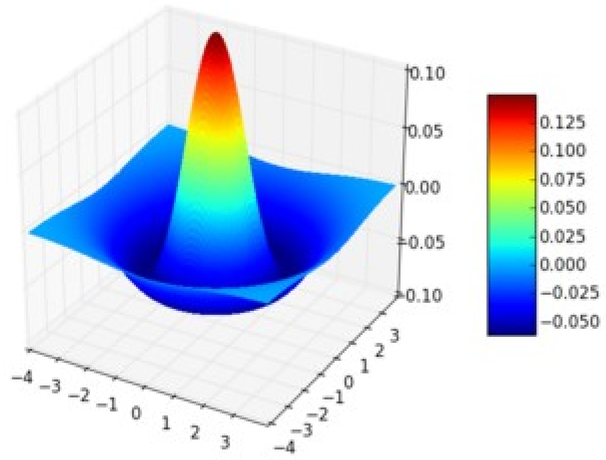
Probabilistic interpretation of dynamic neural fields



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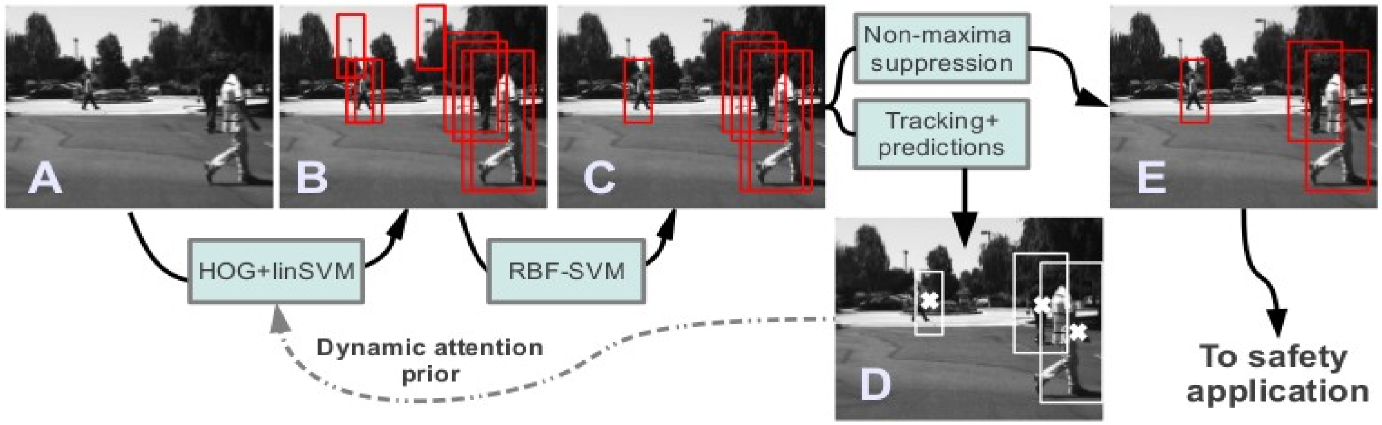
About my research

- Object recognition under hard conditions



- New neural learning methods for object recognition

- Neural fields

Object recognition and probability

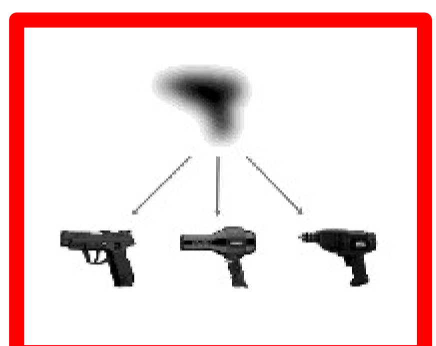
- For many researchers, perception is strongly related to probabilistic inference
  - Probabilistic inference passes around **distributions,** takes a decision only at the end of processing
  - P.I. tries **infer** a probabilistic interpretation
    - from sensory input : pixels, features, ..
    - from context prior: scene, time of day, location, other humans, ..
  - In probabilistic terms : try to find posterior **distribution** $p(int_i \mid input)$
  - possibly multiple combination of distributions using **Bayes' rule**

$$p(A \mid B) = \frac{p(B \mid A)\, p(B)}{p(A)}$$

Object recognition and probability

- Typically many possible interpretations for one image pattern

- Only context can meaningfully disambiguate !



$p(int = \text{'gun'} | pattern) \sim$

$p(pattern | int = \text{'gun'}) \; p(int = \text{'gun'})$

Can neural fields represent distributions ??

?

Basic problems

I. How to infer most probable interpretation of inputs ?
→ **Lyapunov analysis!**

II. Attractor states do not carry any information except place.
How to express probability ?
→ **latency !**

III. Lateral interactions do not allow to represent distributions
→ **circuit building!**

Lyapunov analysis of dynamic neural fields

- **Problem I : How to infer most probable interpretation of inputs ?**
- Lyapunov functional represents « energy » for a dynamic system
  - can only decrease/stay constant
  - bounded from below
  - stable state ⇔ Lyaponov functional constant
- Allows to understand what a system does
- Not every dynamic system has L.F.

→ but dynamic neural fields do[1] !

[1] Kubota, S and Aihara, K. Analyzing the global dynamics of a neural field model. Neural Processing Letters, 2004.

Lyapunov analysis of dynamic neural fields, II

- Lyapunov functional for neural fields :

$$\tau \dot{z}(\vec{x},t) = -z(\vec{x},t) + I(\vec{x},t) + \int w(x,x')z(\vec{x},t)dx' + h$$

$$E = -\int z(\vec{x},t)\left[I(\vec{x},t) - h\right] - \frac{1}{2}\int z(\vec{x},t)\int w(x,y)z(\vec{y},t)d\vec{x}d\vec{y} + \int\int_0^{z(\vec{x},t)} f^{-1}(z')dz'd\vec{x}$$

$$E = -\langle z, I-h\rangle - \frac{1}{2}\langle z, w * z\rangle + \langle z, F(z)\rangle$$

- Dynamic neural field finds equilibrium between
  - Maximizing similarity of state and input
  - Maximizing similarity of state and and lateral interaction
  - Minimizing activity

Lyapunov analysis of dynamic neural fields, II

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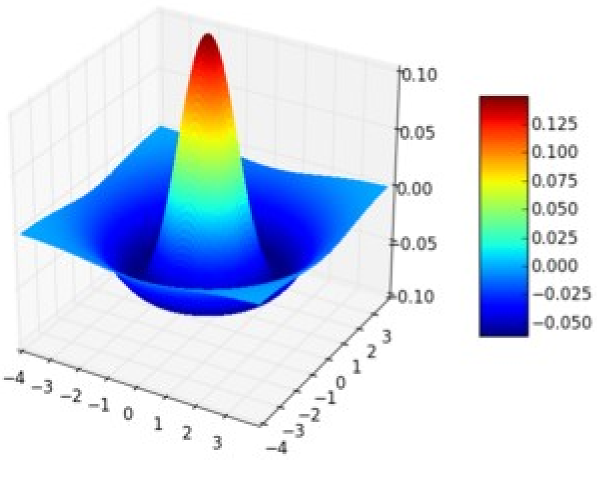
Lyapunov analysis of dynamic neural fields, III

- Lateral connections : data model !
- Neural fields converge to a state which is maximally compatible with data model and input
- Example : standard Mexican hat kernel
  - preferred solutions : single peaks of certain size
  - if multiple peaks in input : fight to the death



Summary problem I

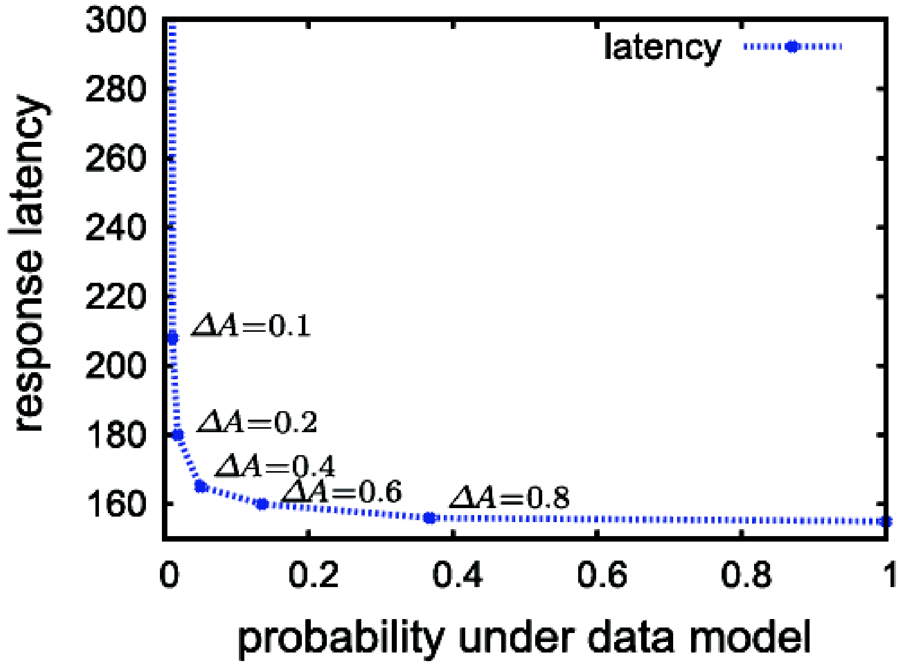
- **How to infer most probable interpretation of inputs ?**
- Have shown that neural fields converge to a state maximally compatible to input and lateral connections
- Lateral connections act as a data model, enforcing their assumption on data statistics on field dynamics
- If lateral connections match data statistics → field converges to most probable state for a given input

Latency

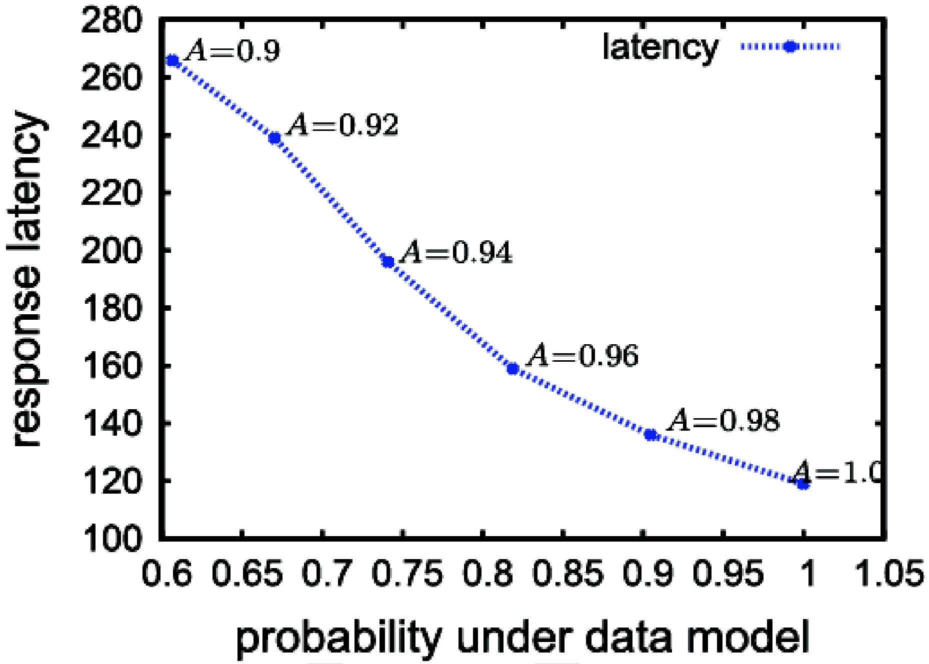
- **Problem II: attractor states do not carry any information except by its position → how to express probability ?**
- Solution : dynamics of attractor formation !
  - Latency : time to attractor solution
  - Expresses probability of that solution
  - Again : determined by data model !
- Lyapunov analysis suggests a connection between convergence time and input
- From experience : inputs that contradict the data model strongly take longer to converge

Latency II

- Simulation results



Violation of data model by adding second Gaussian, amplitude difference ΔA



Violation of data model by reducing amplitude A of single Gaussian

Summmary problem II

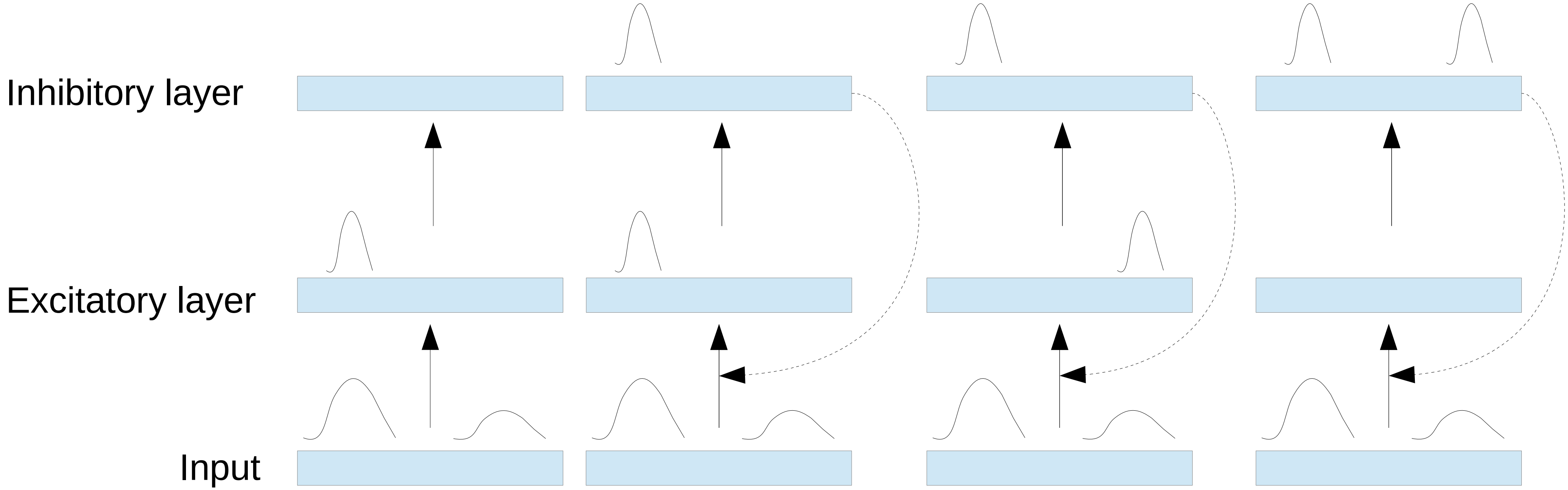
- **Attractor states do not carry any information except by its position → how to express probability ?**
- Probability of an attractor state expressed by its latency (measured from input onset)
- Strongly depends on data model encoded into lateral connections
- Latency is for free !

Circuit building

- **Problem III : Lateral interactions do not allow to represent distributions**
- This is true, nothing we can do
- Best we can do : represent maximum of probability distribution and its probability
- Can we recover the next most probable interpretation ? The next n most probable ones ?

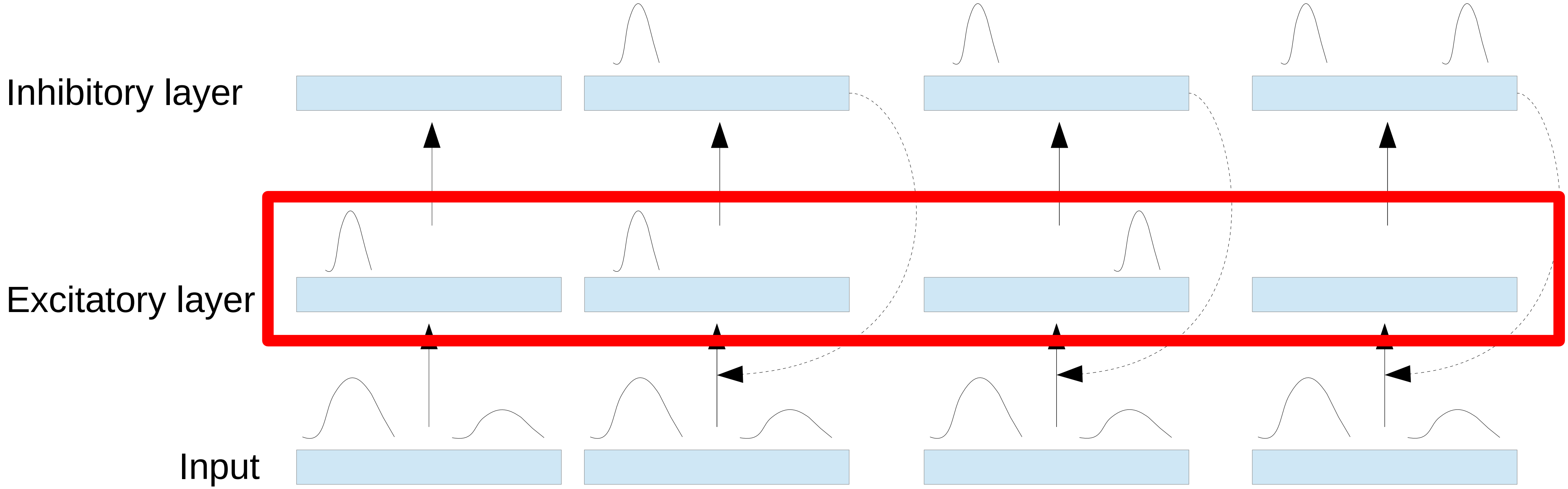
Circuit building

- In vision science : inhibition-of-return effect
- We can do something similar with neural fields !



Circuit building

- Sequence of attractor states approximates input distribution !
- Descending order of probability



Summary

- **Lateral interactions do not allow to represent distributions**
- No way to change that
- But : by building circuits, we can obtain the n most probable interpretations in descending order
- From a practical point of view, this is useful
- Shifted functions from spatial to temporal domain

Probabilistic neural fields

- Proposal :
  - Lateral connections represent a data model
  - Attractor solution represents the most likely interpretation of inputs given the data model
  - Latency is a linked to the probability of the attractor solution !
  - Distributions can be recovered by an inhibition-of-return mechanism

Summary

- Neural fields can indeed approximately represent distributions of probability
- Make use of temporal dimension of dynamics
- No need to change dynamics, just make use of what exists
- From space coding to spatio-temporal coding (large amount of evidence from biology that this is at common principle)
- « Most confident signals come first ! »

Outlook

- Problem : parameterization to make latency apparent
- What about **decoding** distributions ?
- What about **manipulating** distributions ? Bayes' rule ?

  → preshape !

- Research in progress !