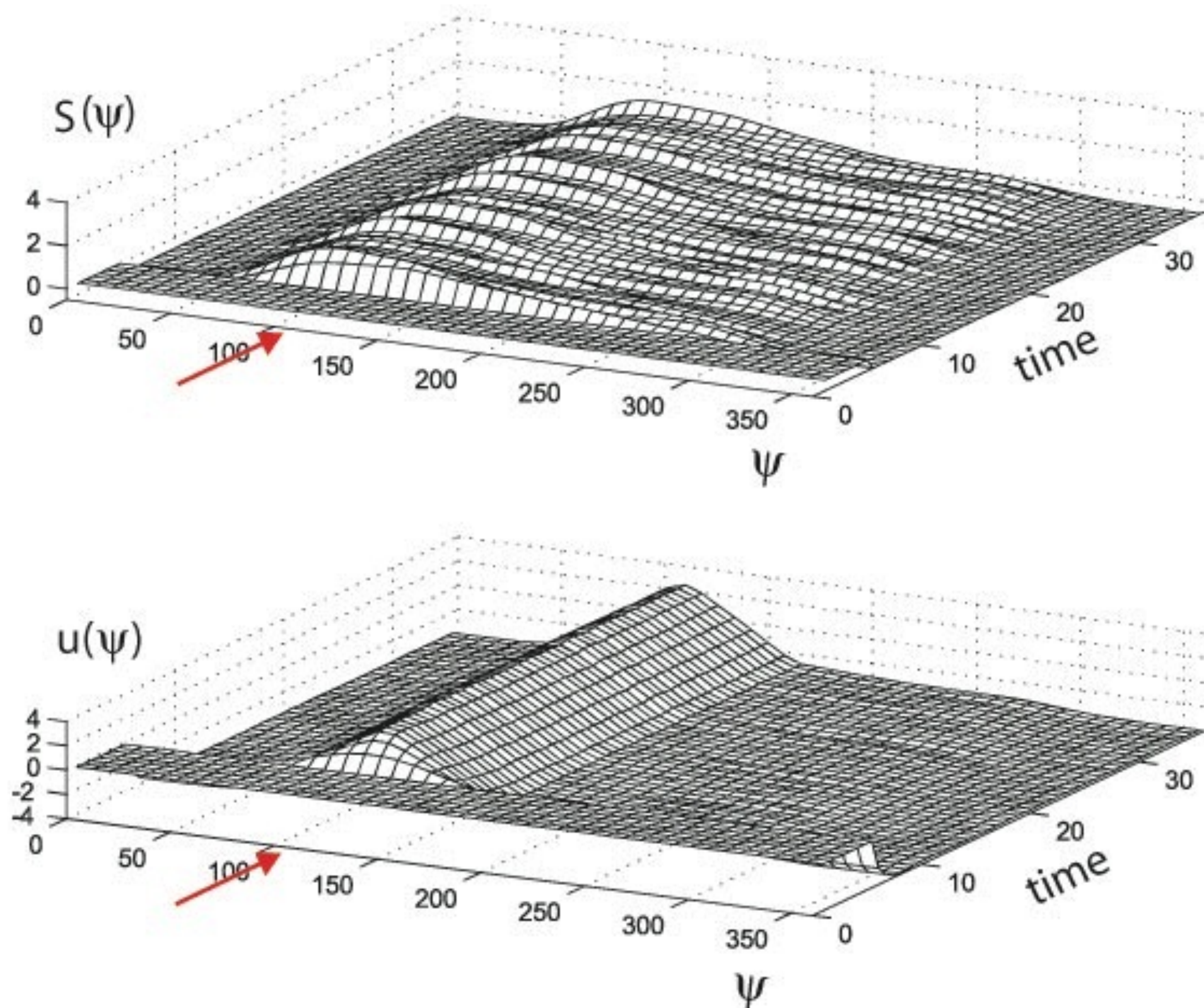


# Target selection in the presence of two sources

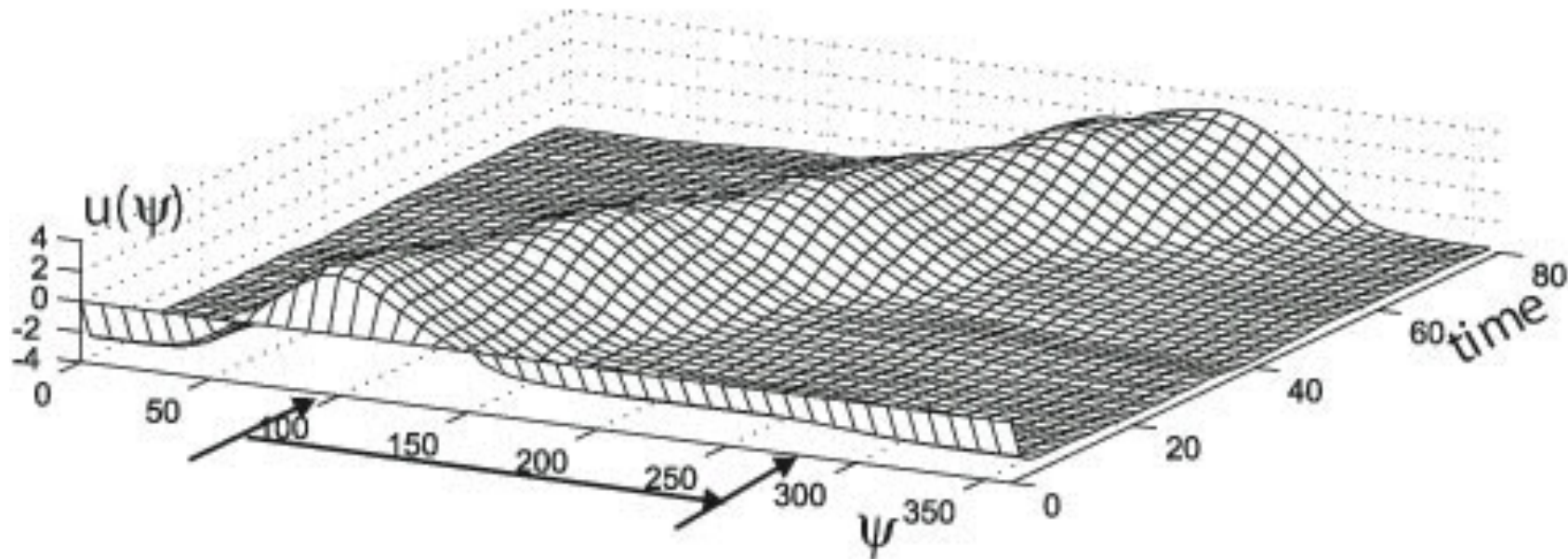
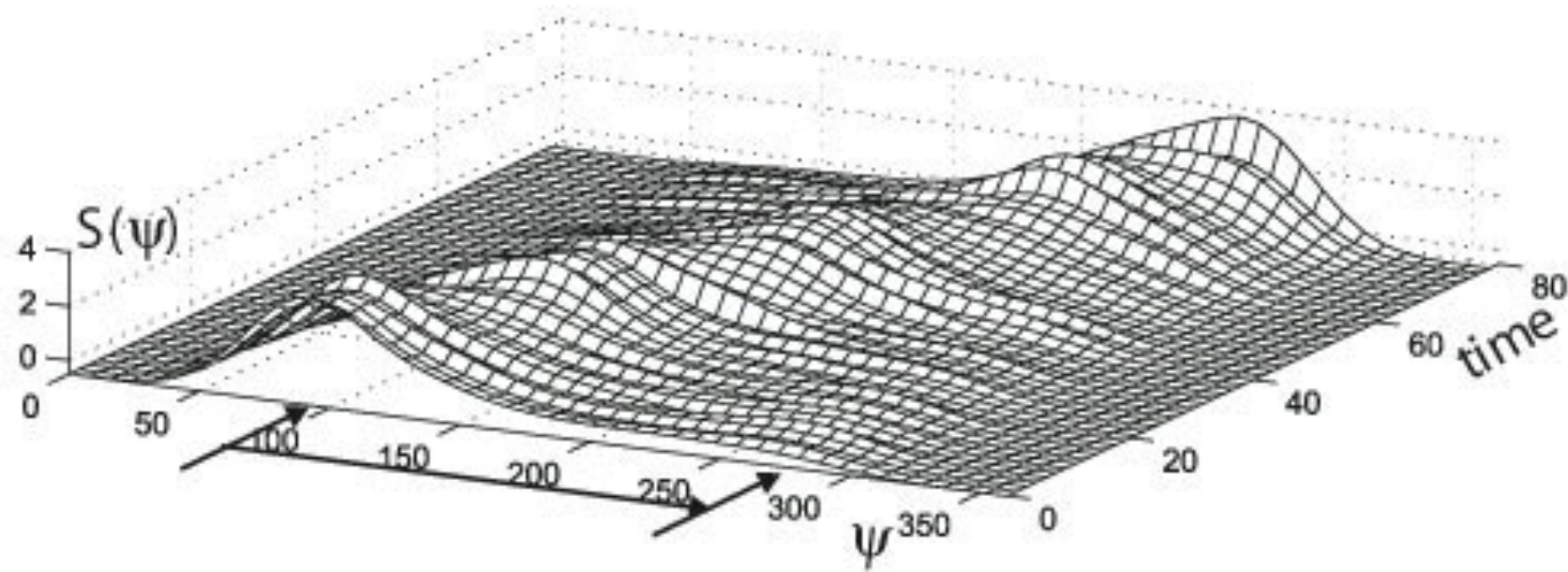




# Robust estimation in the presence of outliers

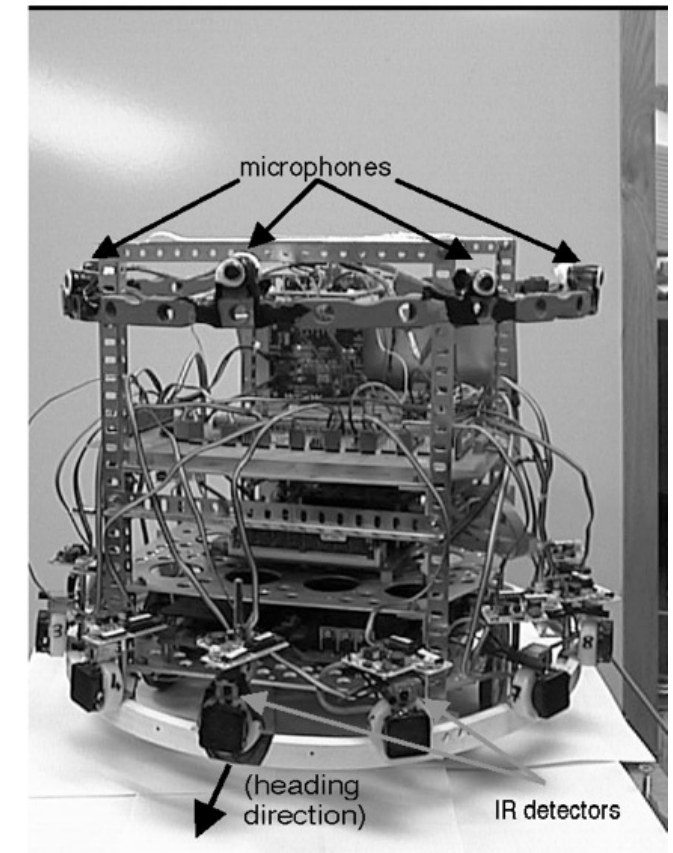
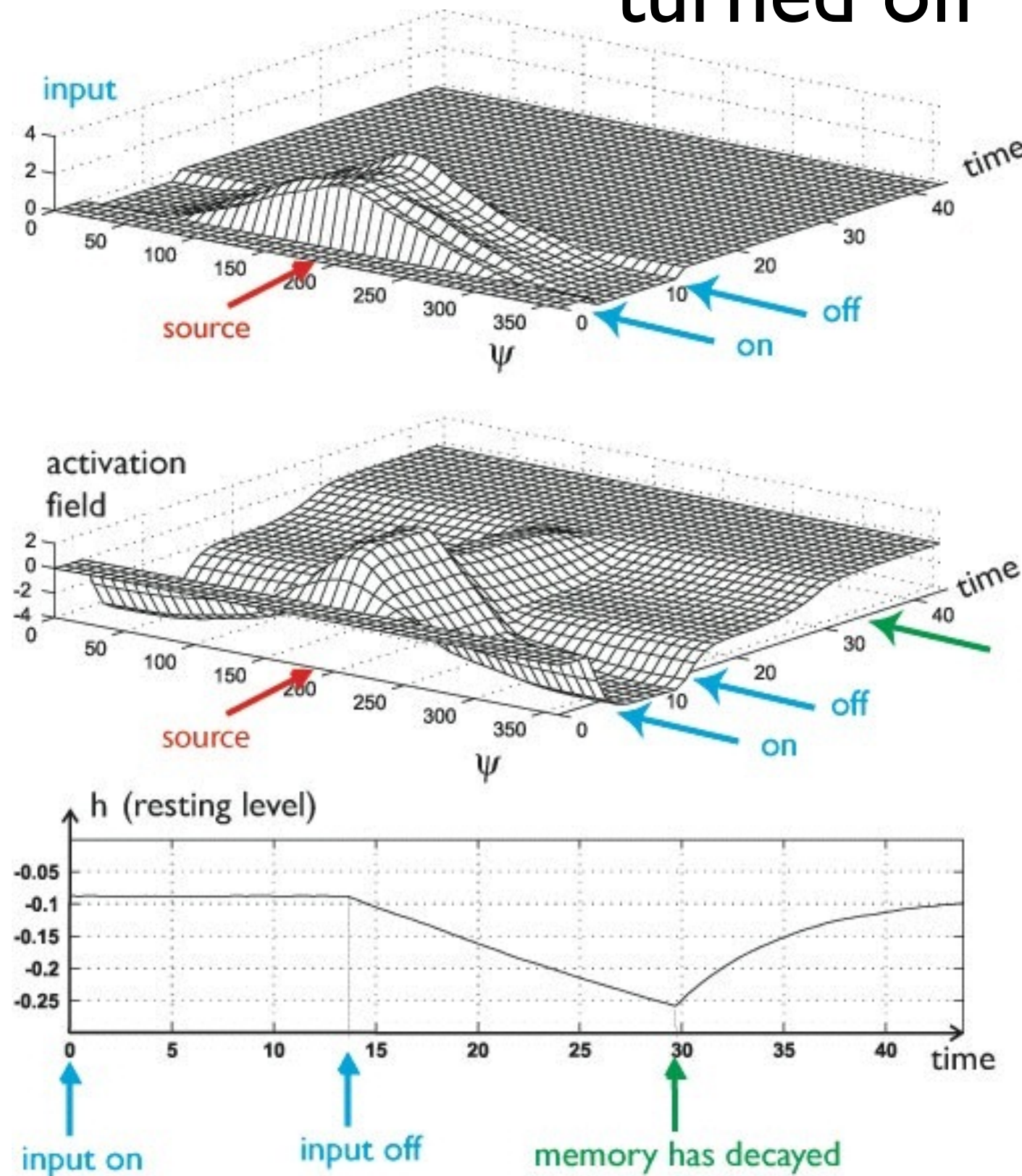


# Tracking when sound source moves





# Memory (and forgetting) when sound source is turned off



[from Bicho, Mallet, Schöner: Int J Rob Res 19:424(2000)]

# Illustration of instabilities

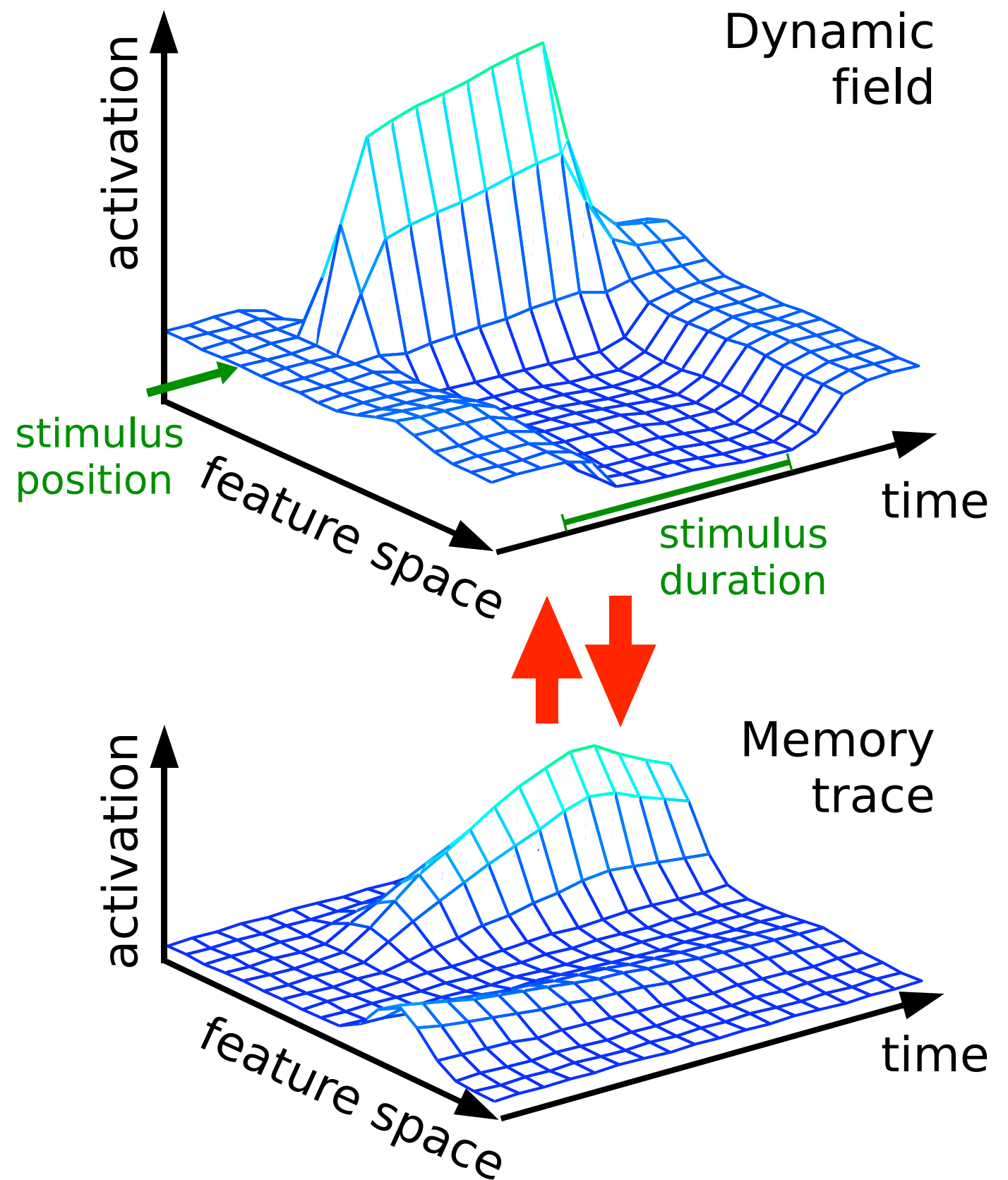




# simplest form of learning: the memory trace

■ William James: habit formation as the simplest form of learning

■ (habituation: same for inhibition)



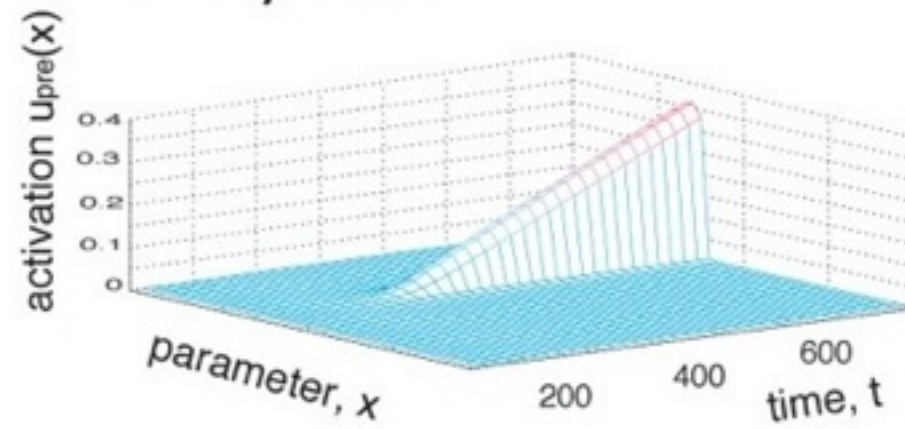
# mathematics of the memory trace

$$\tau \dot{u}(x, t) = -u(x, t) + h + S(x, t) + u_{\text{mem}}(x, t) + \int dx' w(x - x') \sigma(u(x'))$$

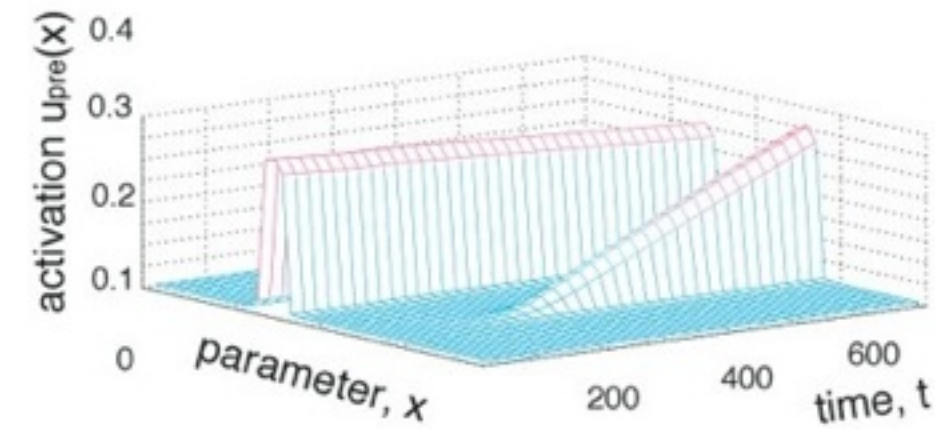
$$\tau_{\text{mem}} \dot{u}_{\text{mem}}(x, t) = -u_{\text{mem}}(x, t) + \int dx' w_{\text{mem}}(x - x') \sigma(u(x', t))$$

- memory trace only evolves while activation is excited
- potentially different growth and decay rates

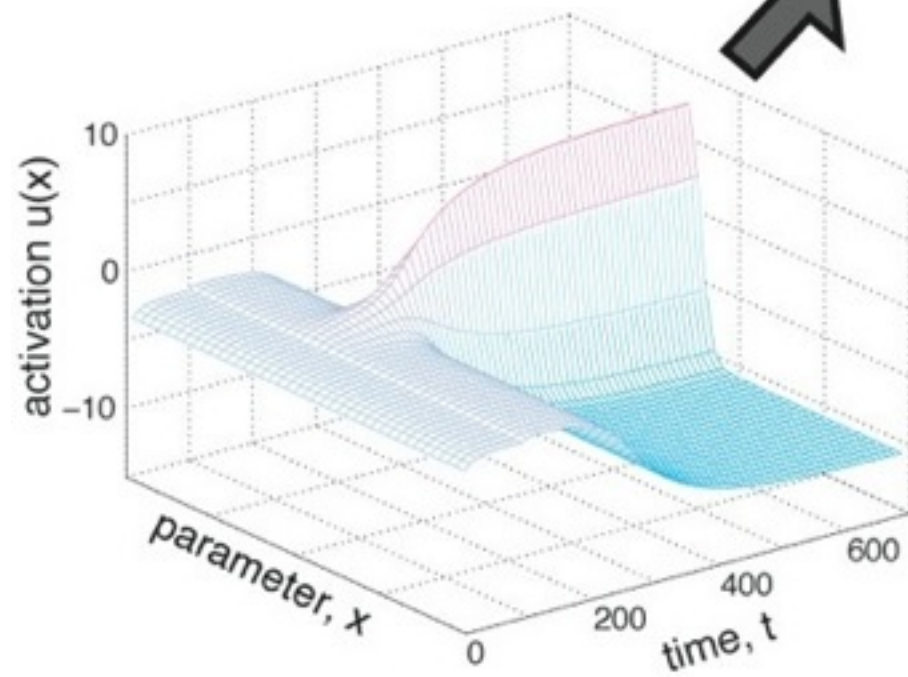
slow memory trace



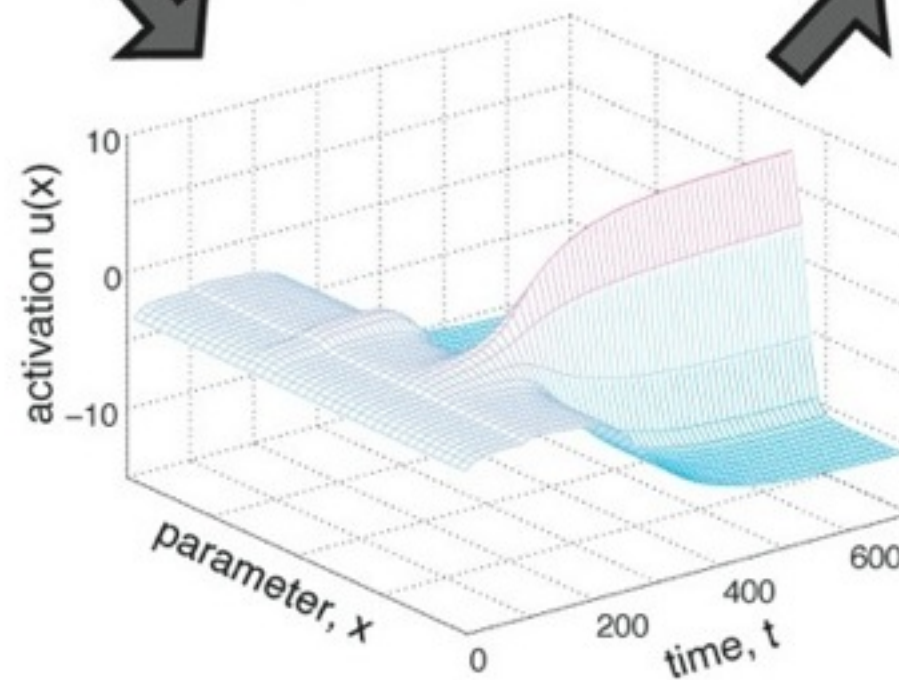
memory trace



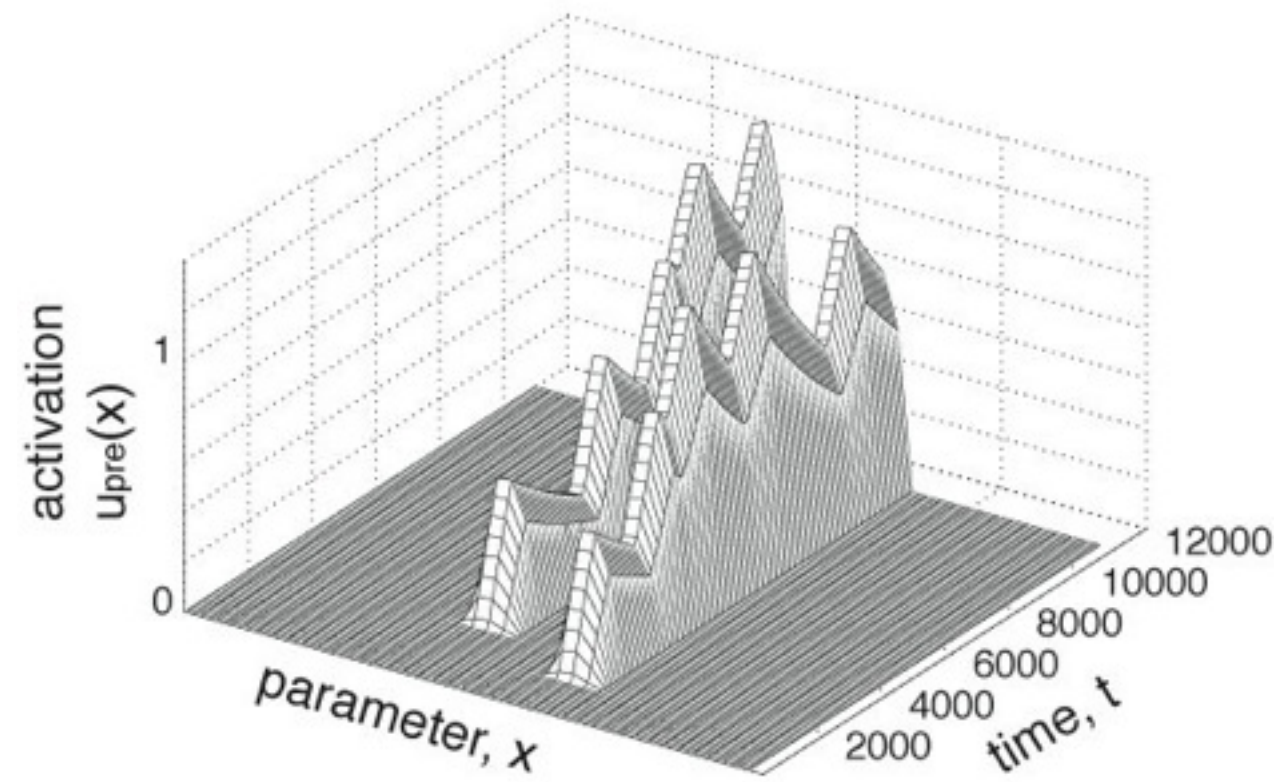
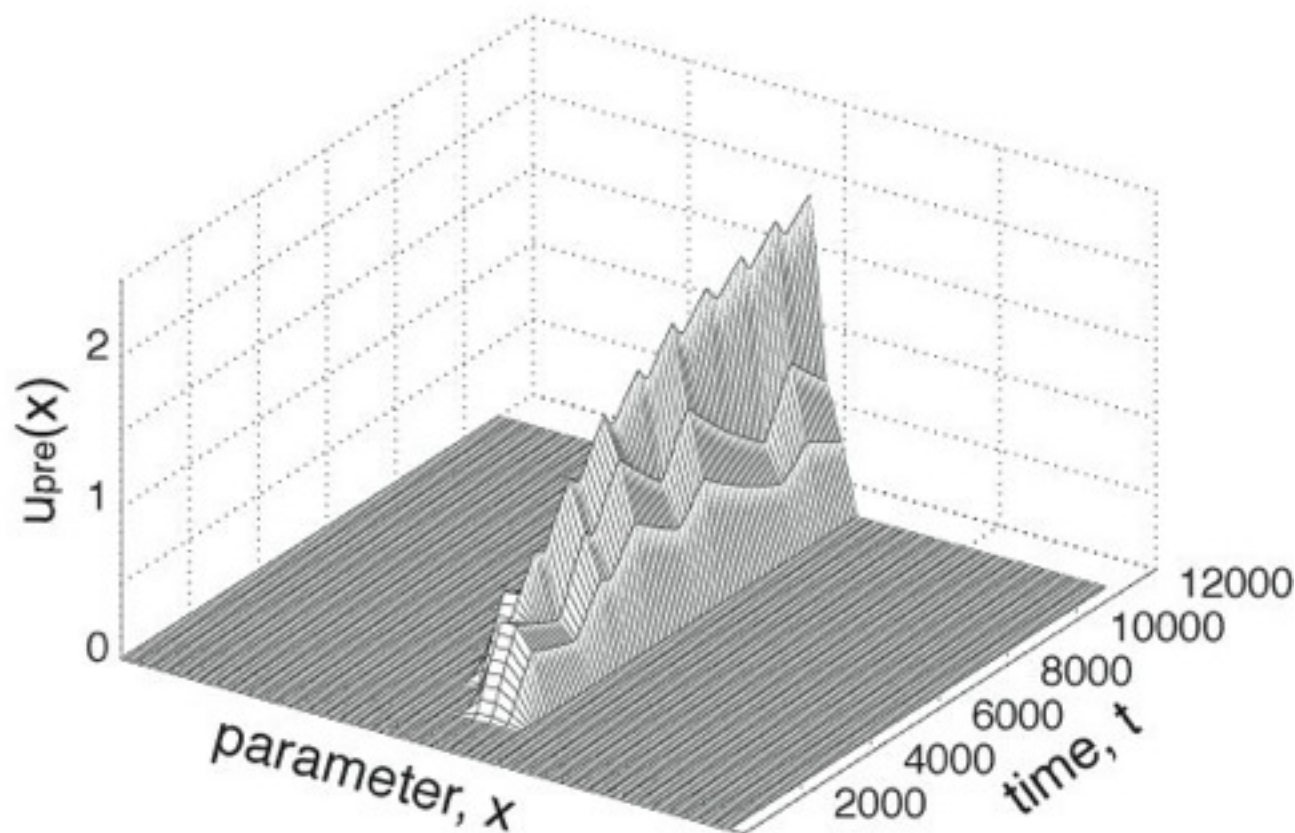
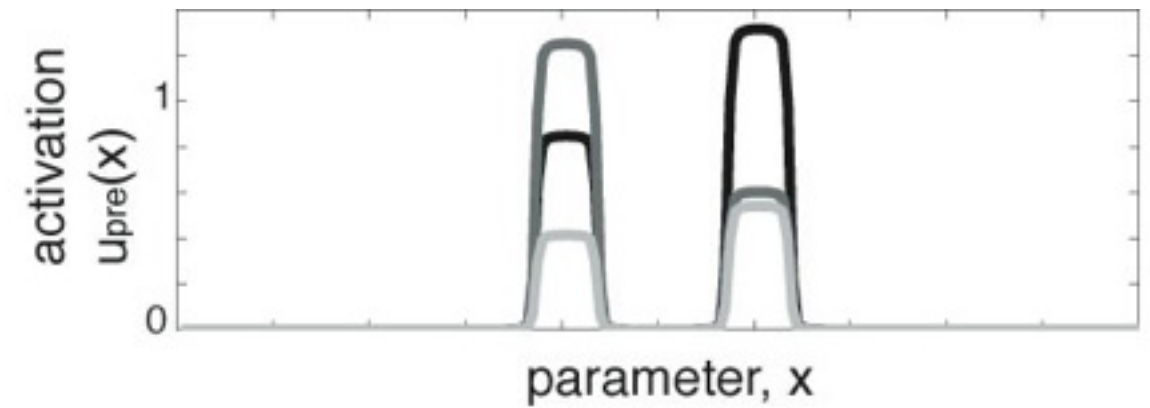
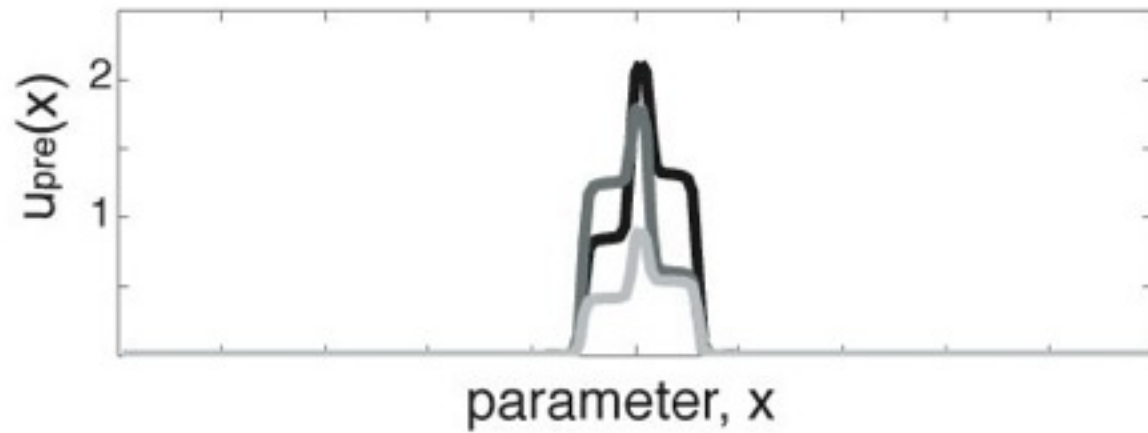
fast activation field



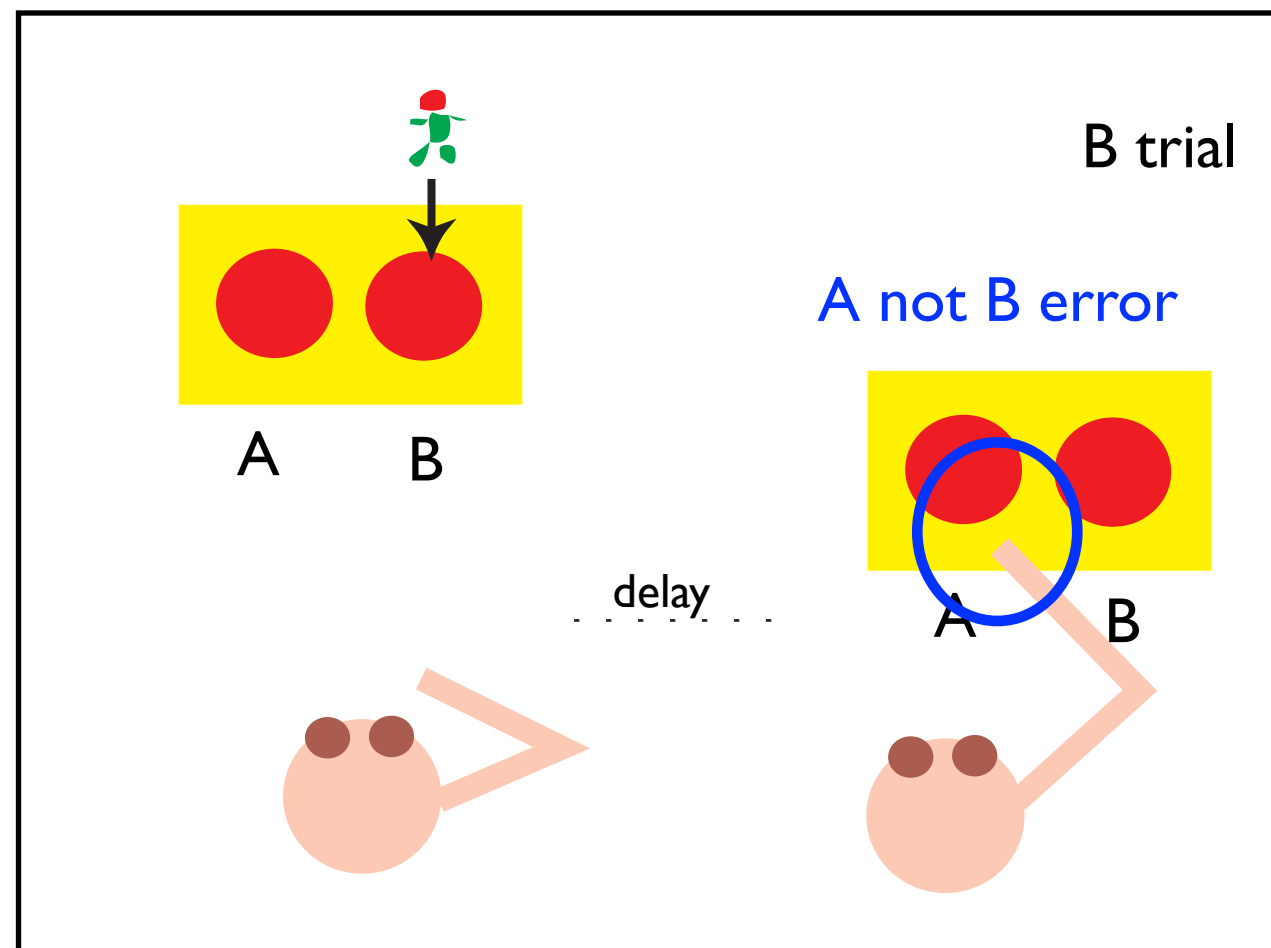
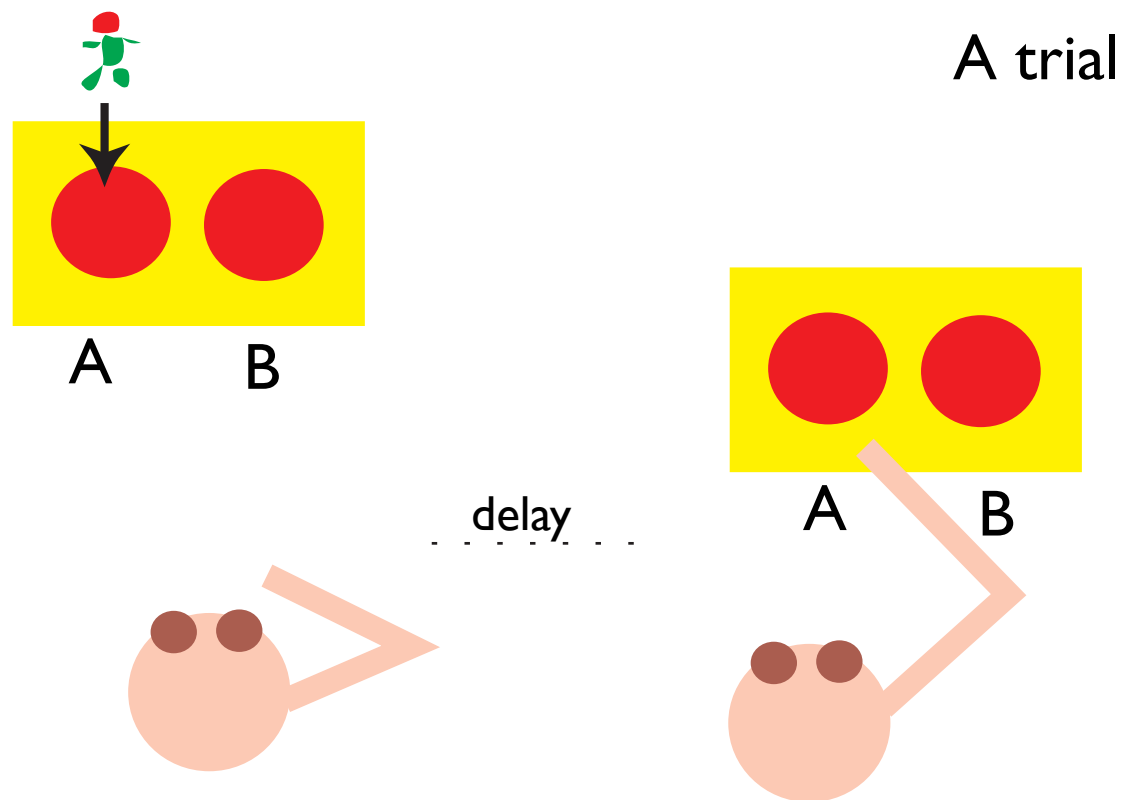
preshapes  
activation field



# categories may emerge ...



# Piaget's A not B paradigm: "out-of-sight -- out of mind"



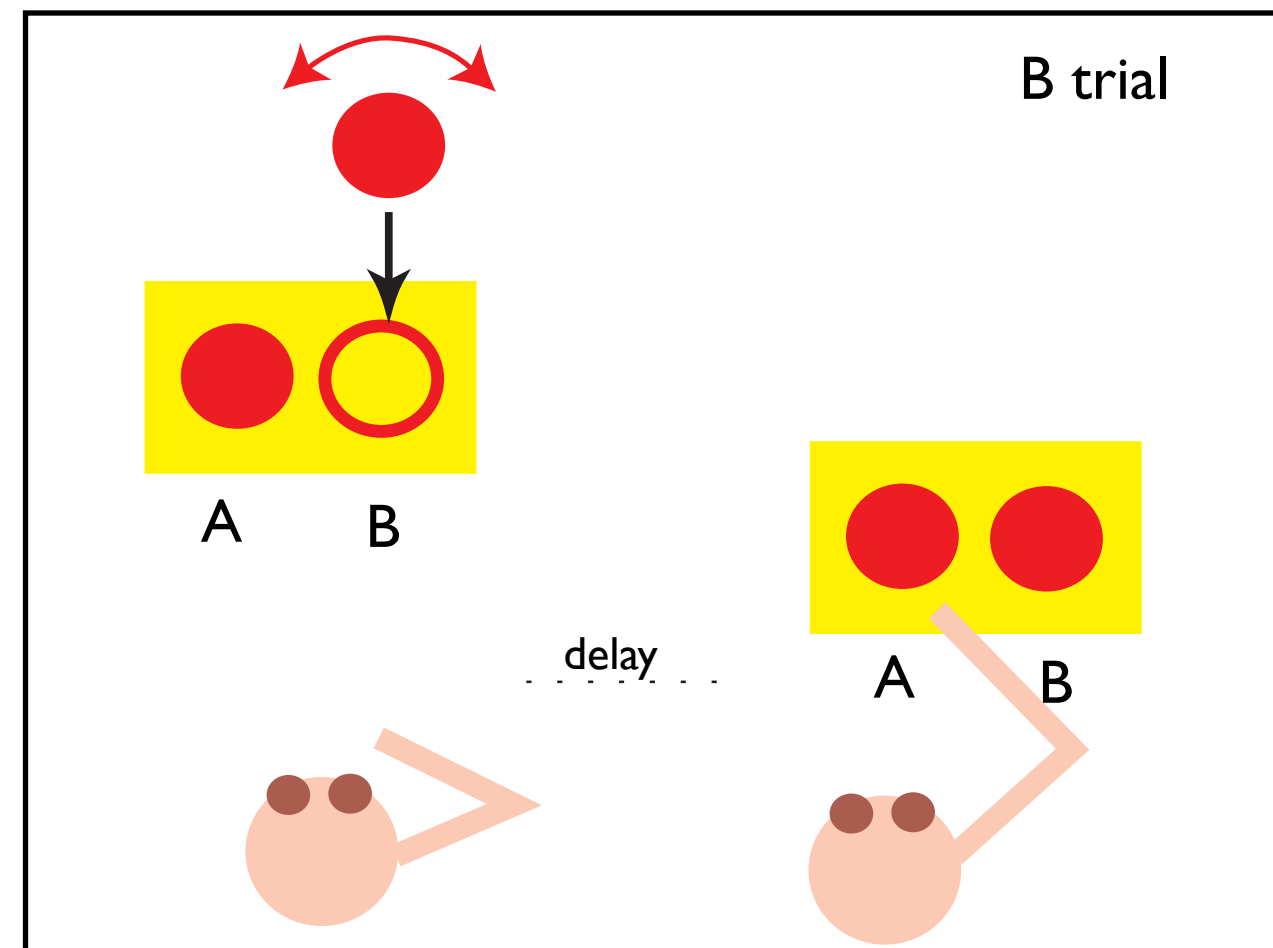
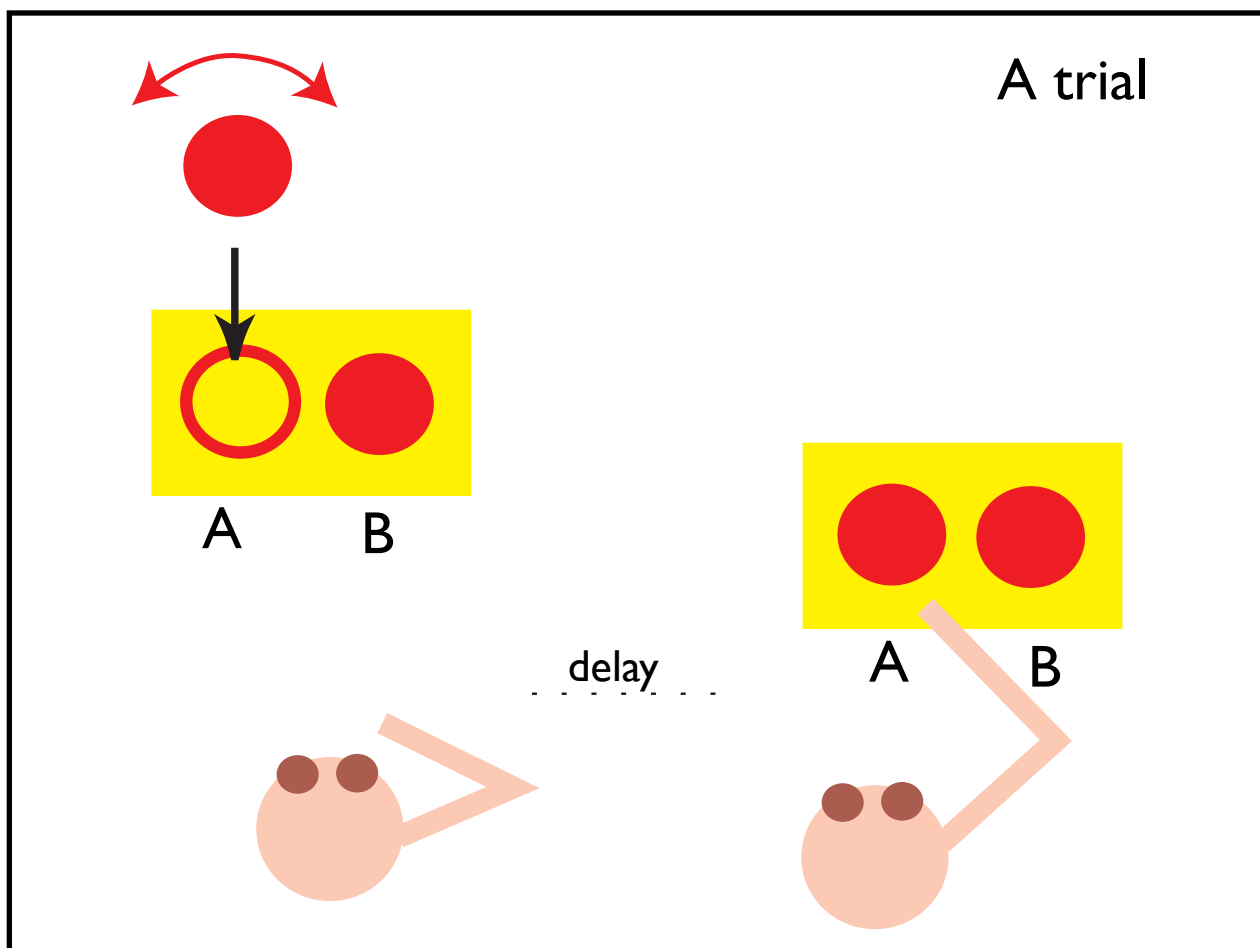


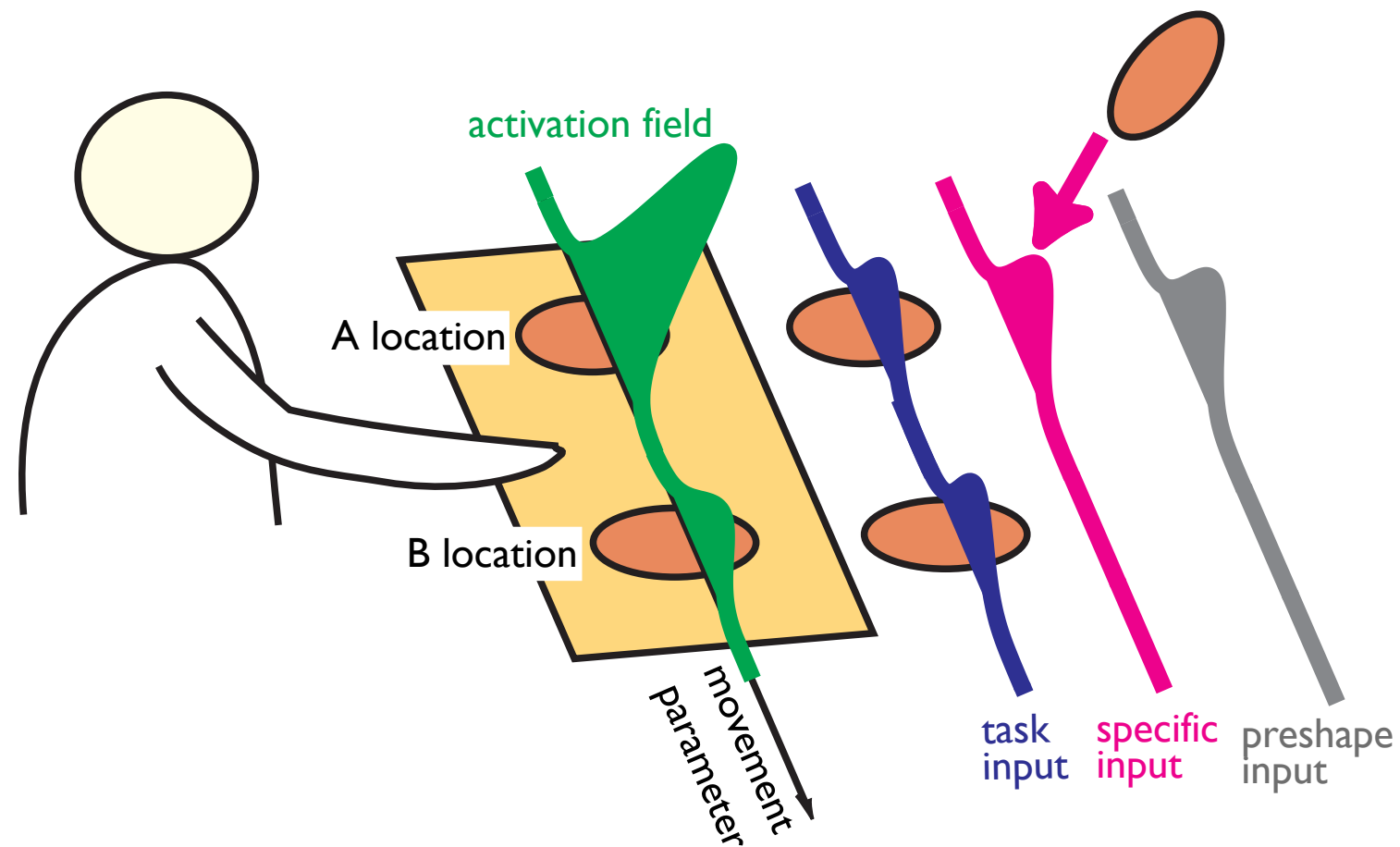
# Toyleless variant of A not B task



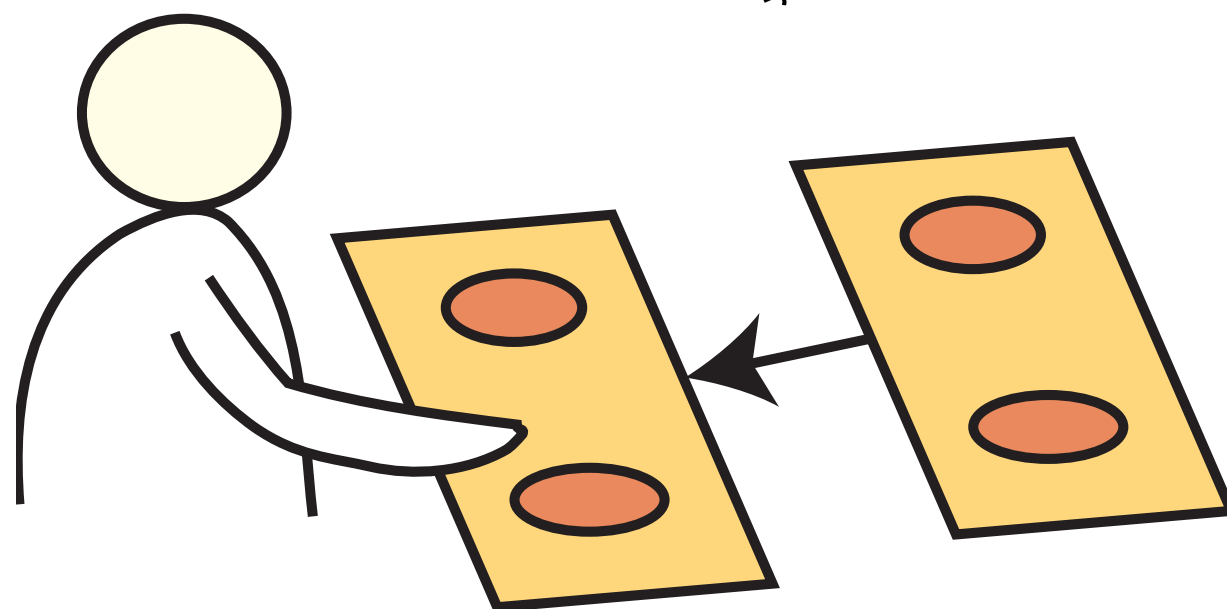
[Smith, Thelen et al.: Psychological Review (1999)]

# Toyleless variant of A not B task reveals that A not B is essentially a decision task!





[Thelen, et al., BBS (2001)]

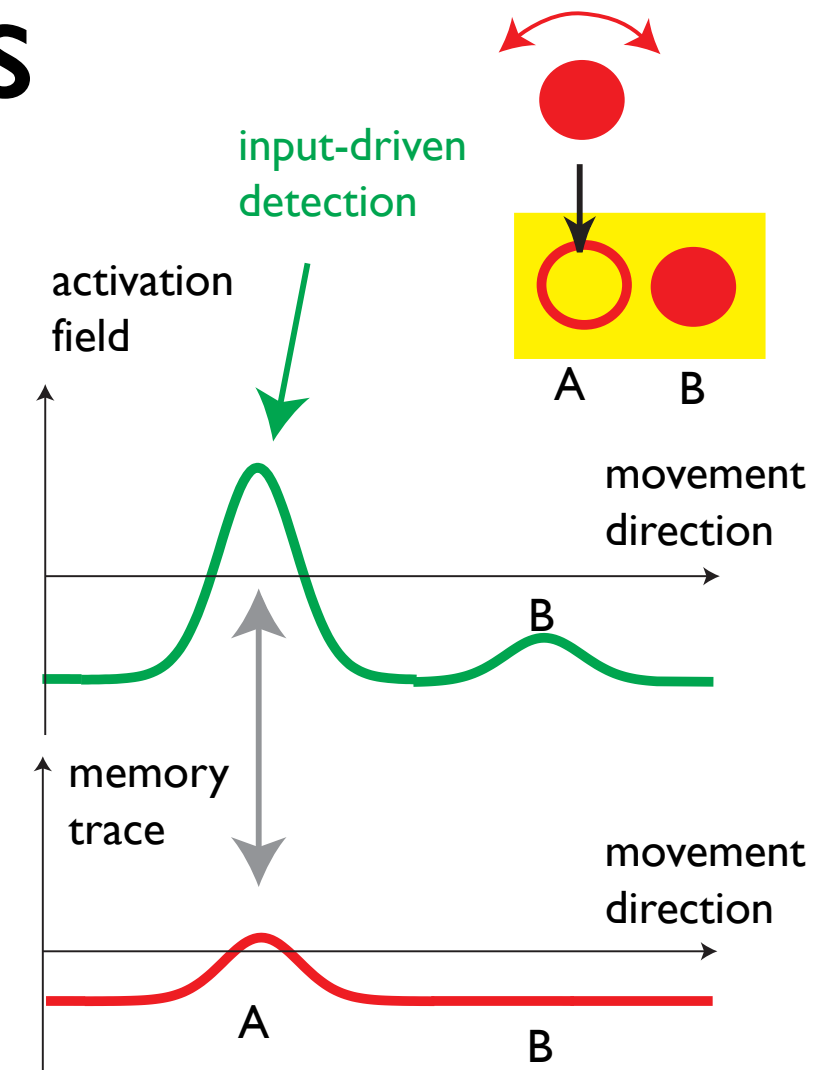


[Dinveva, Schöner, Dev. Science 2007]



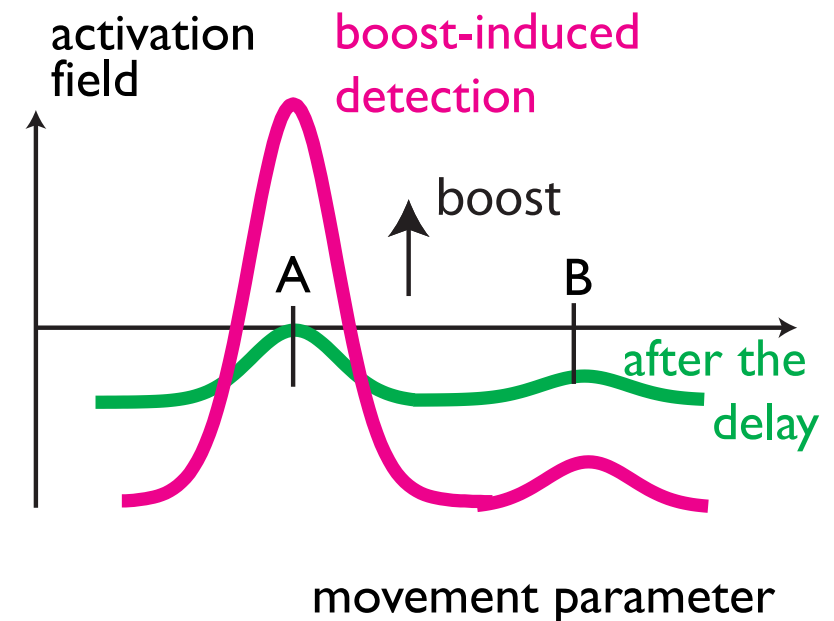
# Instabilities

- detection: forming and initiating a movement goal
- selection: making sensori-motor decisions
- (learning: memory trace)
- boost-driven detection: initiating the action
- memory instability: old infants sustain during the delay, young infants do not



# Instabilities

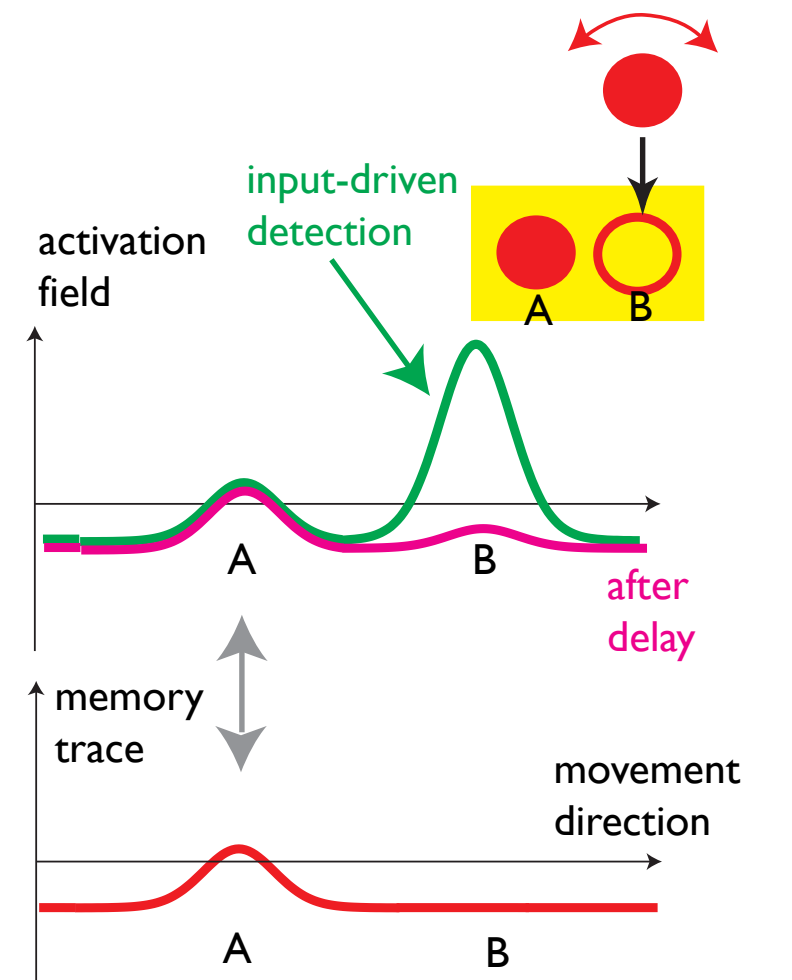
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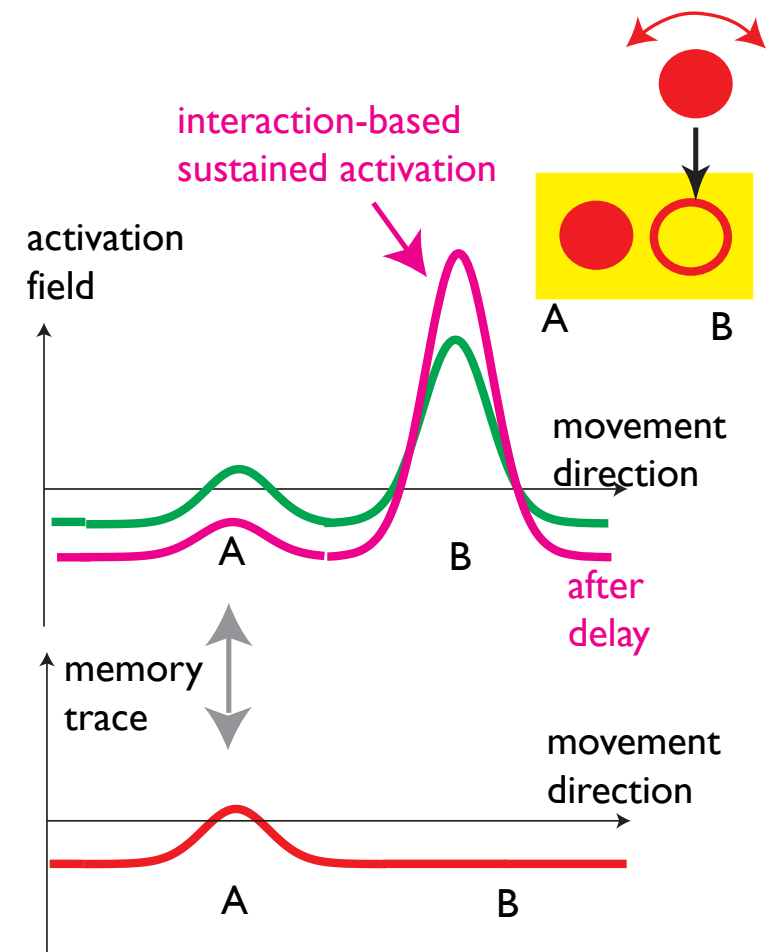
# Instabilities

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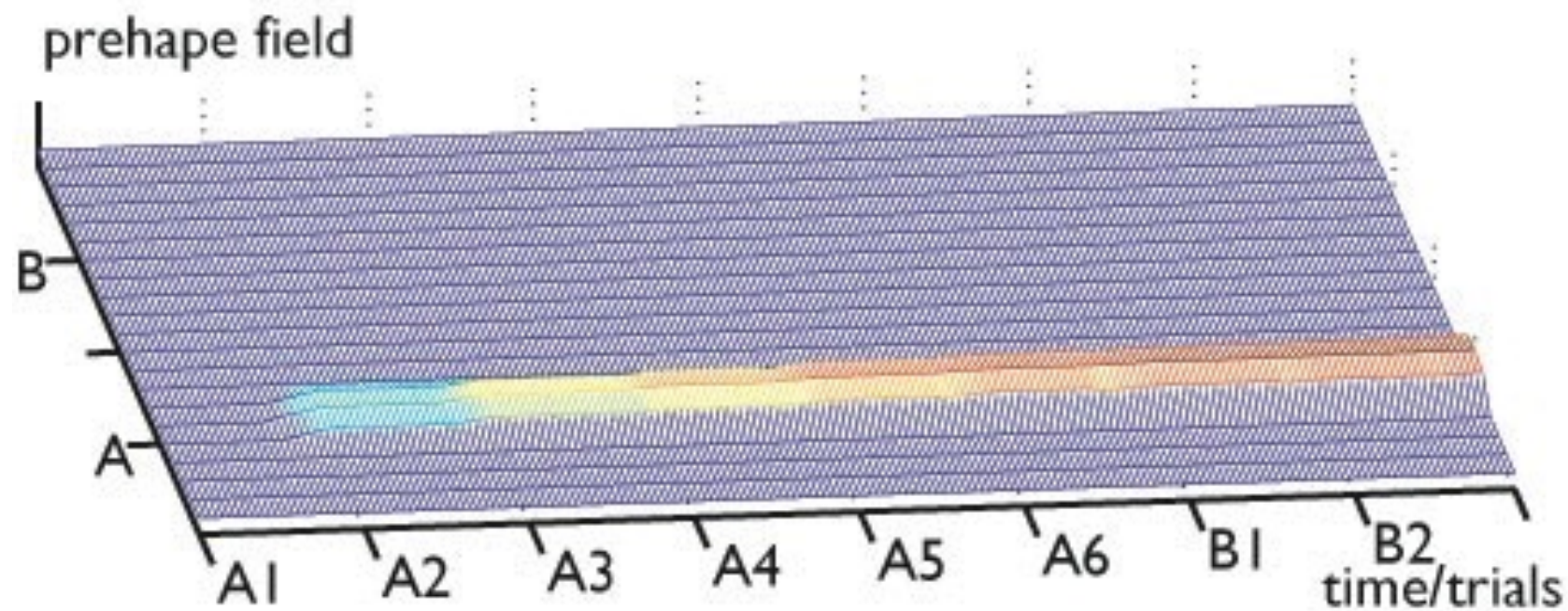
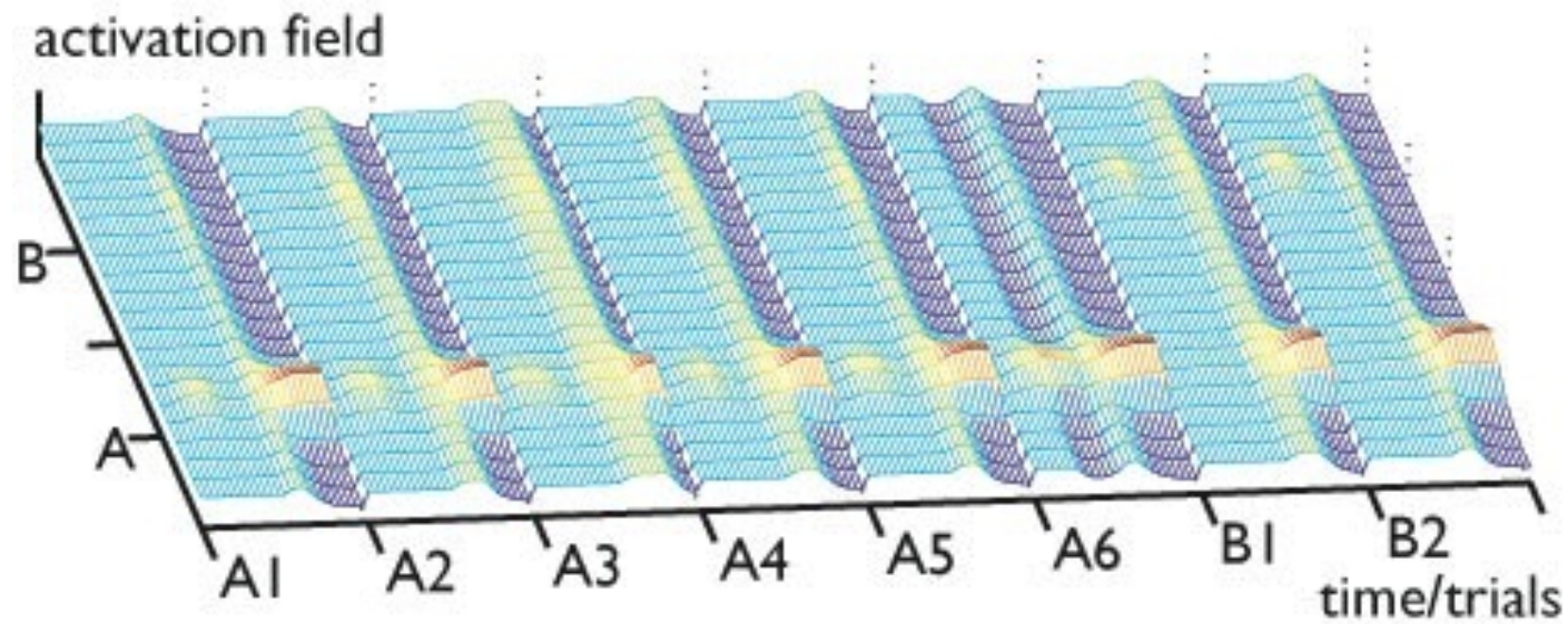
young



old

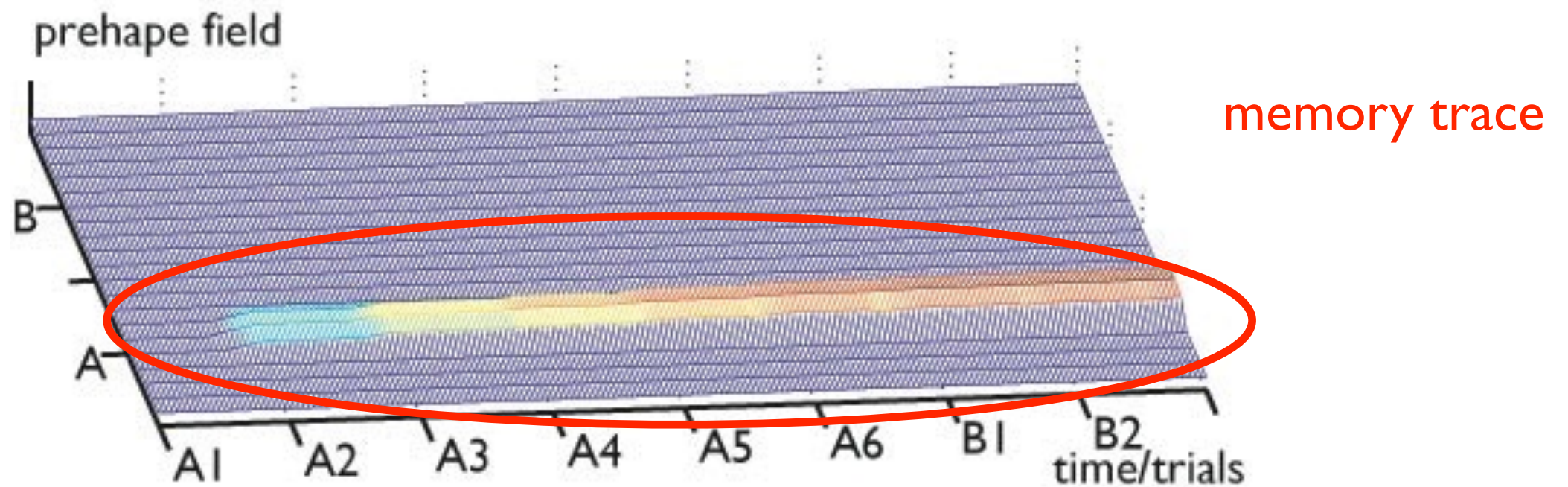
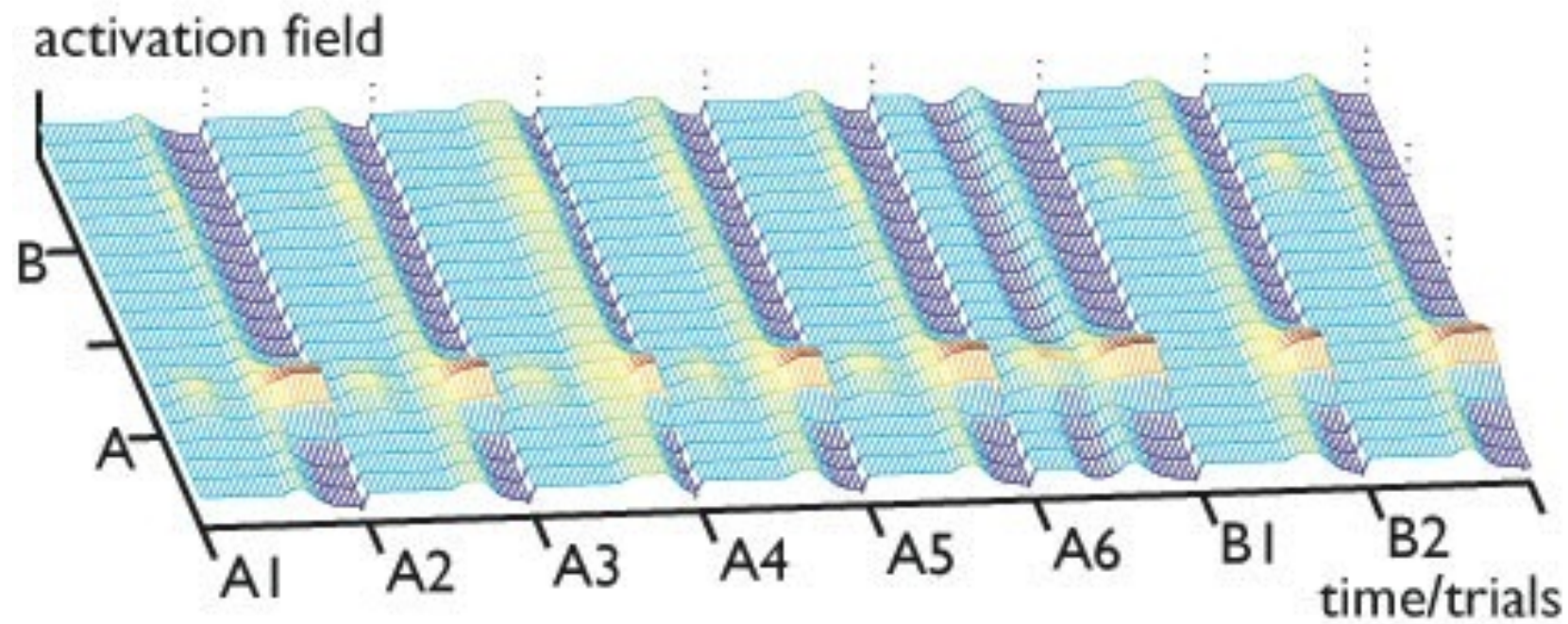


# DFT of infant perseverative reaching



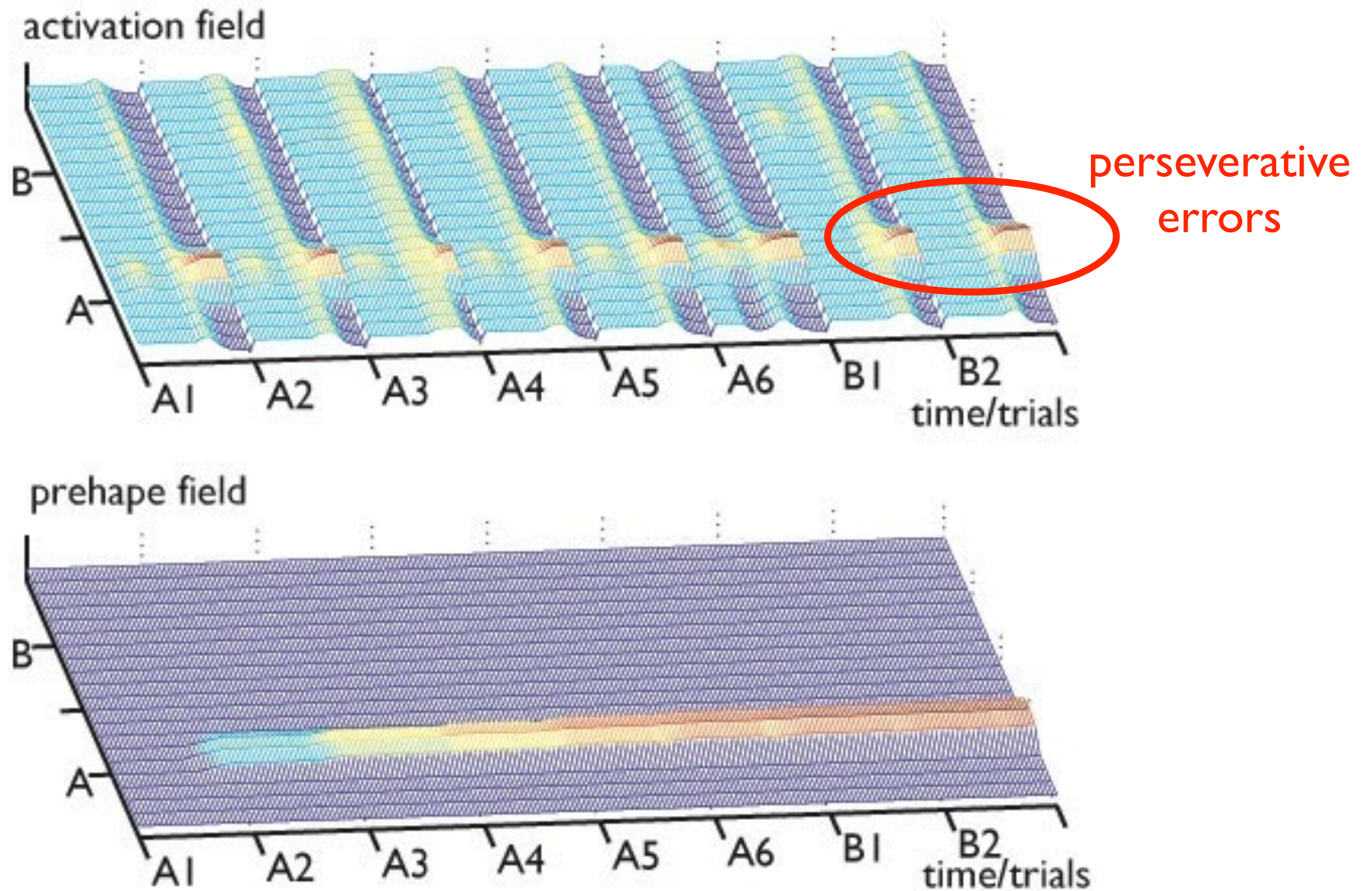


# DFT of infant perseverative reaching





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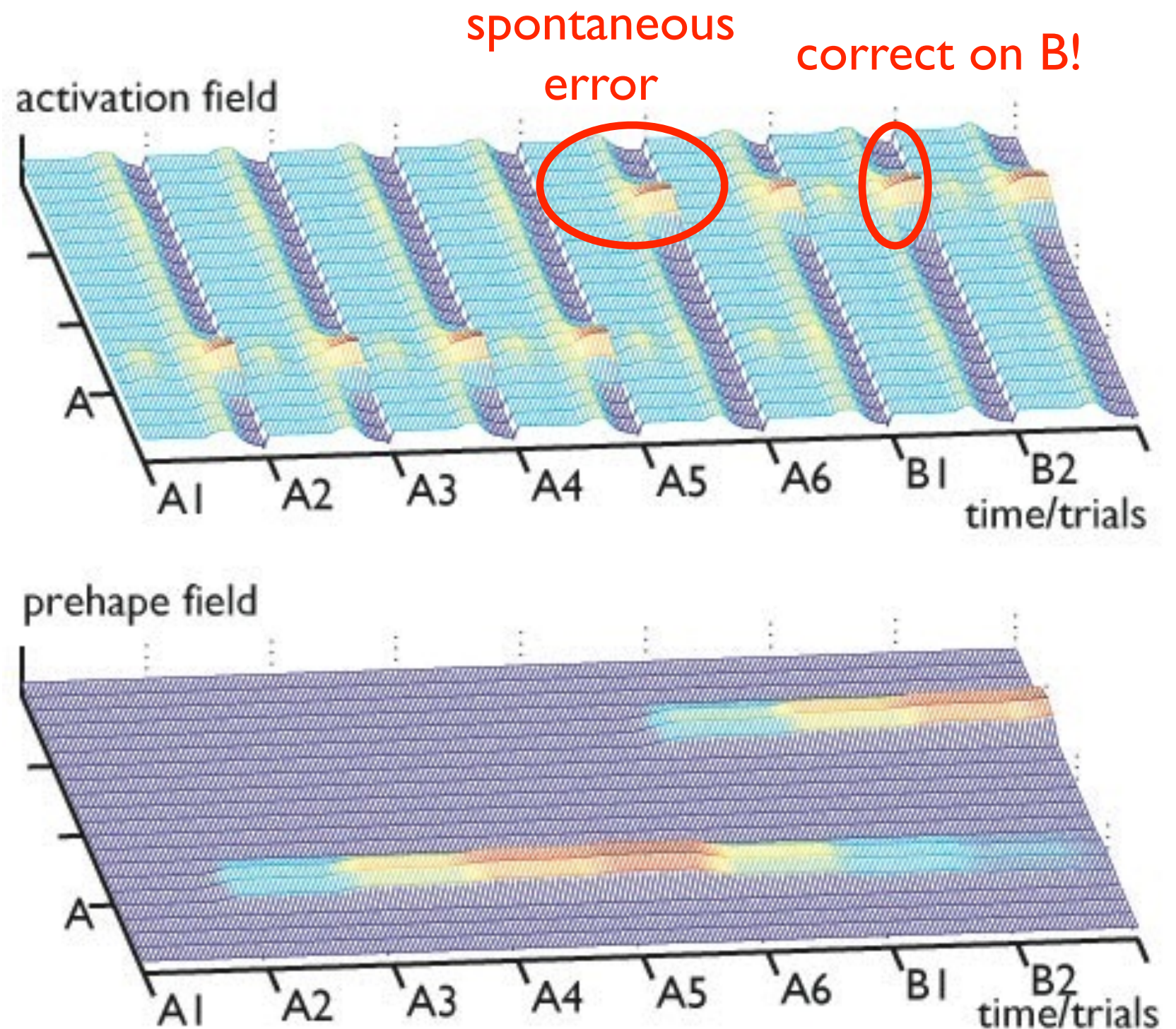




# DFT of infant perseverative reaching

- in spontaneous errors, activation arises at B on an A trial

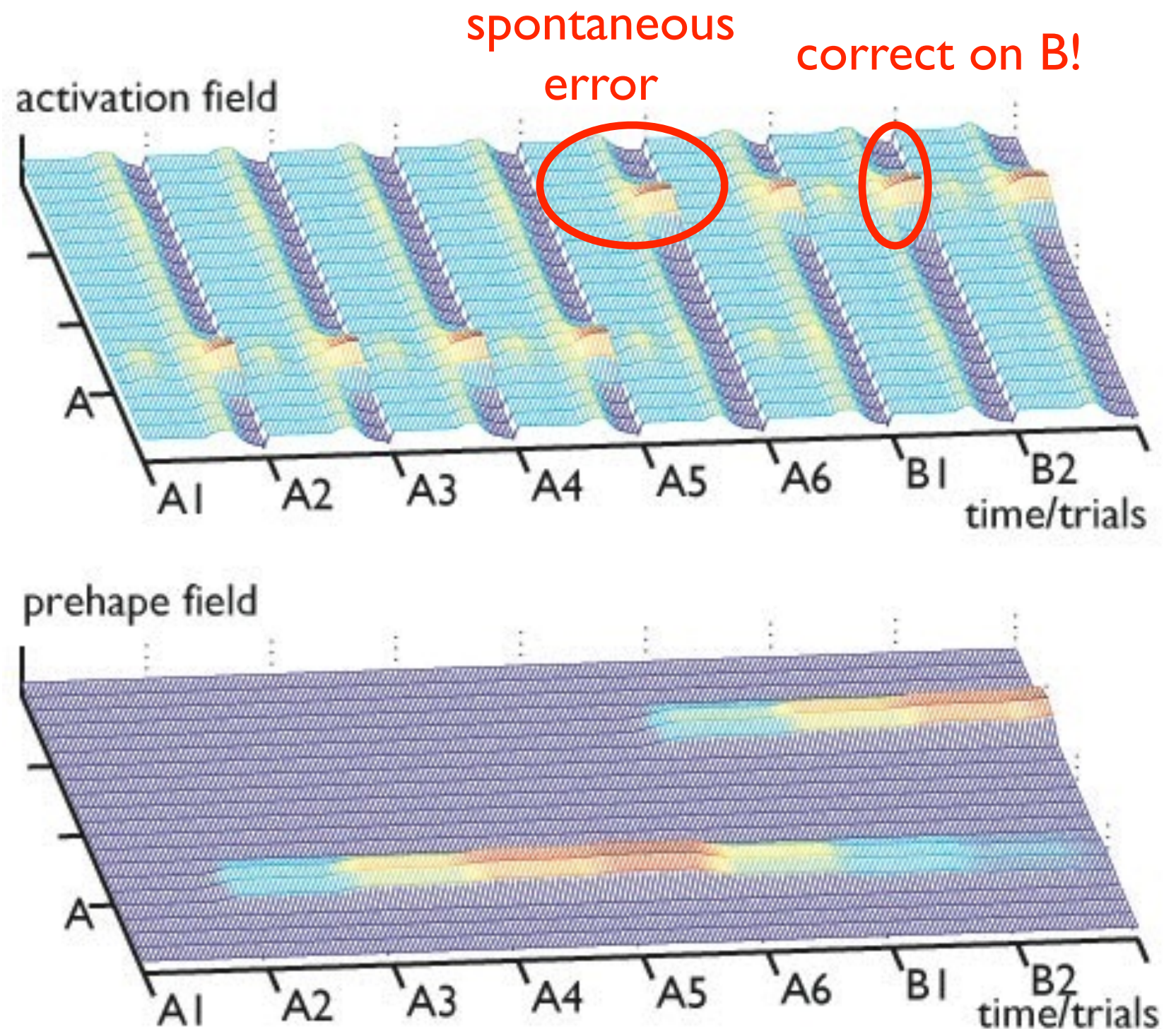
- which leads to correct reaching on B trial





# DFT of infant perseverative reaching

- that is because reaches to B on A trials leave memory trace at B

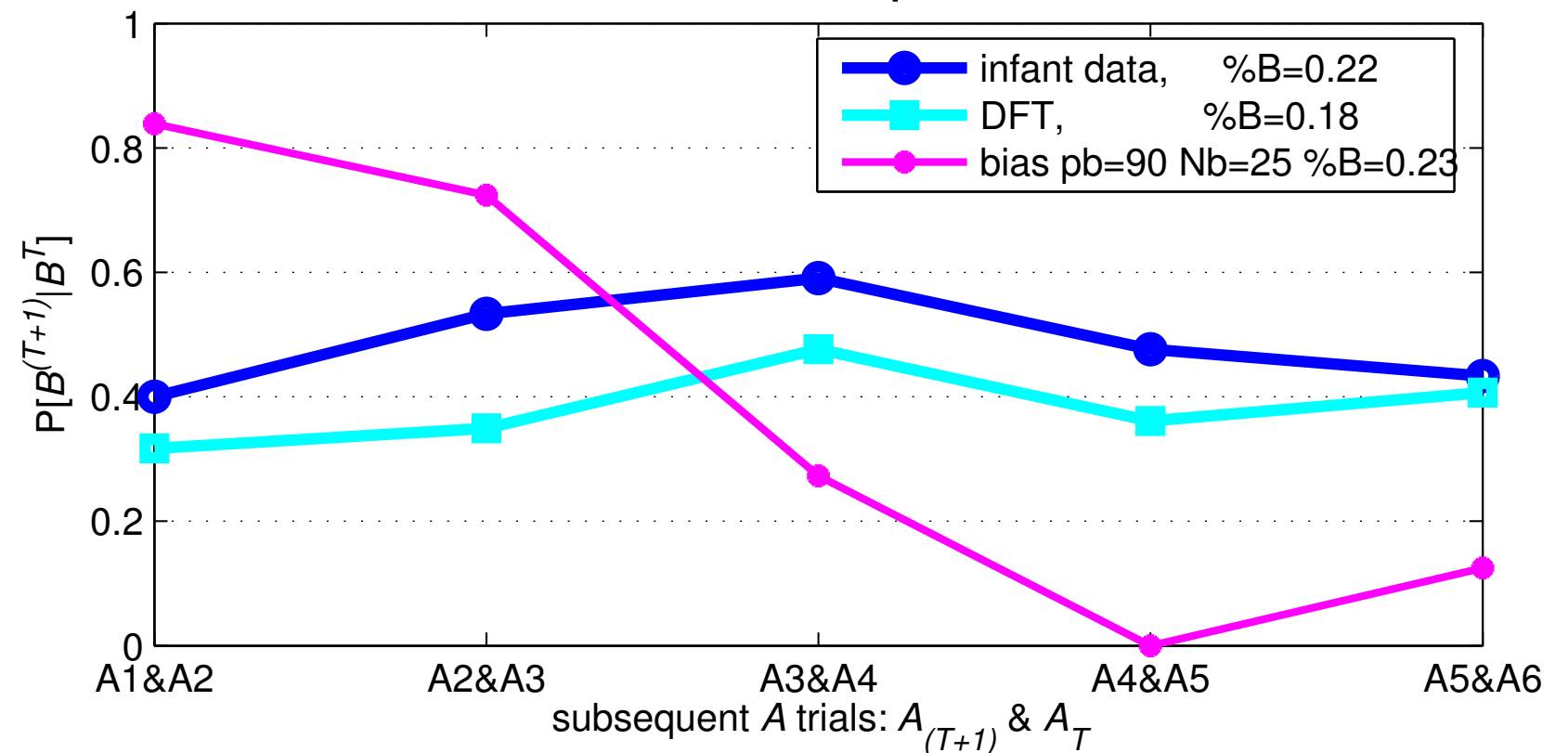




# DFT of infant perseverative reaching

■ spontaneous errors promote spontaneous errors

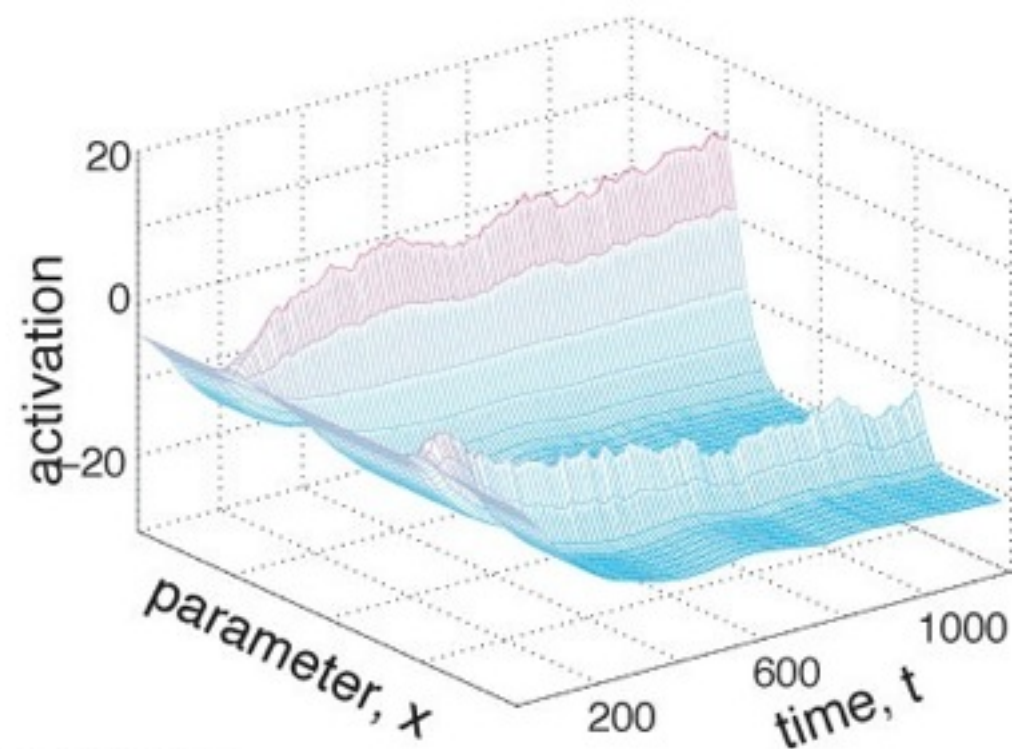
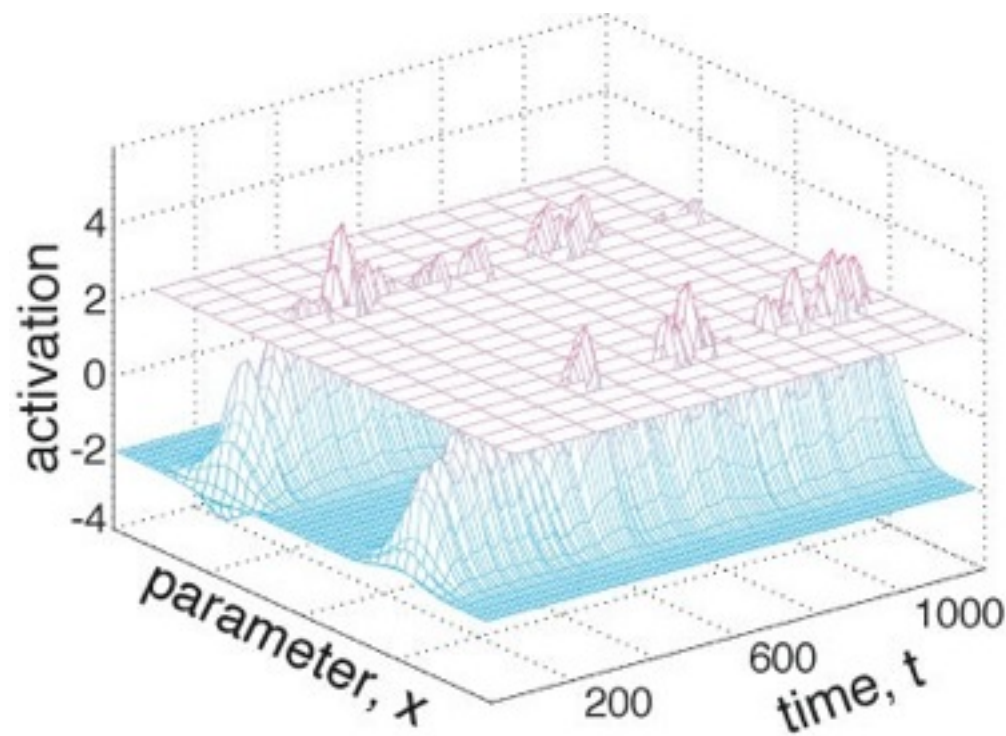
**first** and **second** reaches to  $B$   
are on two subsequent  $A$  trials



[Dinveva, Schöner, Dev. Science 2007]

# DFT is a neural process model

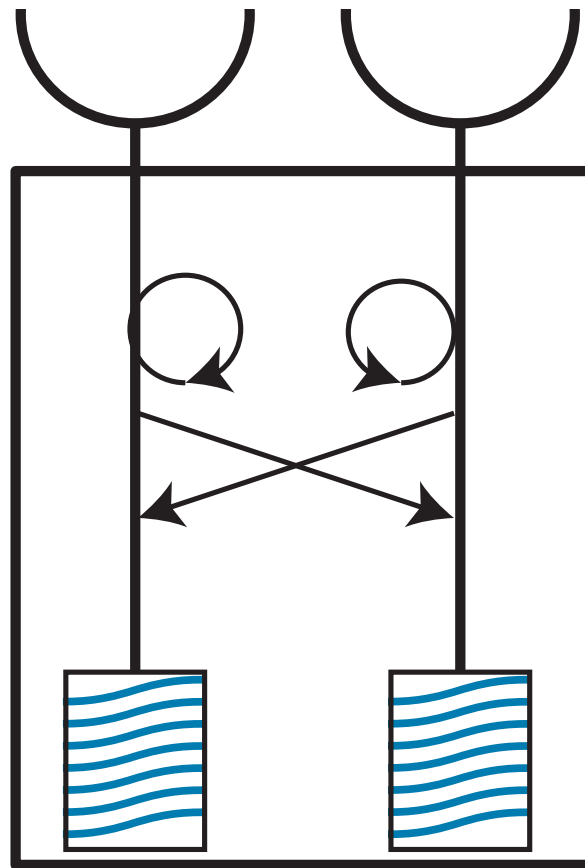
- that makes the decisions in each individual trial, by amplifying small differences into a macroscopic stable state
- and that's how decisions leave traces, have consequences



[Wilimzig, Schöner, 2006]

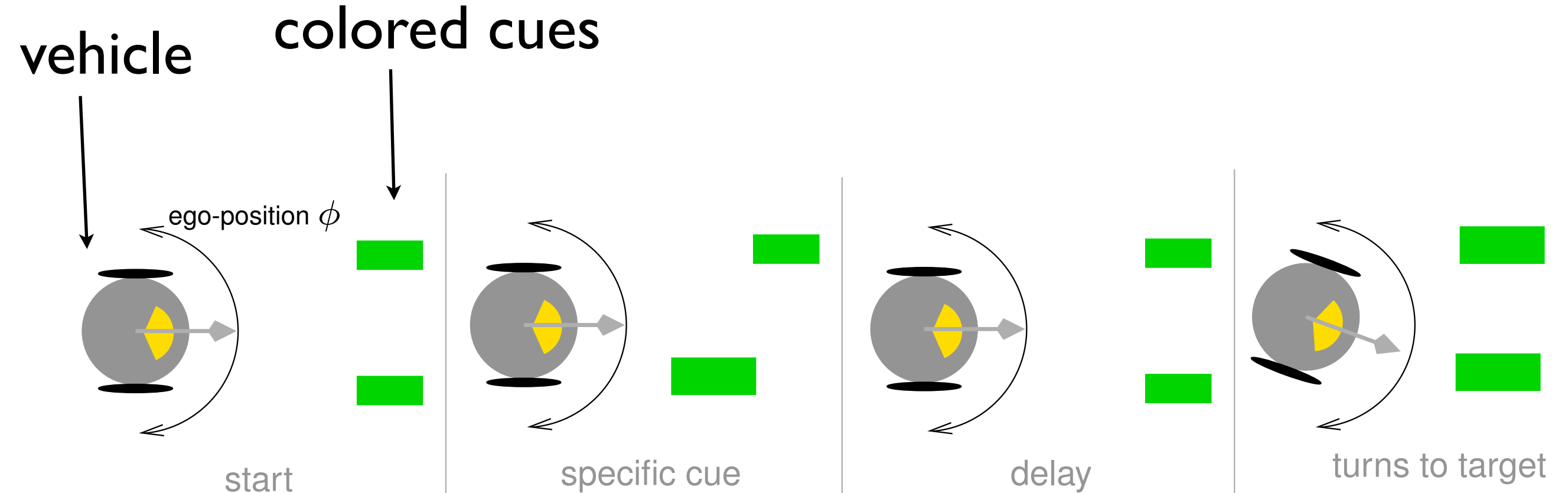
# Combining neural and behavioral dynamics

source<sub>1</sub> ☆ ☆ source<sub>2</sub>



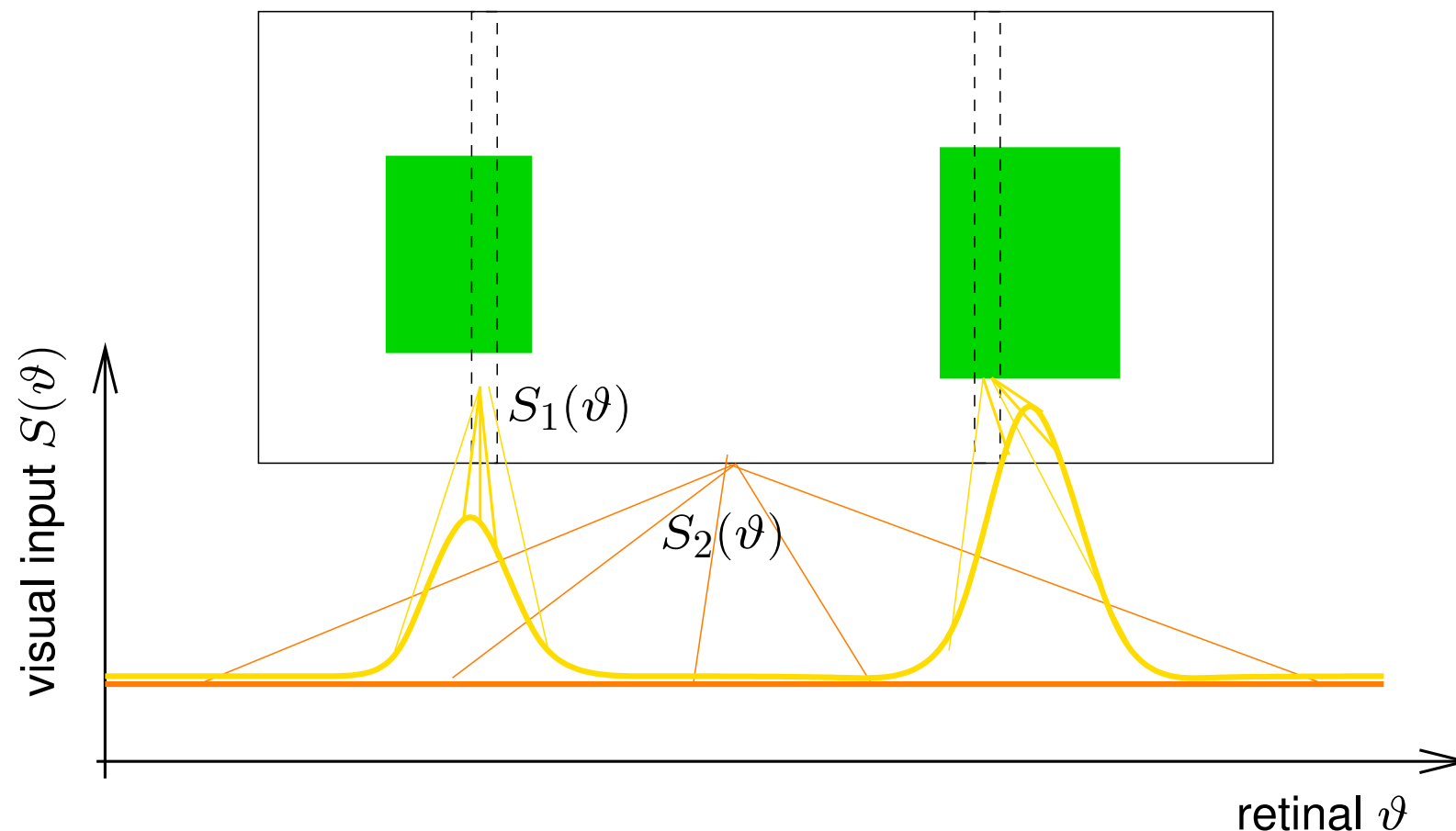
# Embodied A not B

- implementing the A not B model on a autonomous robot with continuous link to sensory and motor surfaces...



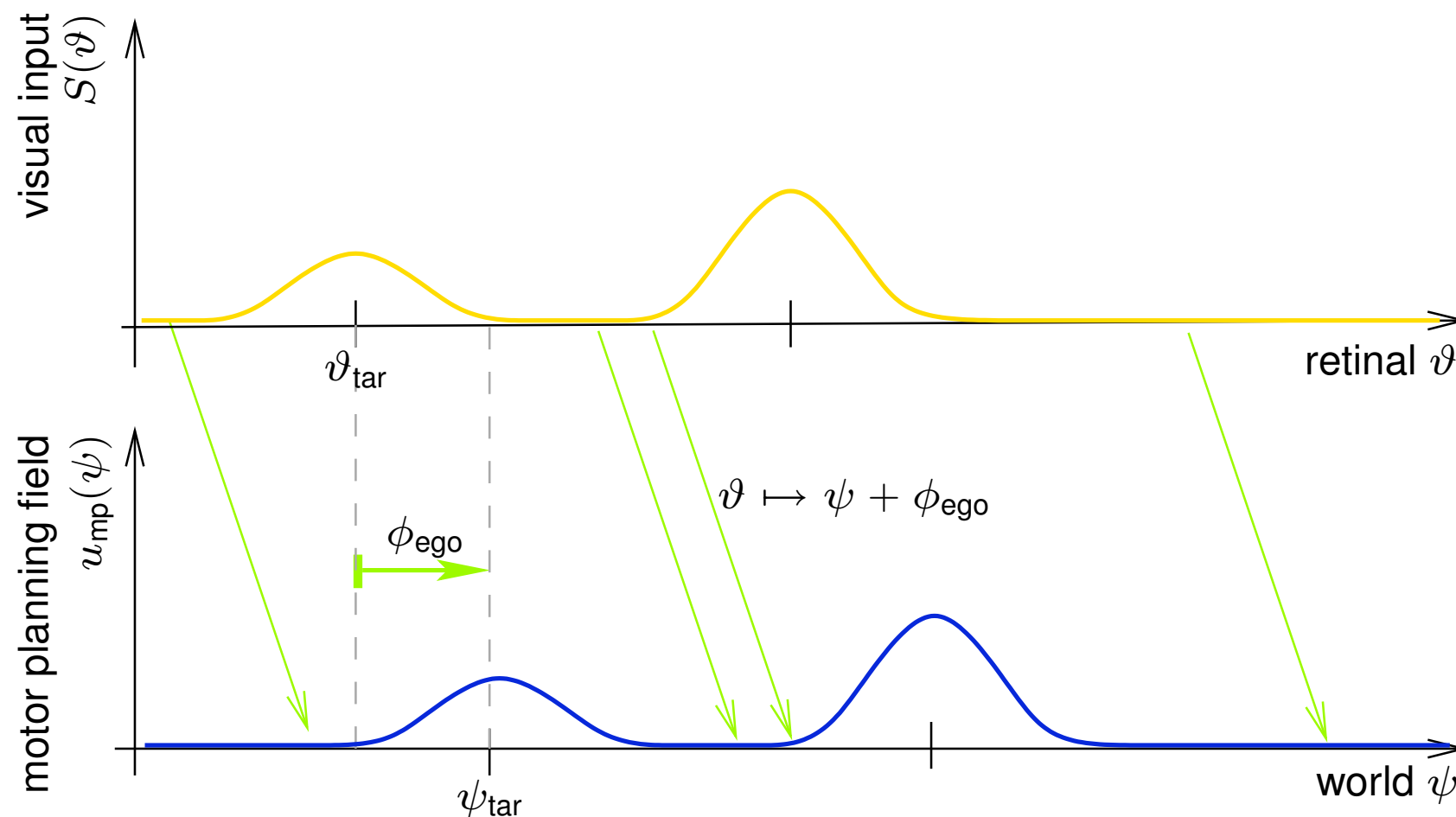
# Visual input

- color-based segmentation
- summing color pixels within color slot along the vertical
- spatially filter at two resolutions



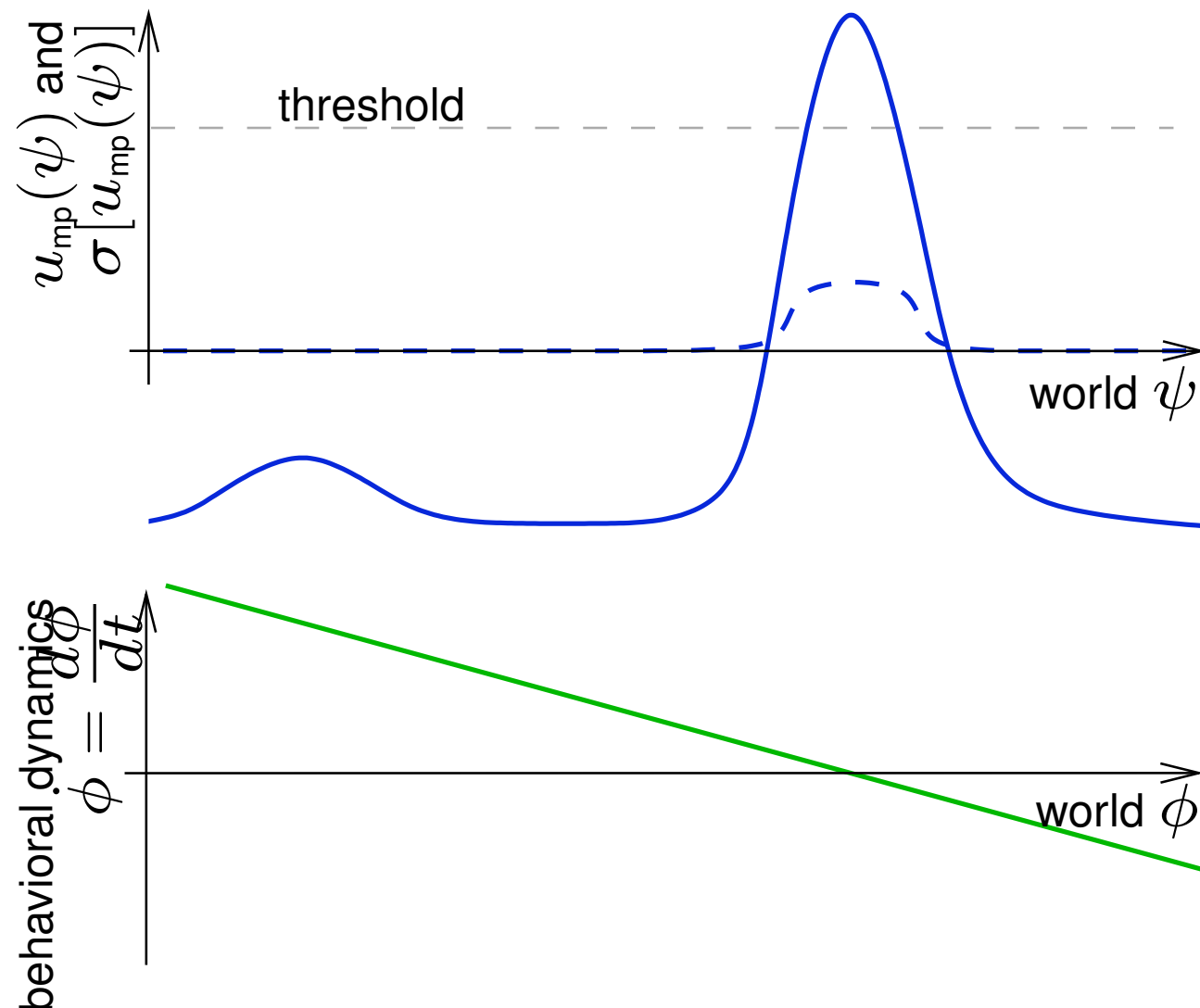
# Dynamic field

- defined over direction in the world
- (requires coordinate transform from retina based on dead-reckoning)



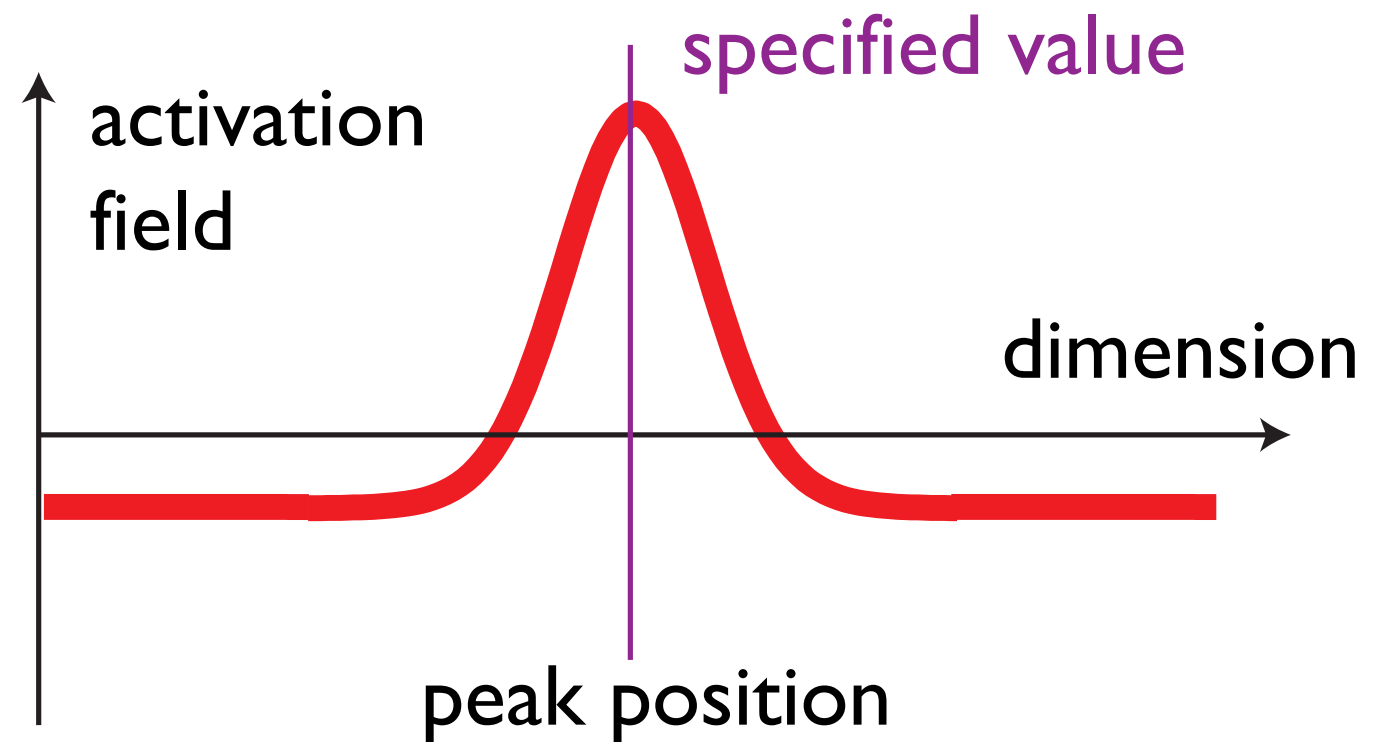
# Motor dynamics

- couple peak in direction field into dynamics of heading direction as an attractor



# “Read-out” by generating attractor dynamics for motor system

- peak specifies value for a dynamical variable that is congruent to the field dimension

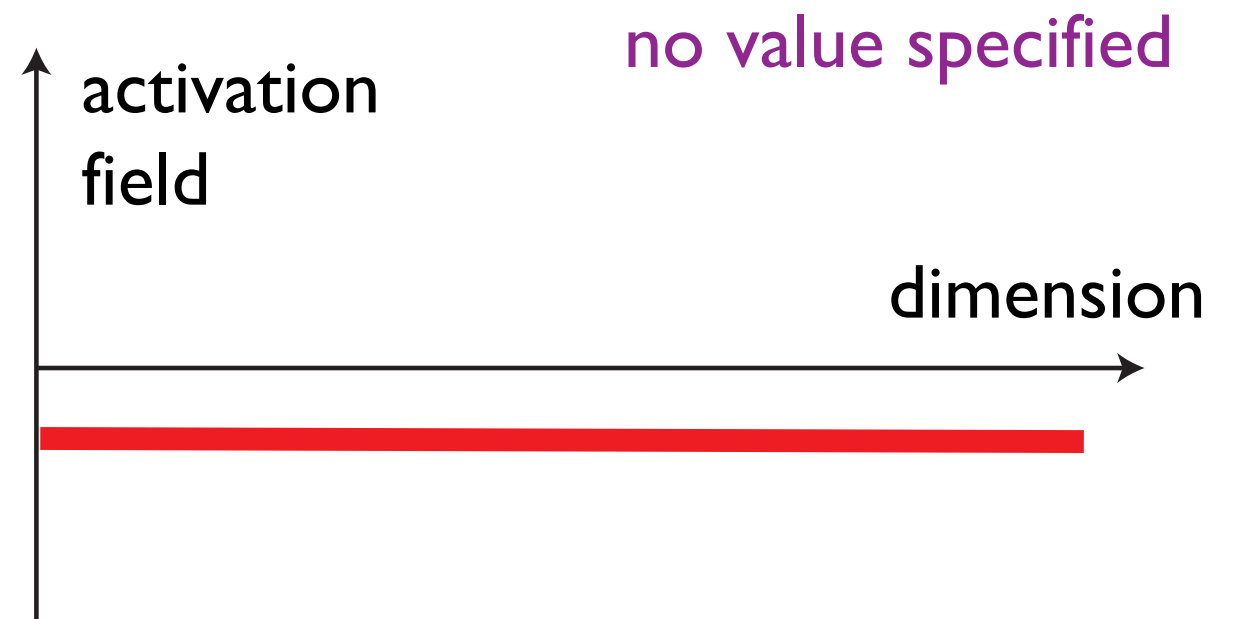
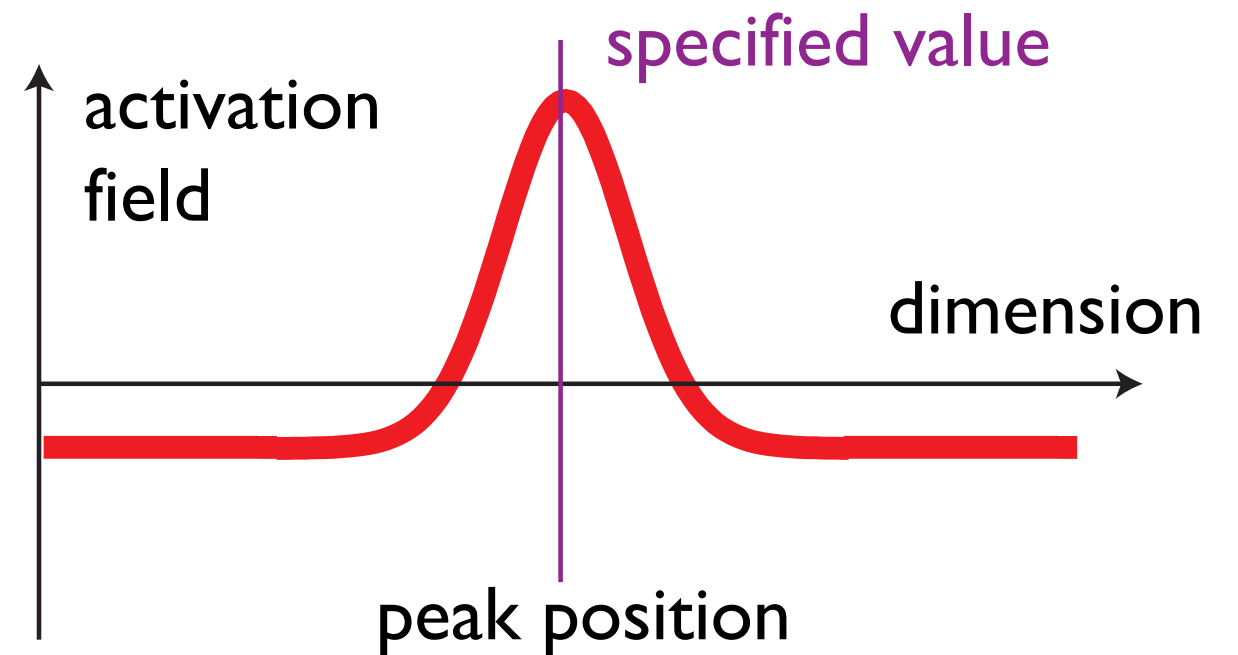




■ treating sigmoided field as probability: need to normalize

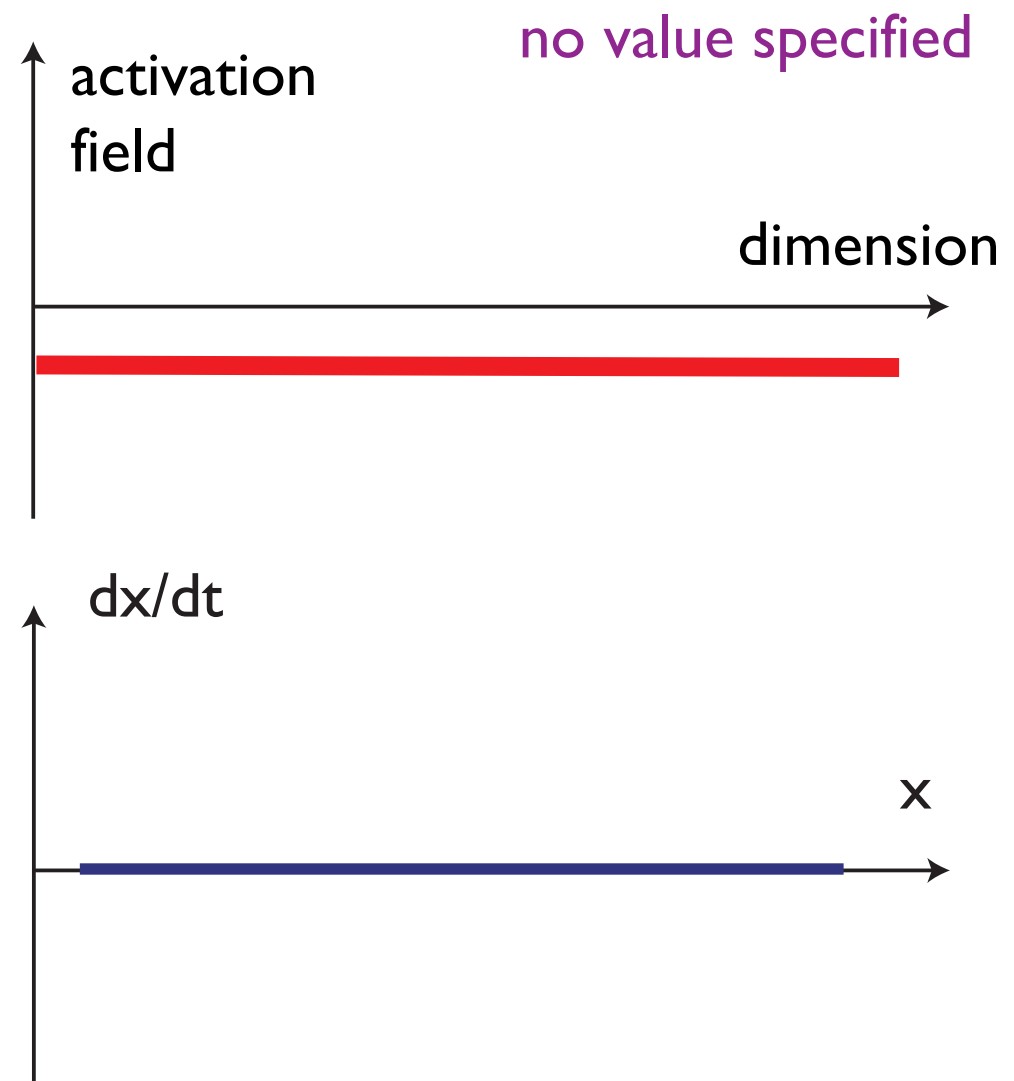
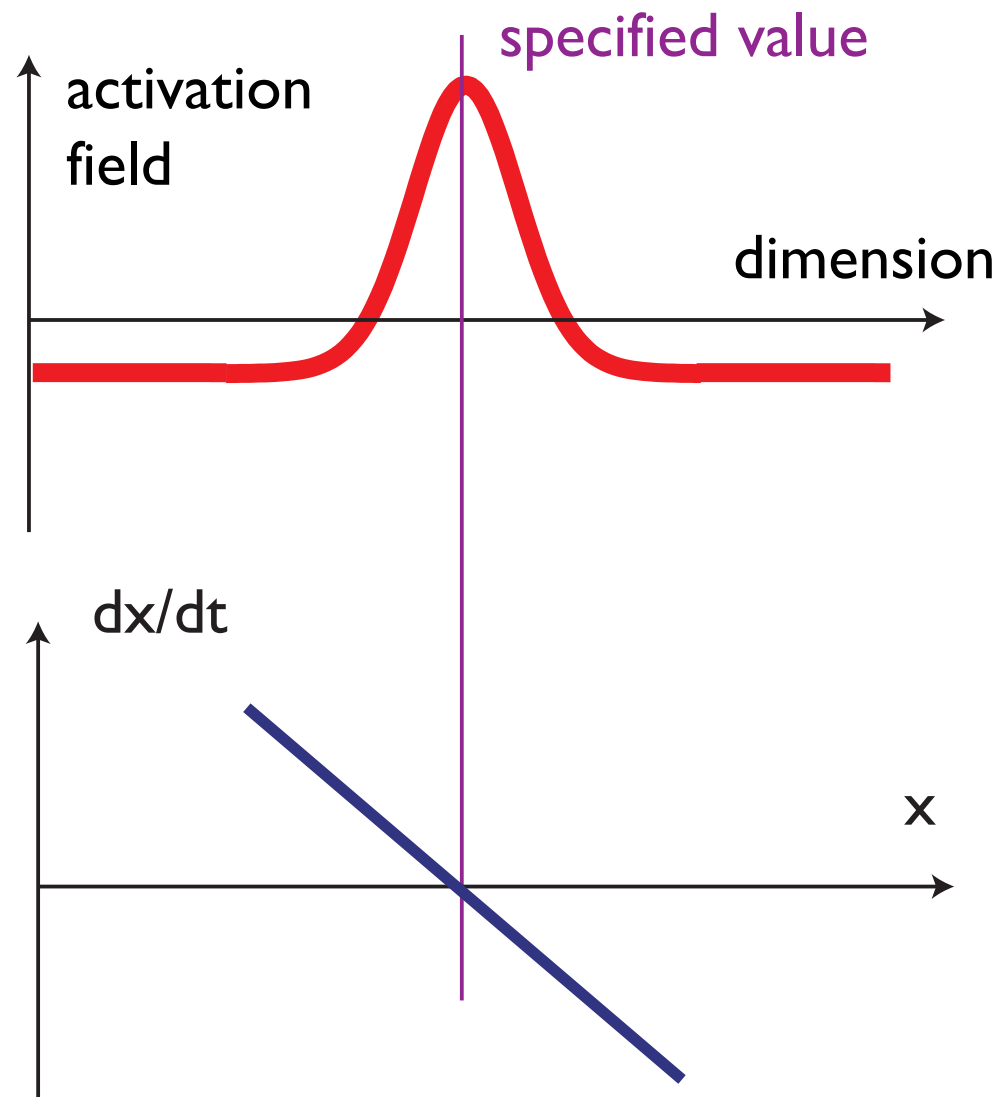
■ => problem when there is no peak: divide by zero!

$$x_{\text{peak}} = \frac{\int dx' \sigma(u(x', t)) x'}{\int dx' \sigma(u(x', t))}$$



# instead:

■ create attractor

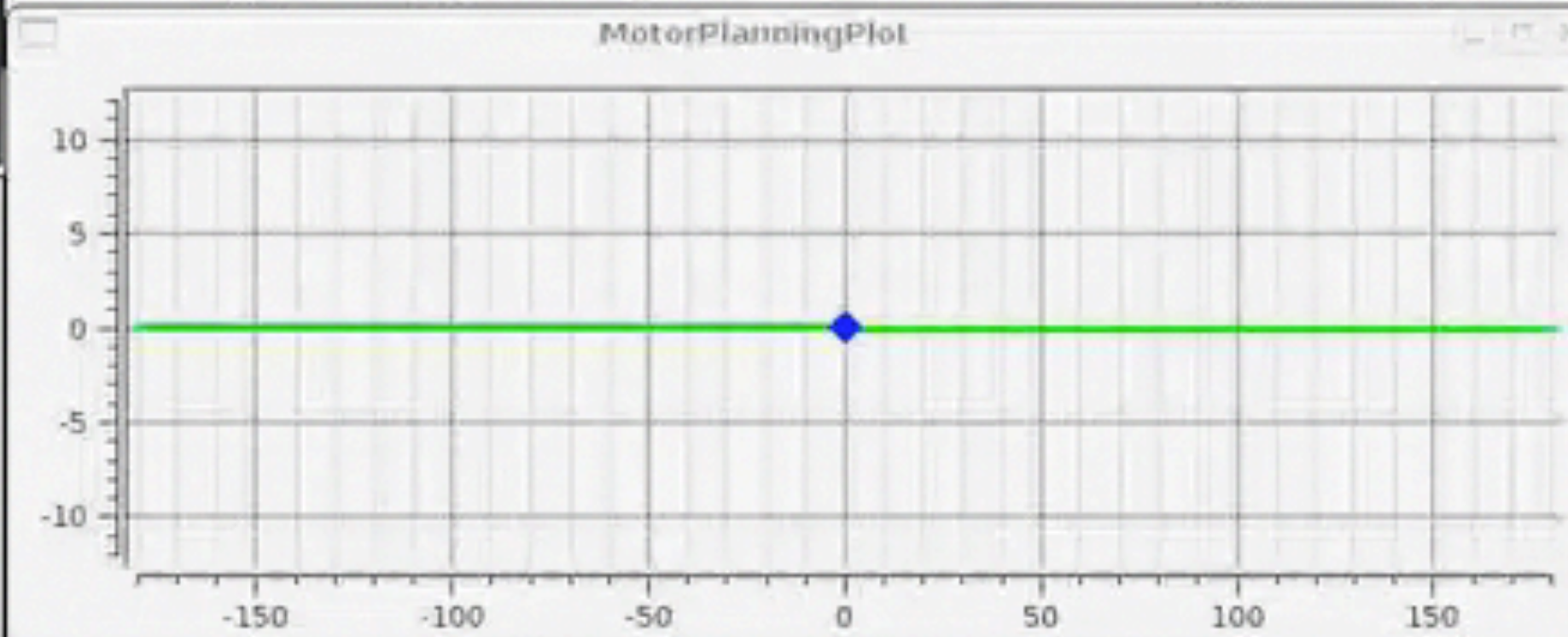
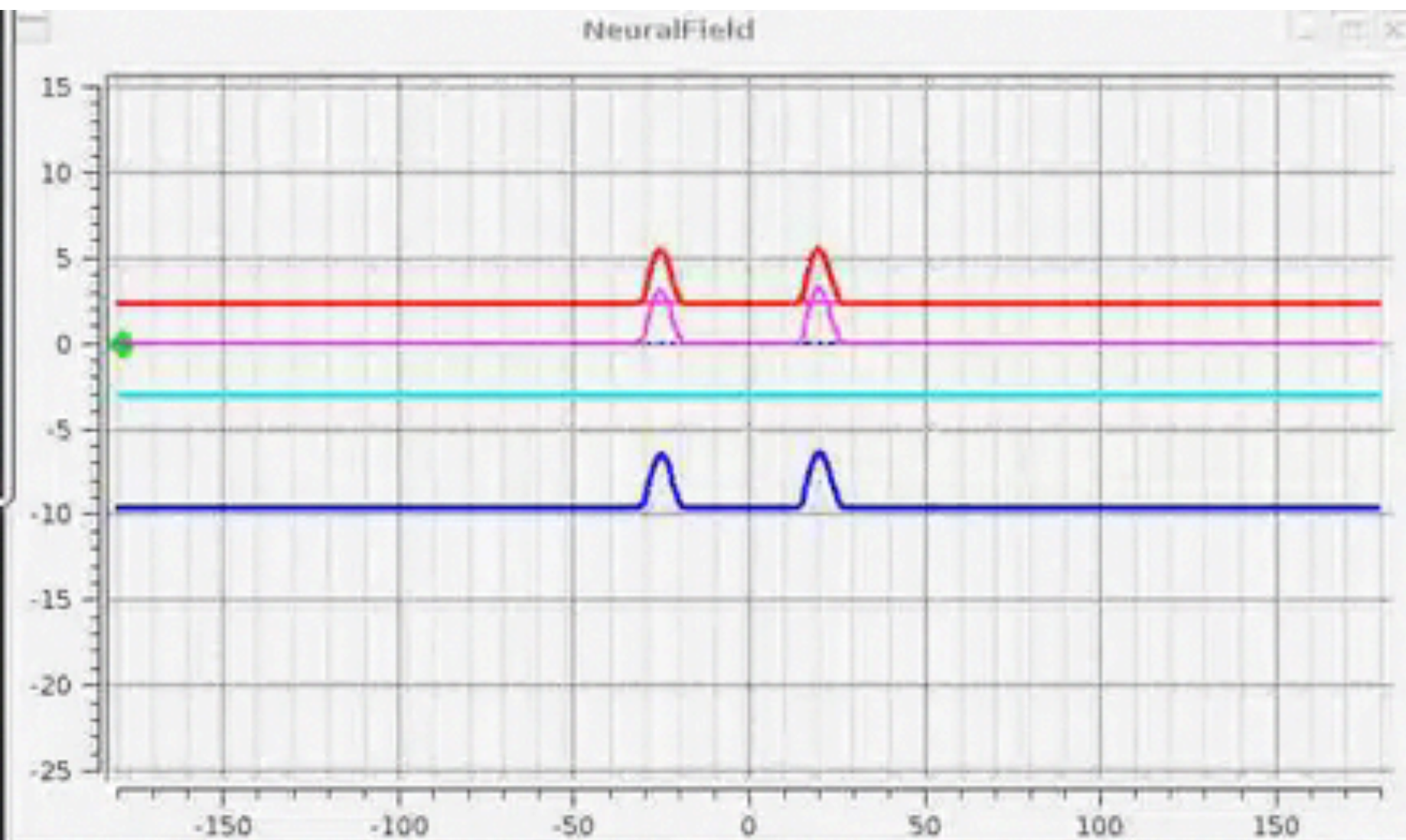
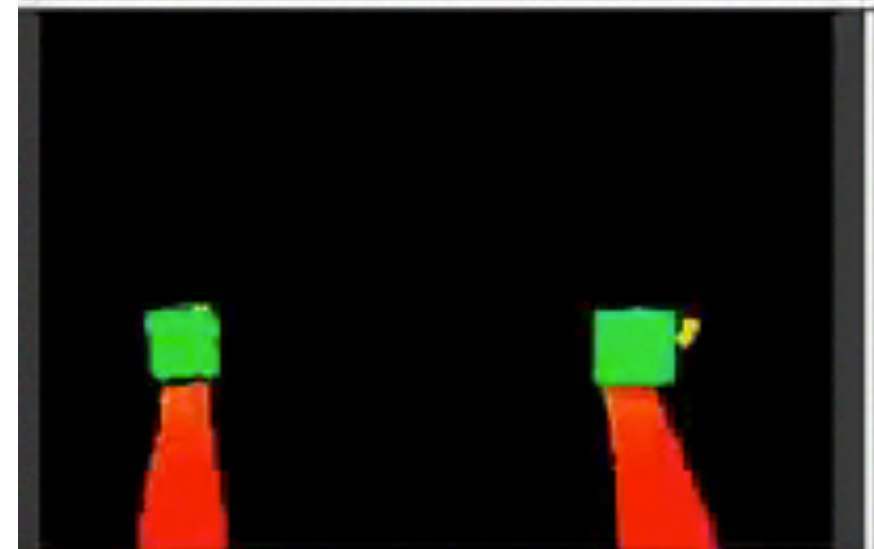
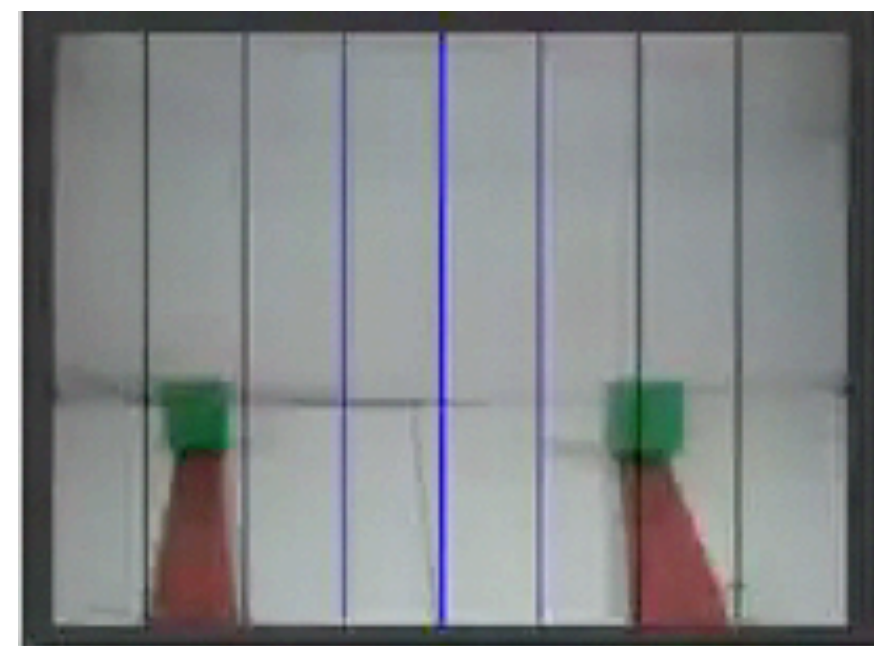


■ solution: peak sets attractor

■ location of attractor: peak location

■ strength of attractor: summed supra-threshold activation

$$\begin{aligned}x_{\text{peak}} &= \frac{\int dx' \sigma(u(x', t)) x'}{\int dx' \sigma(u(x', t))} \\ \dot{x} &= - \int dx' \sigma(u(x', t)) (x - x_{\text{peak}}) \\ &= - \left[ \int dx' \sigma(u(x', t)) x - \int dx' \sigma(u(x', t)) x_{\text{peak}} \right] \\ &= - \left[ \int dx' \sigma(u(x', t)) x - \int dx' \sigma(u(x', t)) x' \right] \\ &= - \int dx' \sigma(u(x', t)) (x - x')\end{aligned}$$



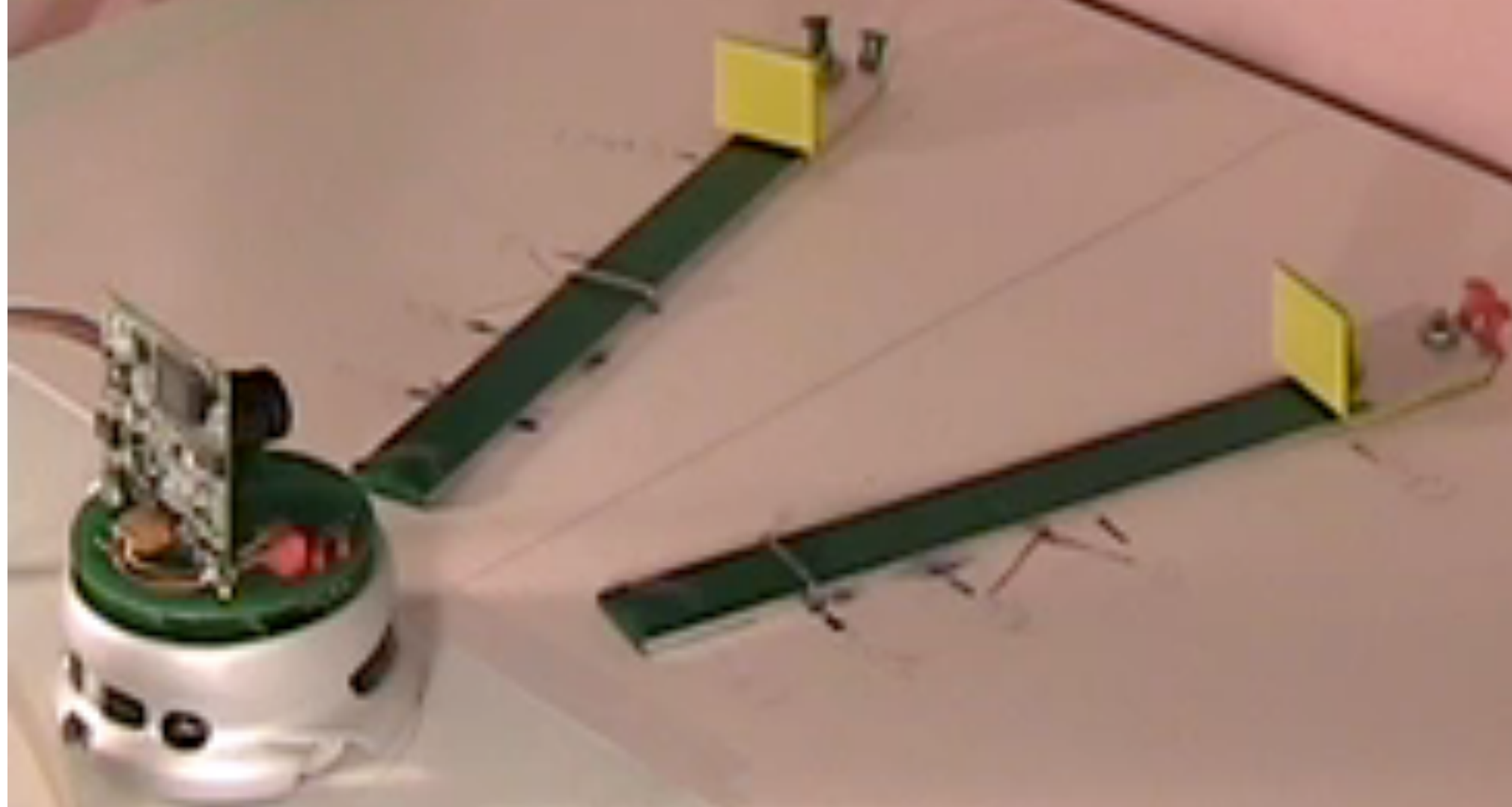


365  
h120  
-1  
p5.6

Exp #3

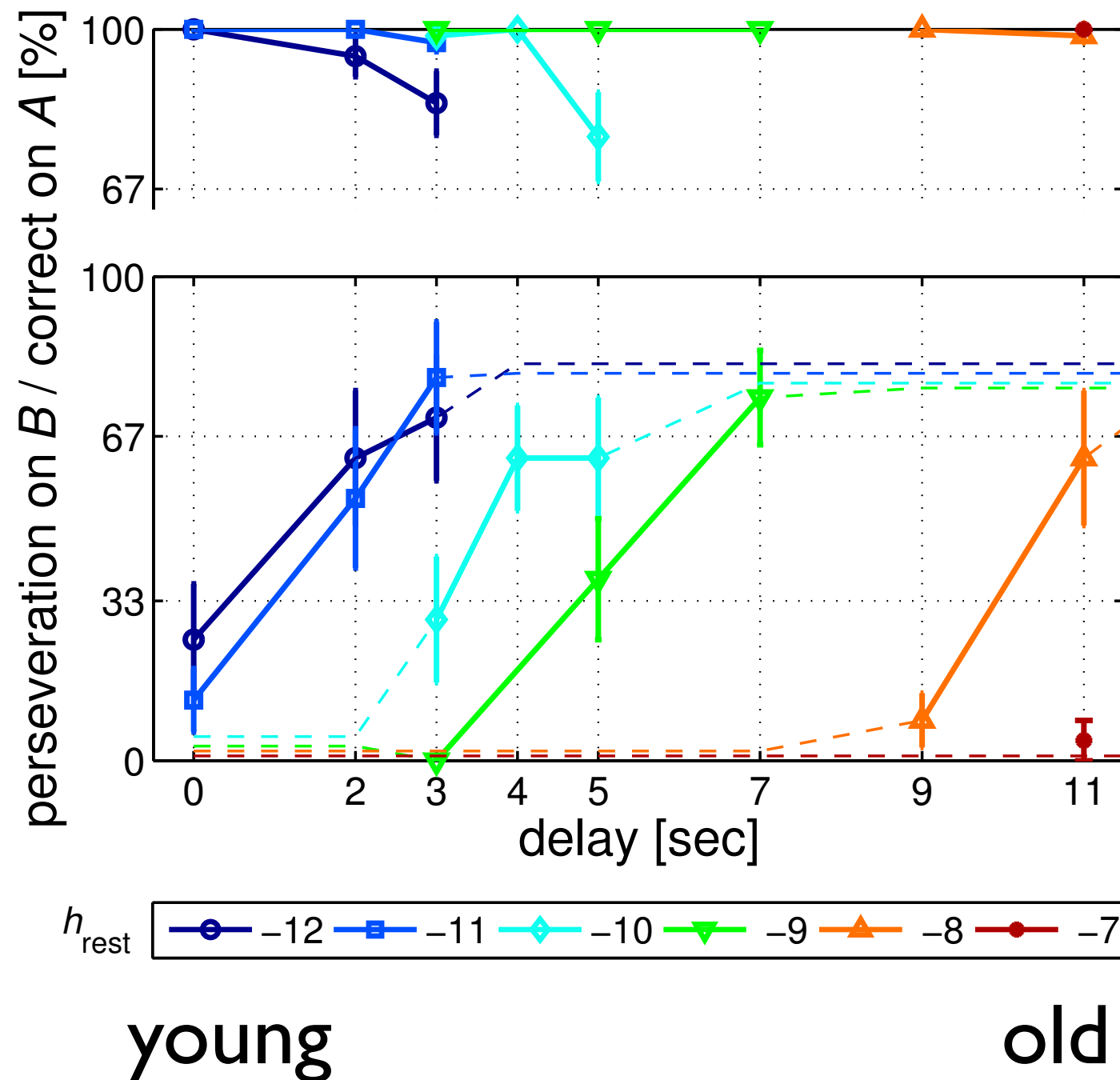
1

R

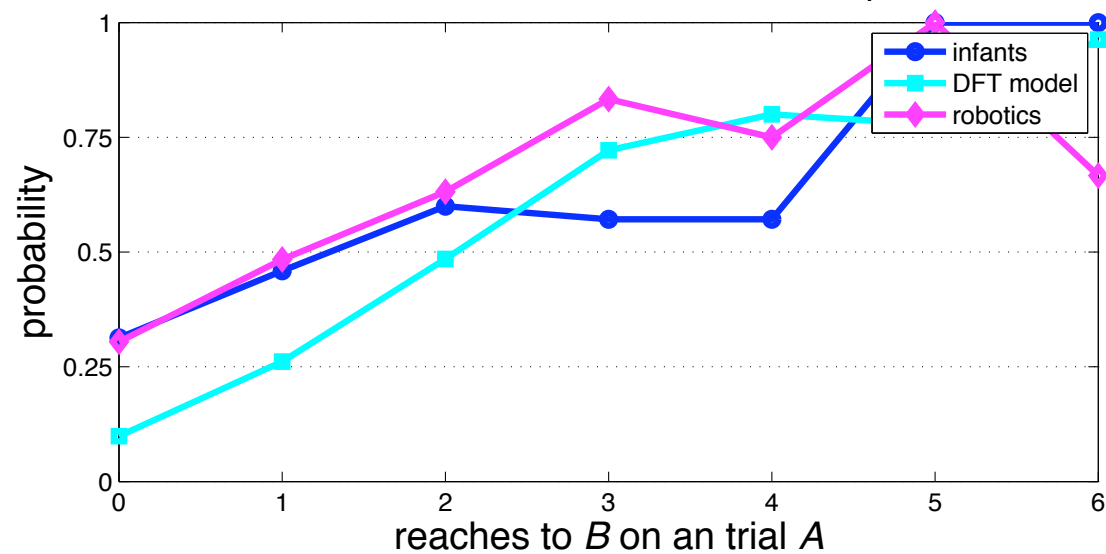




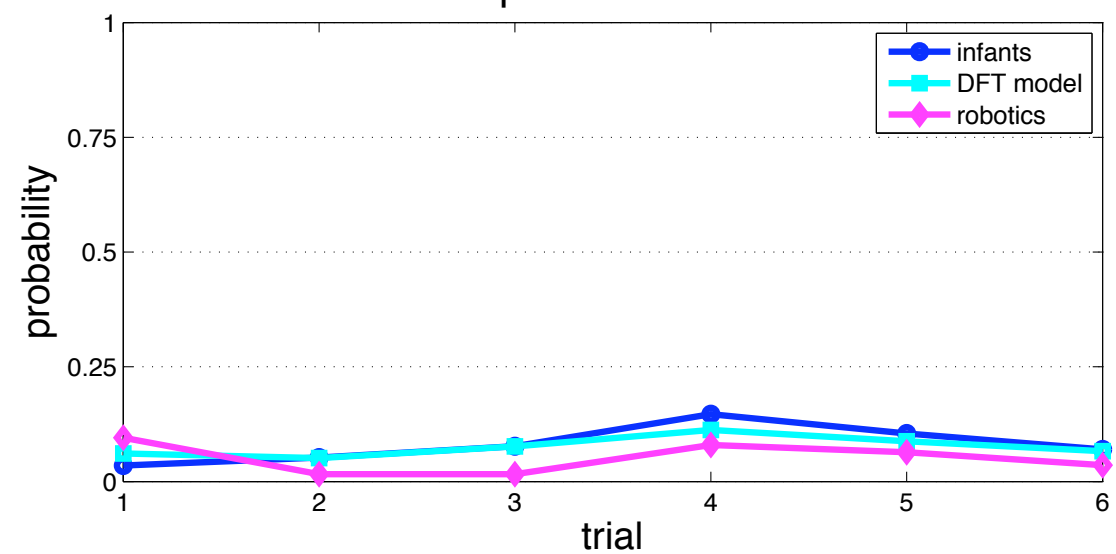
result: reproduce fundamental  
age-delay trade-off in A not B



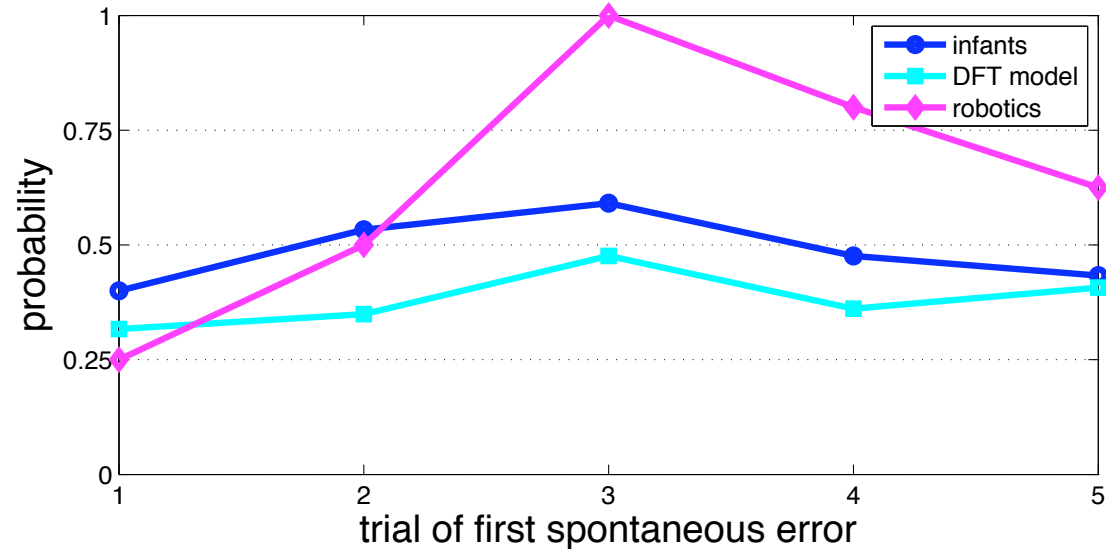
correct responses on trial  $B_1$



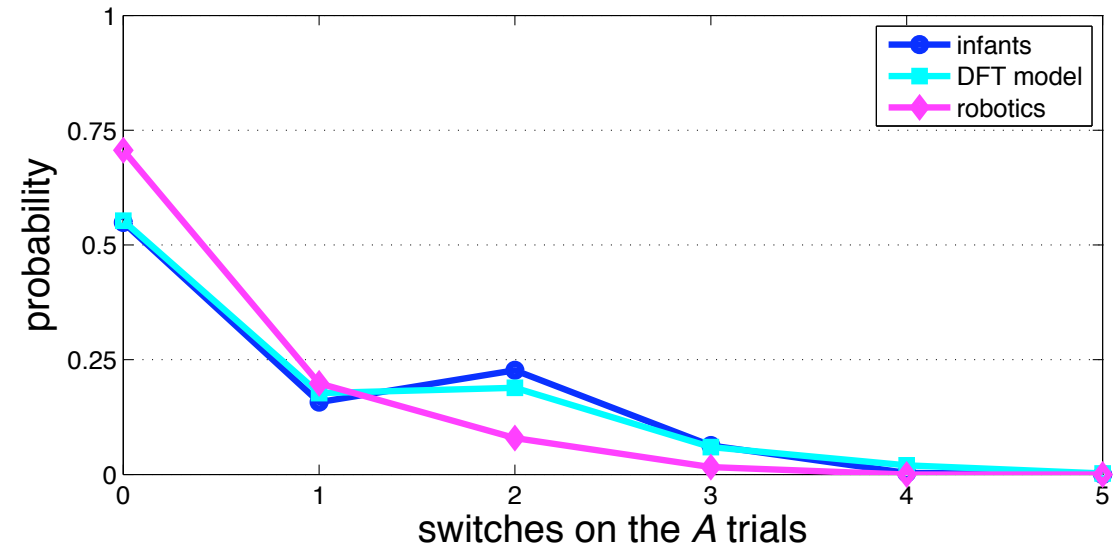
First Spontaneous Errors



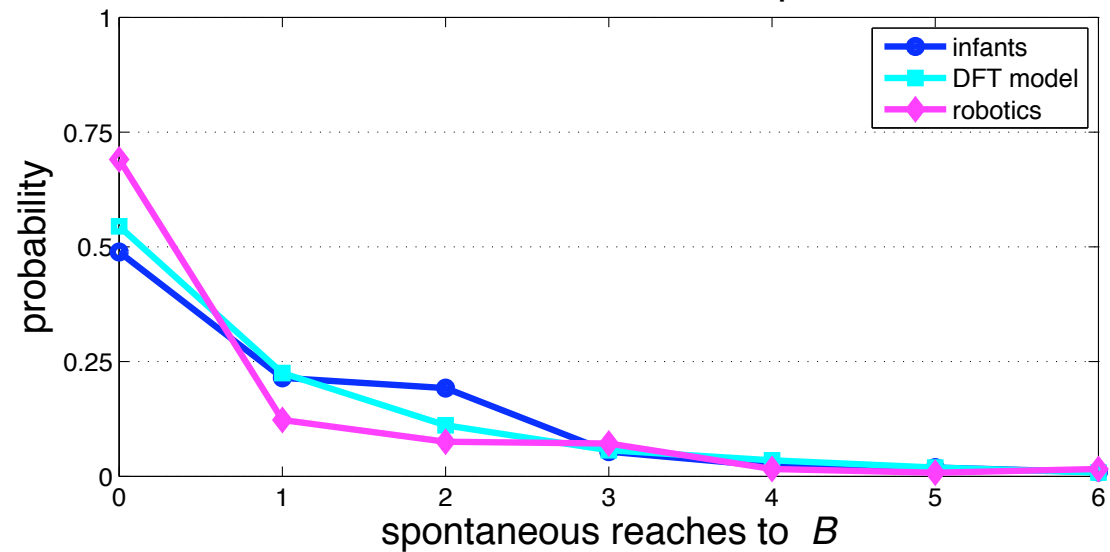
Second Spontaneous Errors



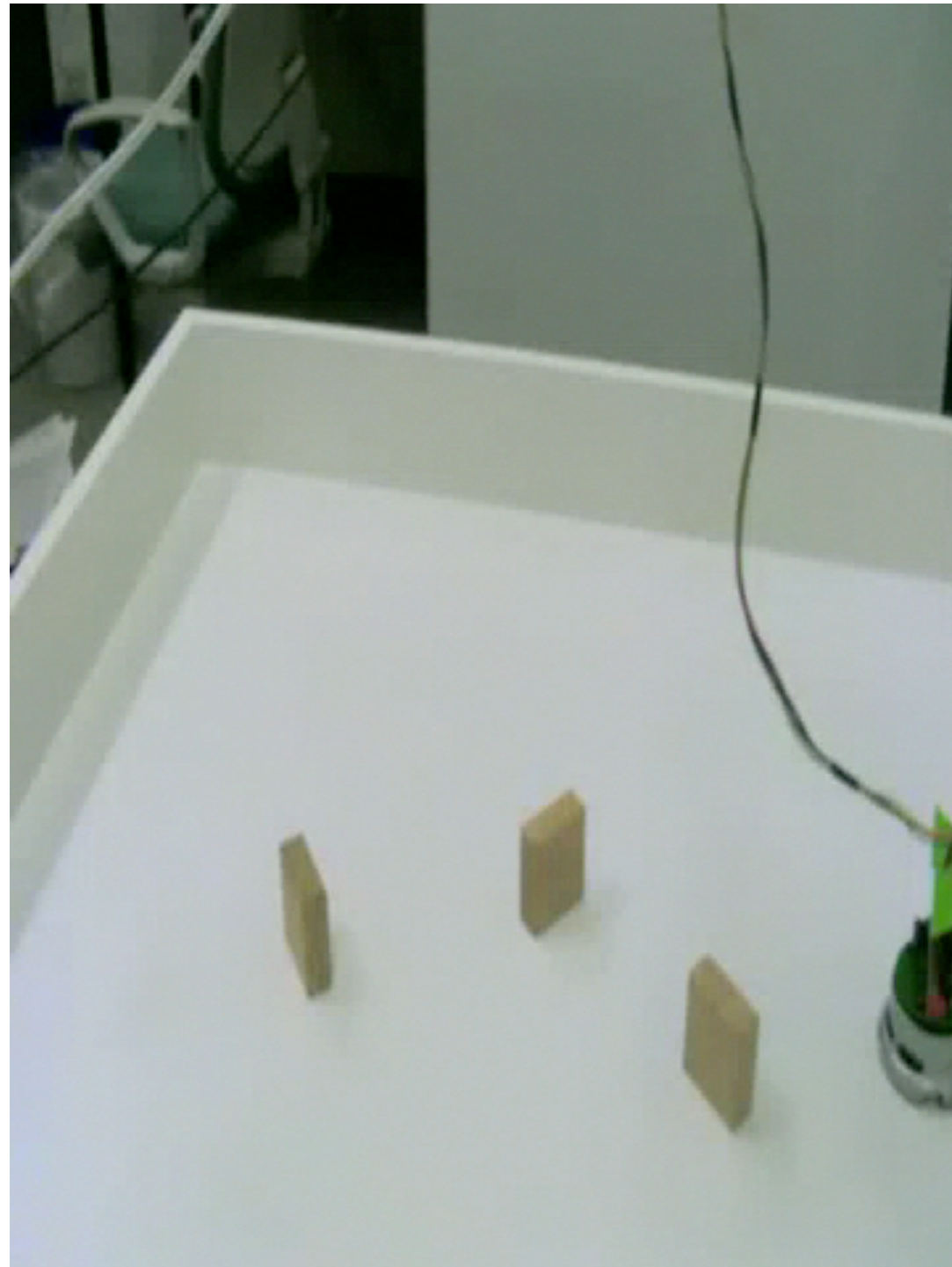
Distributions of Switches



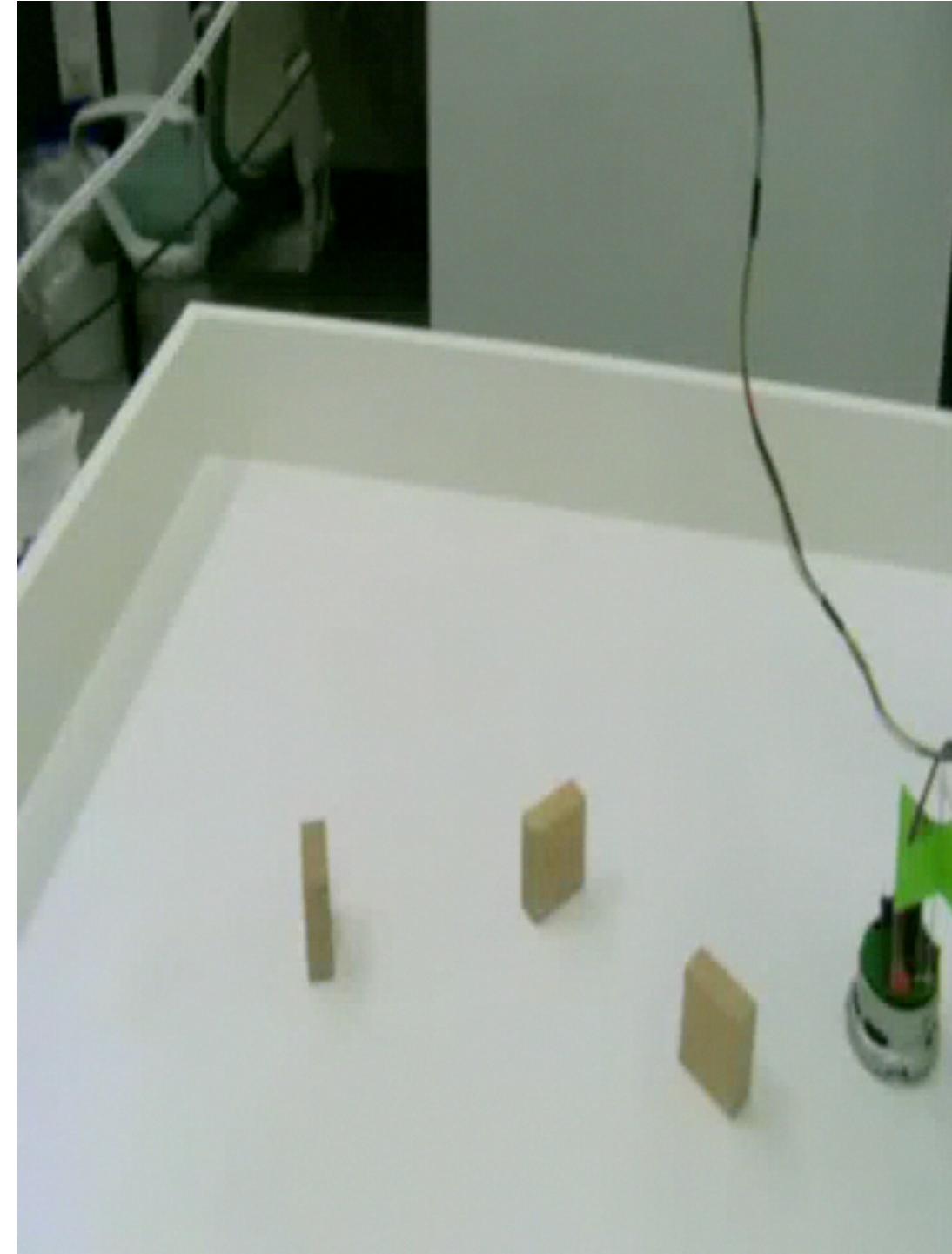
Distributions of Error Frequencies



“young” robot



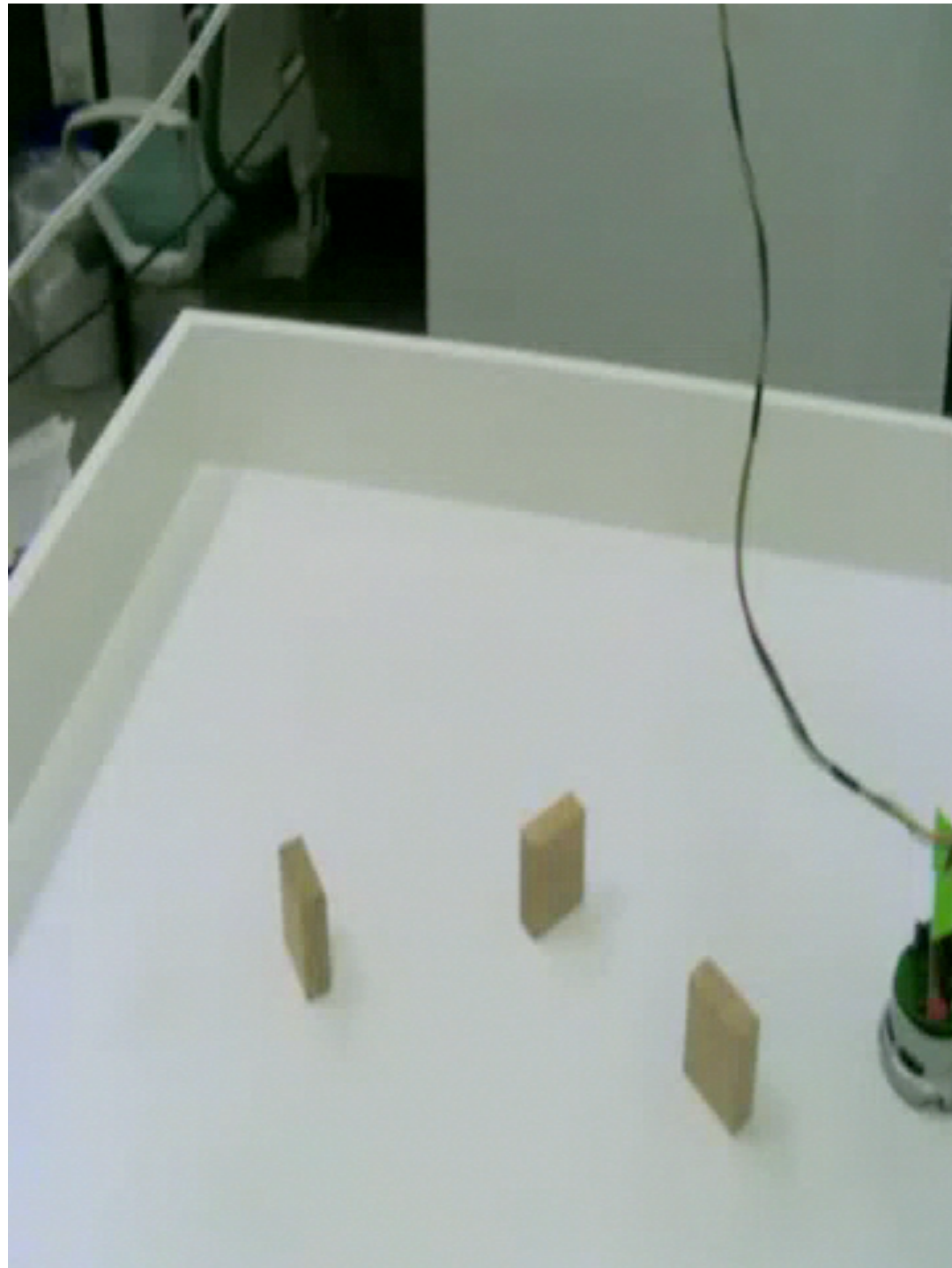
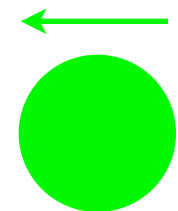
“old” robot





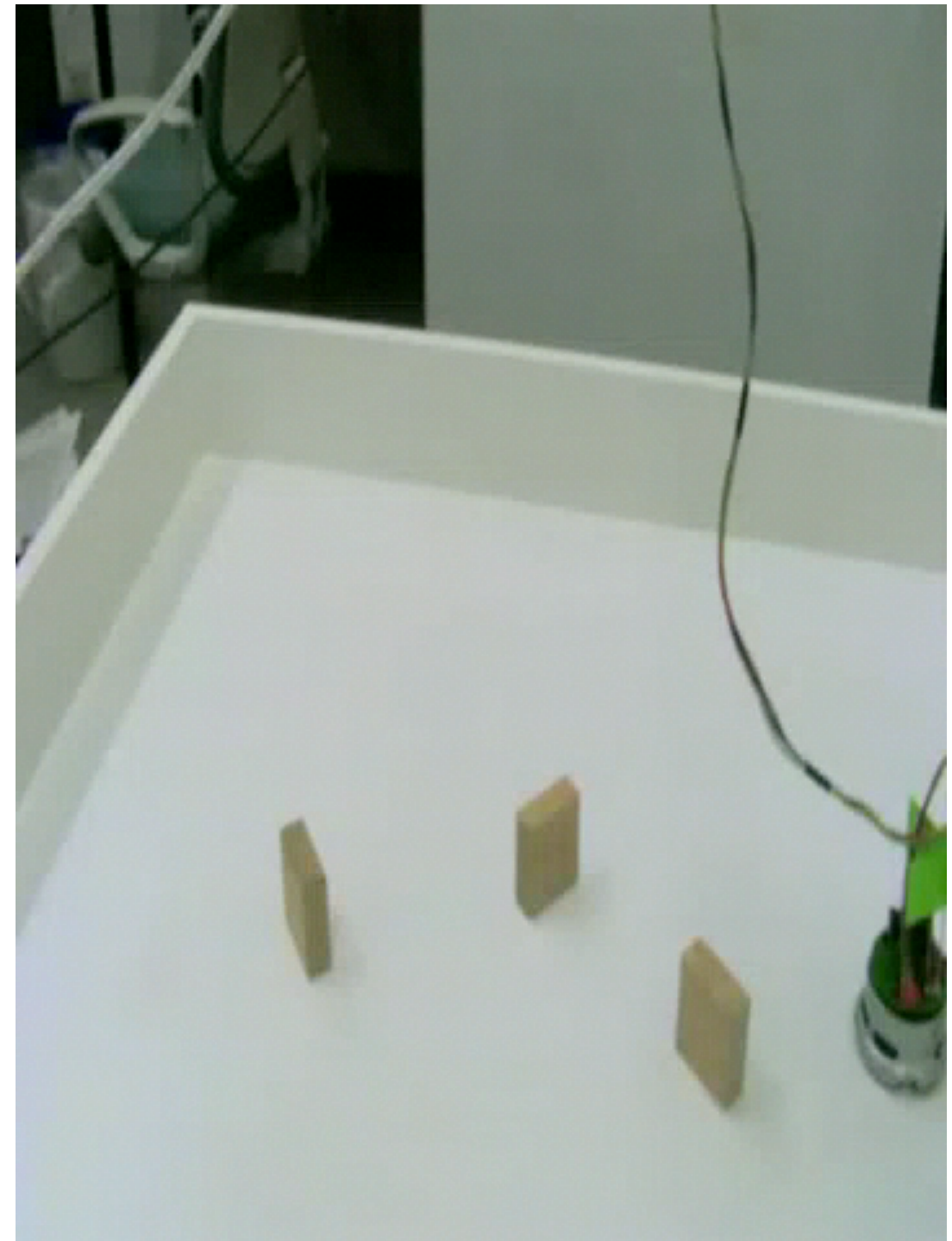
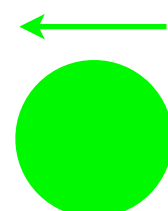
“young” robot

target



“young” robot with  
memory trace

target





# DFT models can be embodied

- stabilization of decisions is critical
- (when we failed to do so, by just “reading out” the location with maximal activation after the delay, that location fluctuate from moment to moment leading to meandering of the robot in an averaged direction)

# Conclusions

- action, perception, and embodied cognition takes place in continuous spaces. peaks = units of representation are attractors of the neural dynamics
- neural fields link neural representations to these continua
- stable activation peaks are the units of neural representation
- peaks arise and disappear through instabilities through which elementary cognitive functions (e.g. detection, selection, memory) emerge

# The conceptual framework of DFT

