Neural Dynamics

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how to represent the inner state of the Central Nervous System?

=> activation concept
Activation

- neural state variables
  - membrane potential of neurons?
  - spiking rate?
  - ... population activation...
Activation

- activation as a real number, abstracting from biophysical details
  - low levels of activation: not transmitted to other systems (e.g., to motor systems)
  - high levels of activation: transmitted to other systems
  - as described by sigmoidal threshold function
  - zero activation defined as threshold of that function
Activation

- compare to connectionist notion of activation:
  - same idea, but tied to individual neurons
- compare to abstract activation of production systems (ACT-R, SOAR)
  - quite different... really a function that measures how far a module is from emitting its output...
Activation dynamics

- activation variables $u(t)$ as time continuous functions...

$$\tau \frac{du(t)}{dt} = f(u)$$

- what function $f$?
Activation dynamics

- start with $f=0$

$$\tau \dot{u} = \xi_t$$
Activation dynamics

need stabilization

\[ \tau \dot{u} = -u + h + \xi_t. \]
In a dynamical system, the present predicts the future: given the initial level of activation $u(0)$, the activation at time $t$: $u(t)$ is uniquely determined

$$\frac{du(t)}{dt} = \dot{u}(t) = -u(t) + h \quad (h < 0)$$
tutorial on mental simulation
Neural dynamics

- stationary state = fixed point = constant solution
- stable fixed point: nearby solutions converge to the fixed point = attractor

\[ \frac{du(t)}{dt} = \dot{u}(t) = -u(t) + h \quad (h < 0) \]
Neural dynamics

- exponential relaxation to fixed-point attractors
- $\tau \frac{du}{dt} = f(u)$

$u(t)$

$u(0)$

$u(0)/e$

$u(\tau)$

Time

Resting level

Vector-field

$\tau \frac{du}{dt} = -u(t) + h$
solutions = trajectories vs. dynamical system

A fixed point, \( u_0 \), is formally defined as a solution of

\[ f(u_0) = 0 \quad (B1.3) \]

As illustrated in Figure 1.6. Because the function \( f \) does not depend on time, the fixed point, \( u_0 \), is constant over time as well, so that

\[ \dot{u}_0 = 0, \]

and thus:

\[ \dot{u}_0 = 0 = (B1.3). \]

In other words, the fixed point, \( u_0 \), is a constant solution of the differential equation.

A fixed point is “asymptotically stable” if the solutions of the dynamical system that start from initial conditions nearby converge over time to the fixed point. When the dynamics, \( f \), has a negative slope at the fixed point,

\[ \frac{df}{du} u_0 = 0 < 0, \]

then the fixed point is stable. The arrows in

Time, \( t \)

...
Neural dynamics

- attractor structures ensemble of solutions = flow

\[ \frac{d\mathbf{u}}{dt} = f(\mathbf{u}) \]

- resting level

\[ \tau \mathbf{u}(t) = -\mathbf{u}(t) + h \]
Neuronal dynamics

- **inputs** = contributions to the rate of change
  - positive: excitatory
  - negative: inhibitory
- $\tau \dot{u}(t) = -u(t) + h + \text{inputs}(t)$
=> simulation
tutorial on numerics

- dynamical system
  - continuous time

- differential
  - quotient
  - approximates the derivative in discrete time

- Euler iteration
  - equation in discrete time

\[ \ddot{u} = f(u). \]

\[ \dot{u}(t_i) \approx \frac{u(t_i) - u(t_{i-1})}{\Delta t} \]

\[ u(t_i) = u(t_{i-1}) + \Delta t f(u(t_{i-1})). \]
Matlab code
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]

\[ g(u) \]

\[ \text{resting level, } h \]

\[ \beta \]

\[ \Rightarrow \text{nonlinear dynamics!} \]
Neuronal dynamics with self-excitation

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]
Neuronal dynamics with self-excitation

- at intermediate stimulus strength: bistable
- “on” vs “off” state

\[ \tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t)) \]
Neuronal dynamics with self-excitation

increasing input strength =>
detection instability

resting level, h

stimulus strength

stable
unstable
Neuronal dynamics with self-excitation

- Decreasing input strength => reverse detection instability
Neuronal dynamics with self-excitation

- The detection and the reverse detection instability create discrete events out of input that changes continuously in time.
simulator
Neuronal dynamics with competition

\[ \tau \dot{u}_1(t) = -u_1(t) + h - \sigma(u_2(t)) + S_1 \]

\[ \tau \dot{u}_2(t) = -u_2(t) + h - \sigma(u_1(t)) + S_2 \]
The rate of change of activation at one site depends on the level of activation at the other site.

**Mutual inhibition**

\[
\begin{align*}
\tau \dot{u}_1(t) &= -u_1(t) + h - \sigma(u_2(t)) + S_1 \\
\tau \dot{u}_2(t) &= -u_2(t) + h - \sigma(u_1(t)) + S_2
\end{align*}
\]

**Sigmoidal nonlinearity**
to visualize, assume that \( u_2 \) has been activated by input to positive level

\[ \Rightarrow \text{then } u_1 \text{ is suppressed} \]
why would $u_2$ be positive before $u_1$ is? E.g., it grew faster than $u_1$ because its inputs are stronger/inputs match better

$\Rightarrow$ input advantage translates into time advantage which translates into competitive advantage
Neuronal dynamics with competition

vector-field in the absence of input

\[ \frac{du}{dt} = f(u) \]

resting state

1D cut through vector-field

resting level
Neuronal dynamics with competition

vector-field (without interaction) when both neurons receive input

\[ du/dt = f(u) \]

1D cut through vector-field

stimulus determined state

input

activated level
Neuronal dynamics with competition

only activated neurons participate in interaction!

sigmoidal nonlinearity
Neuronal dynamics with competition

- vector-field of mutual inhibition

site 1 inhibits site 2

site 2 inhibits site 1

interaction combined
Neuronal dynamics with competition

Vector-field with strong mutual inhibition: bistable
Neuronal dynamics with competition

before input is presented

after input is presented
Neuronal dynamics with competition

=> biased competition

stronger input to site 1:
attractor with activated $u_{\text{-}1}$ stronger,
attractor with activated $u_{\text{-}2}$ weaker, may become unstable
Neuronal dynamics with competition

=> biased competition

before input is presented

after input is presented
=> simulation
next

where do activation variables come from?

=> DFT lecture