Neural Dynamics

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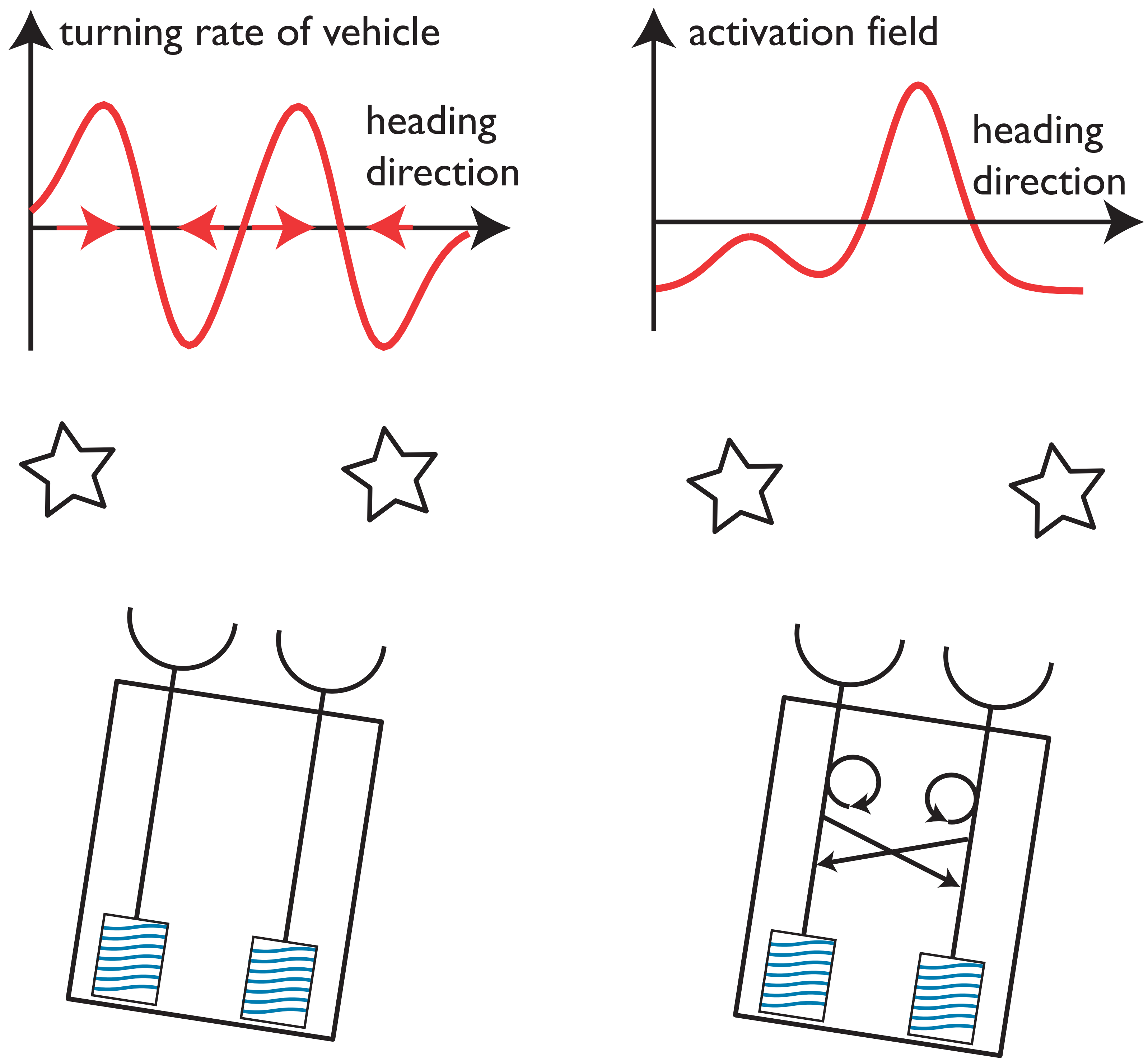
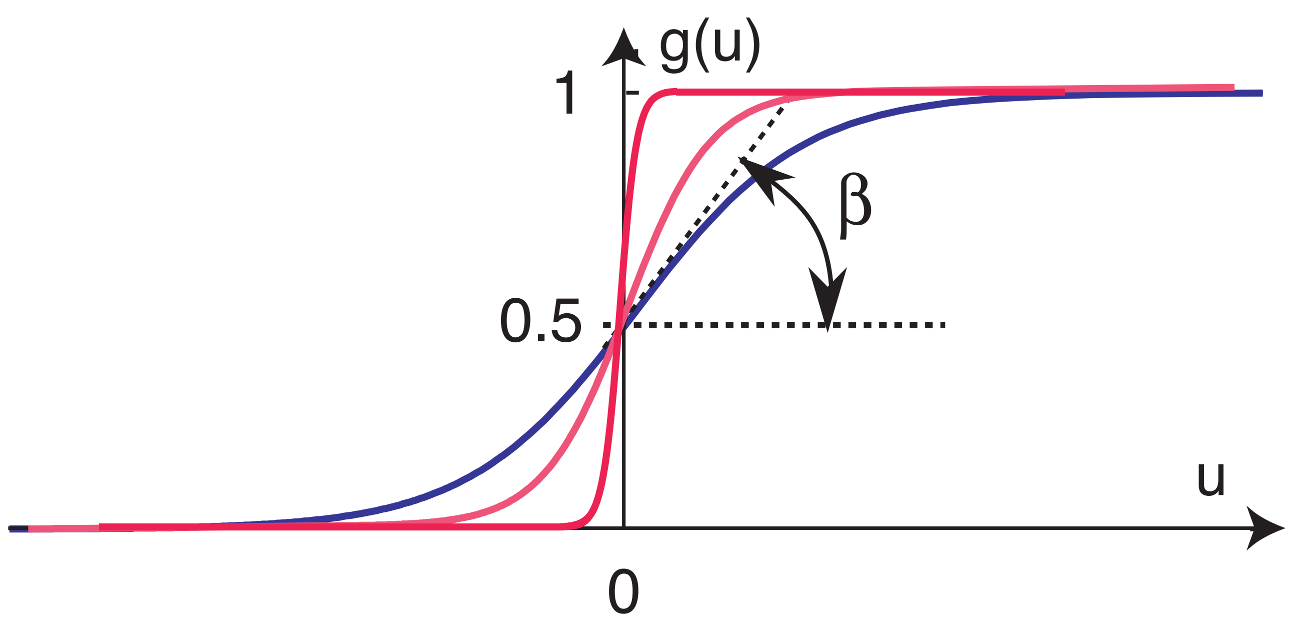
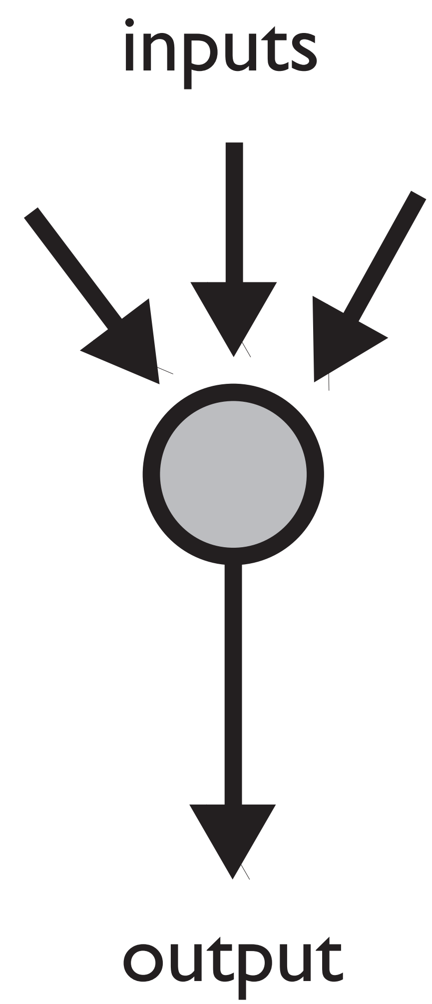
… from behavioral to neural dynamics



Neurons as input-output threshold elements

- activation state (membrane potential or spiking rate)
- summing inputs and generating output through a sigmoidal threshold function

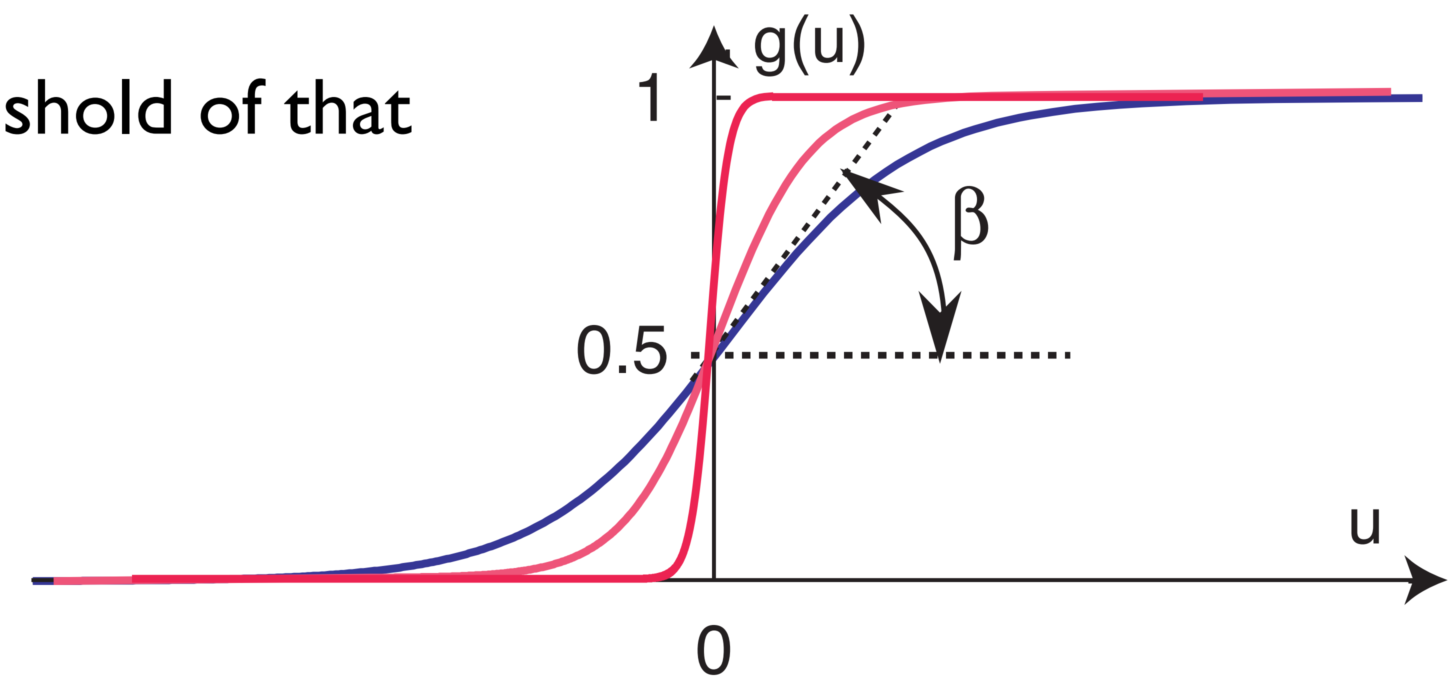




$$\text{output} = g\left(\sum(\text{inputs})\right)$$

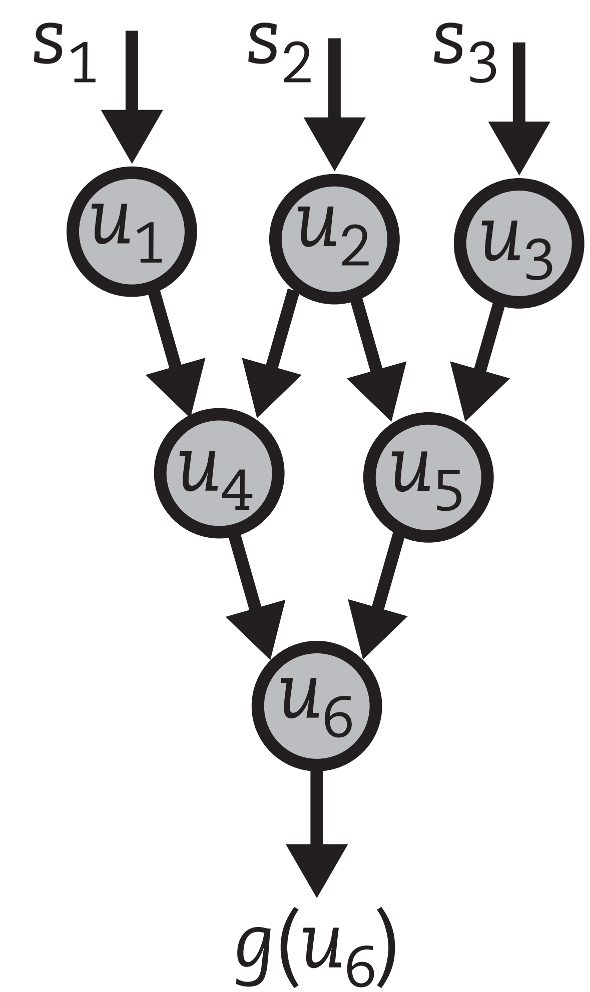
Activation

- activation as a real number, abstracting from biophysical details
  - low levels of activation: not transmitted to other systems (e.g., to motor systems)
  - high levels of activation: transmitted to other systems
  - as described by sigmoidal threshold function
  - zero activation defined as threshold of that function



(Forward) neural networks

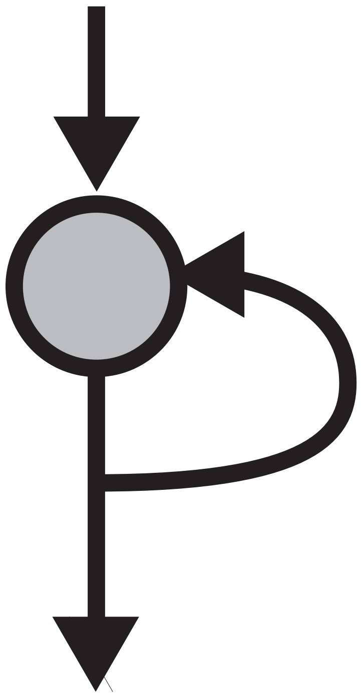
- output from one set of neurons provides input to another set of neuron



Recurrent neural networks

- neural dynamics…

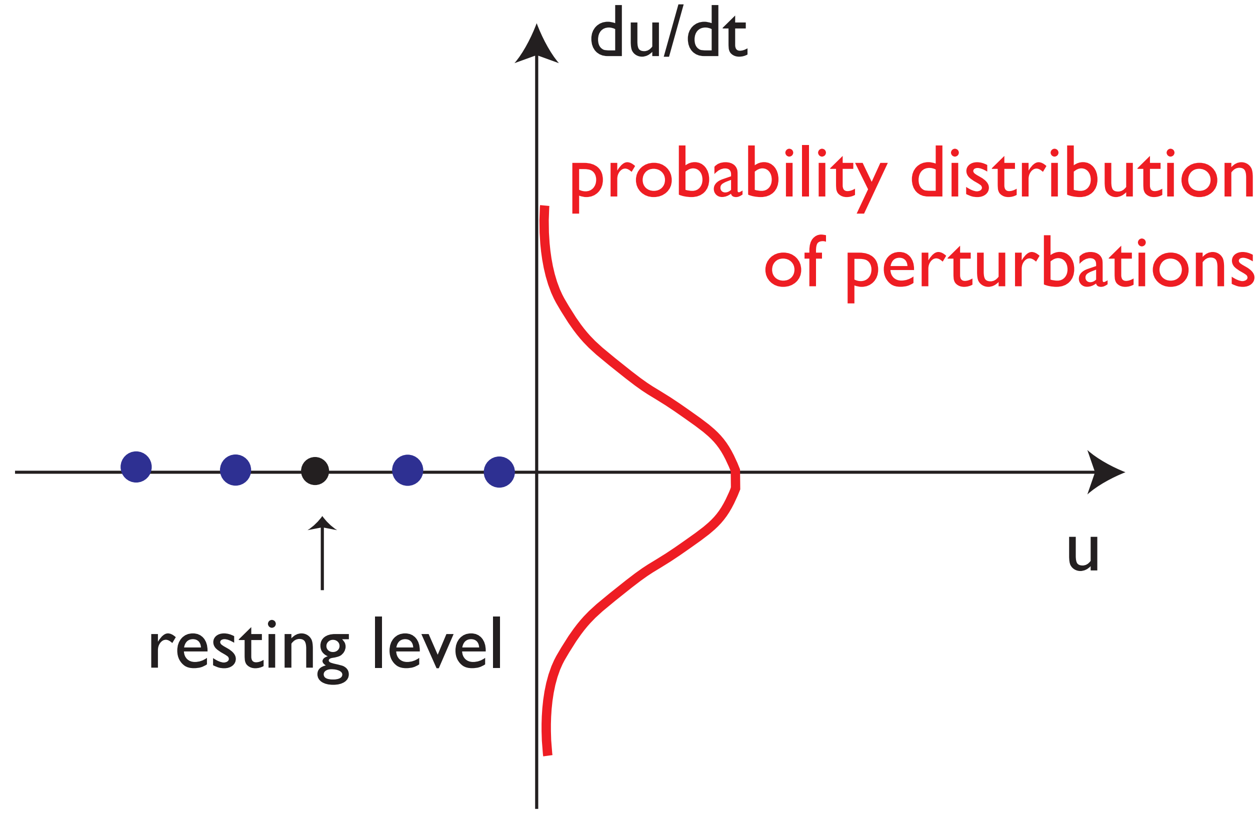
$$\dot{u}(t) = -u(t) + h + \mathrm{input}(t) + g(u(t))$$

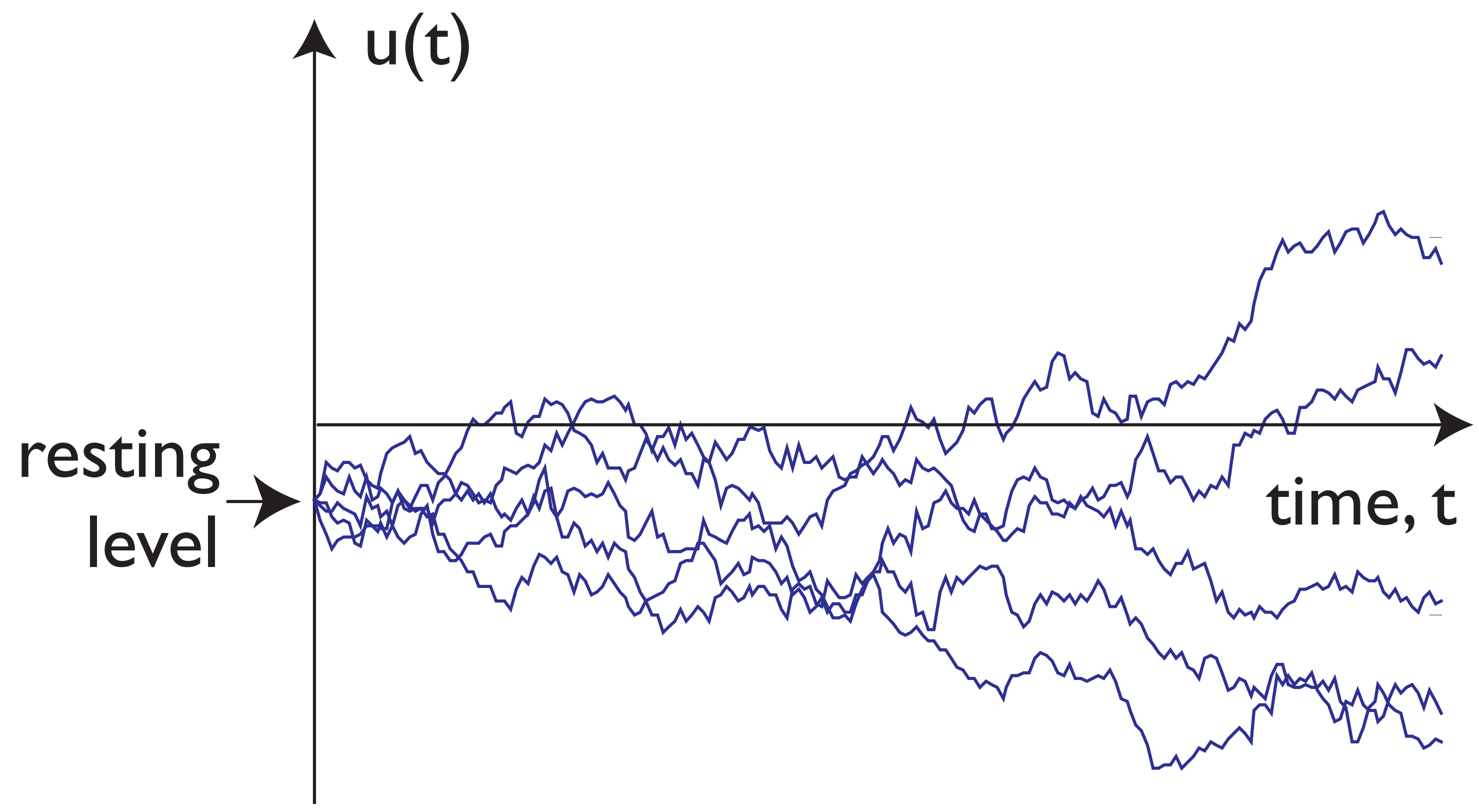


Activation dynamics

- naive neural dynamics

$$\tau \dot{u} = \xi_t$$

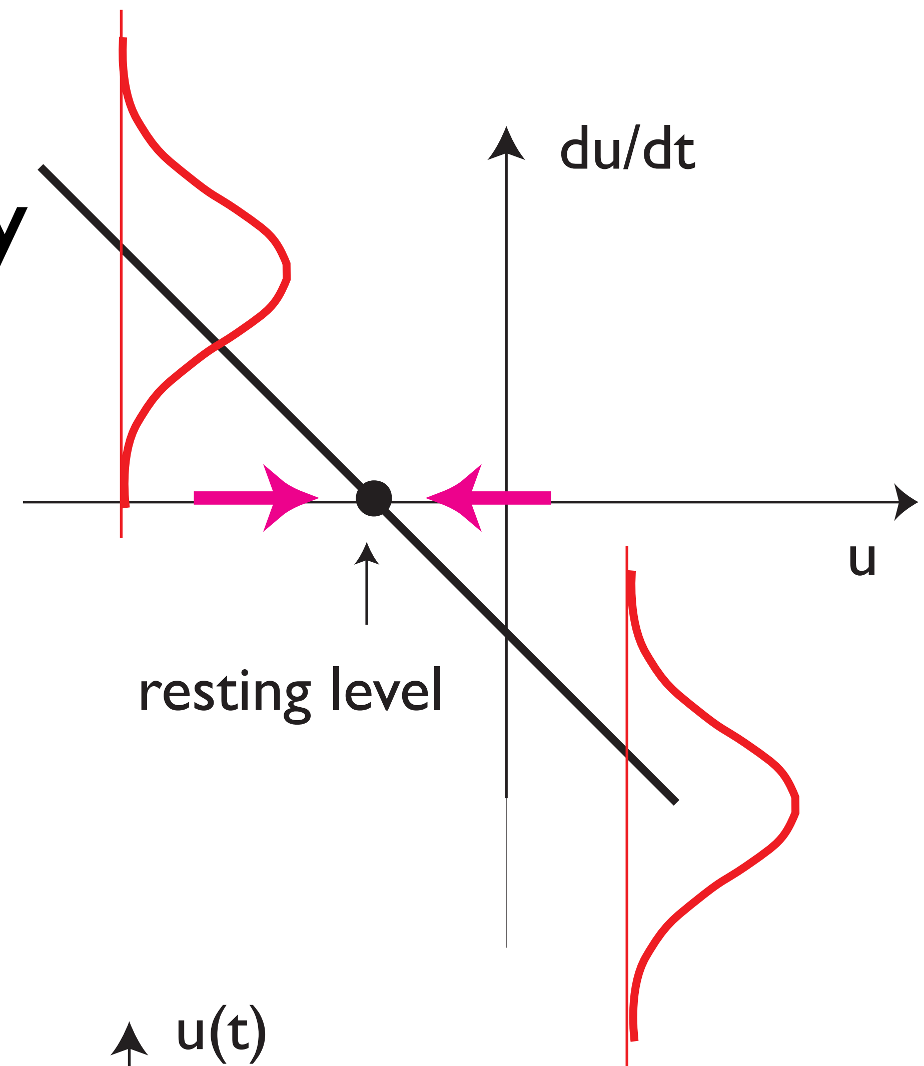


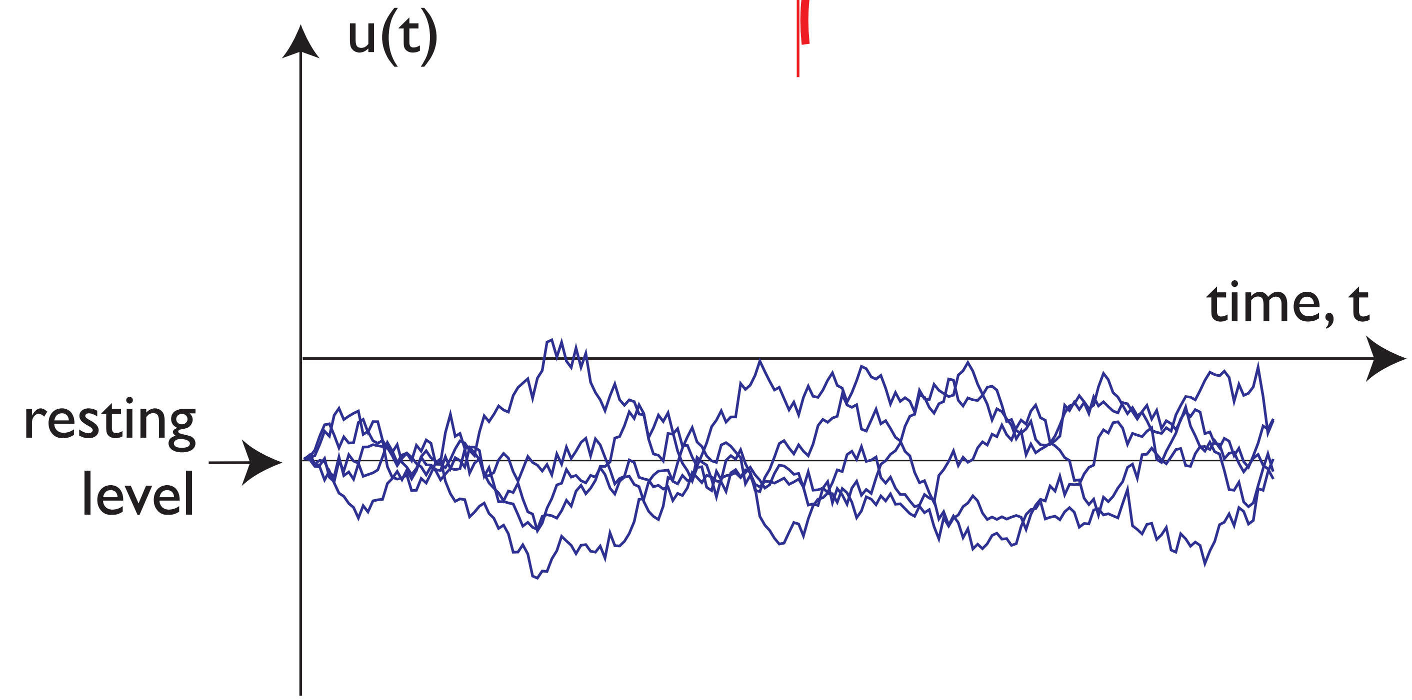


Activation dynamics

- neural dynamics with stability

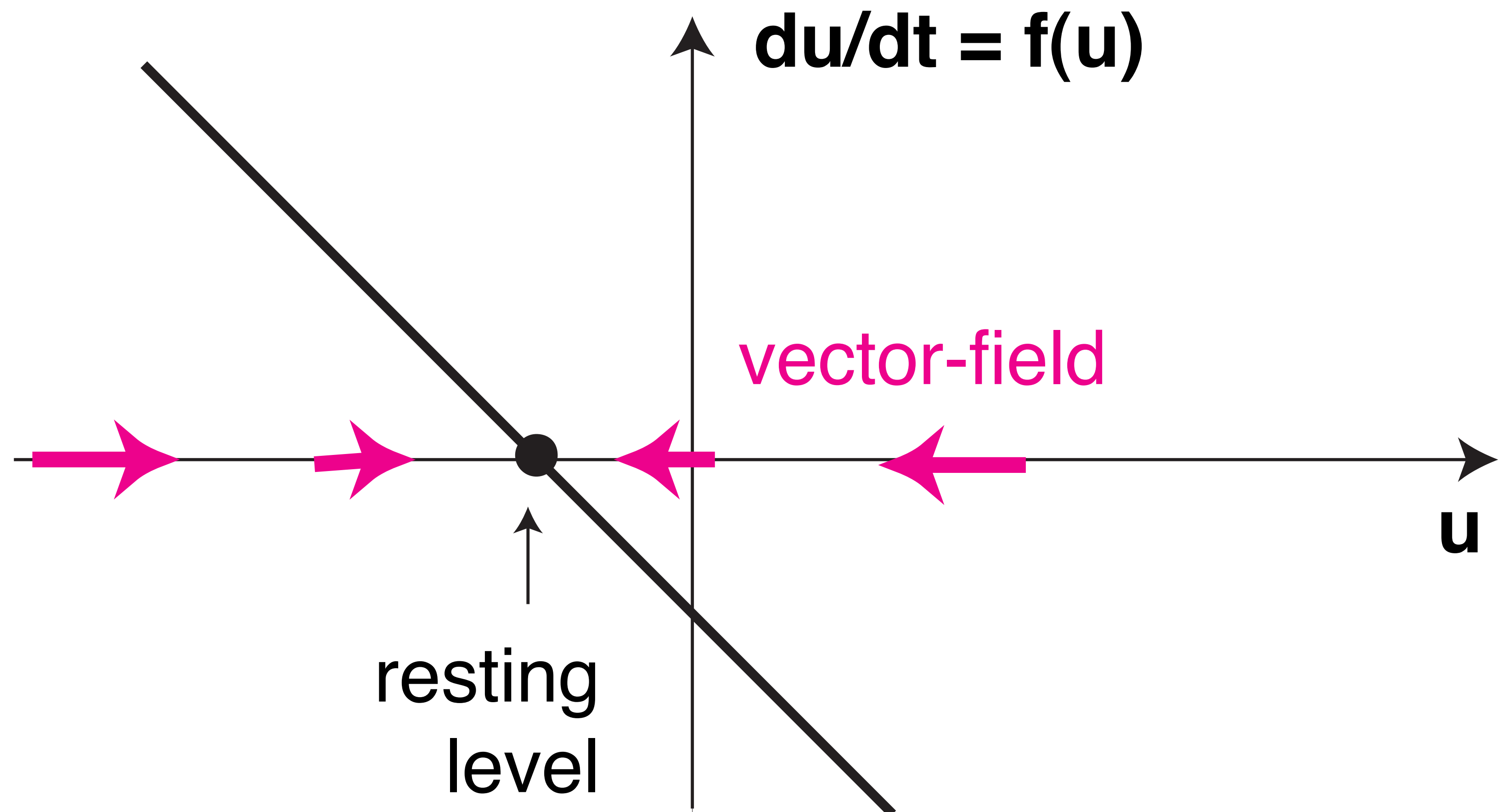
$$\tau \dot{u} = -u + h + \xi_t.$$





Neural dynamics

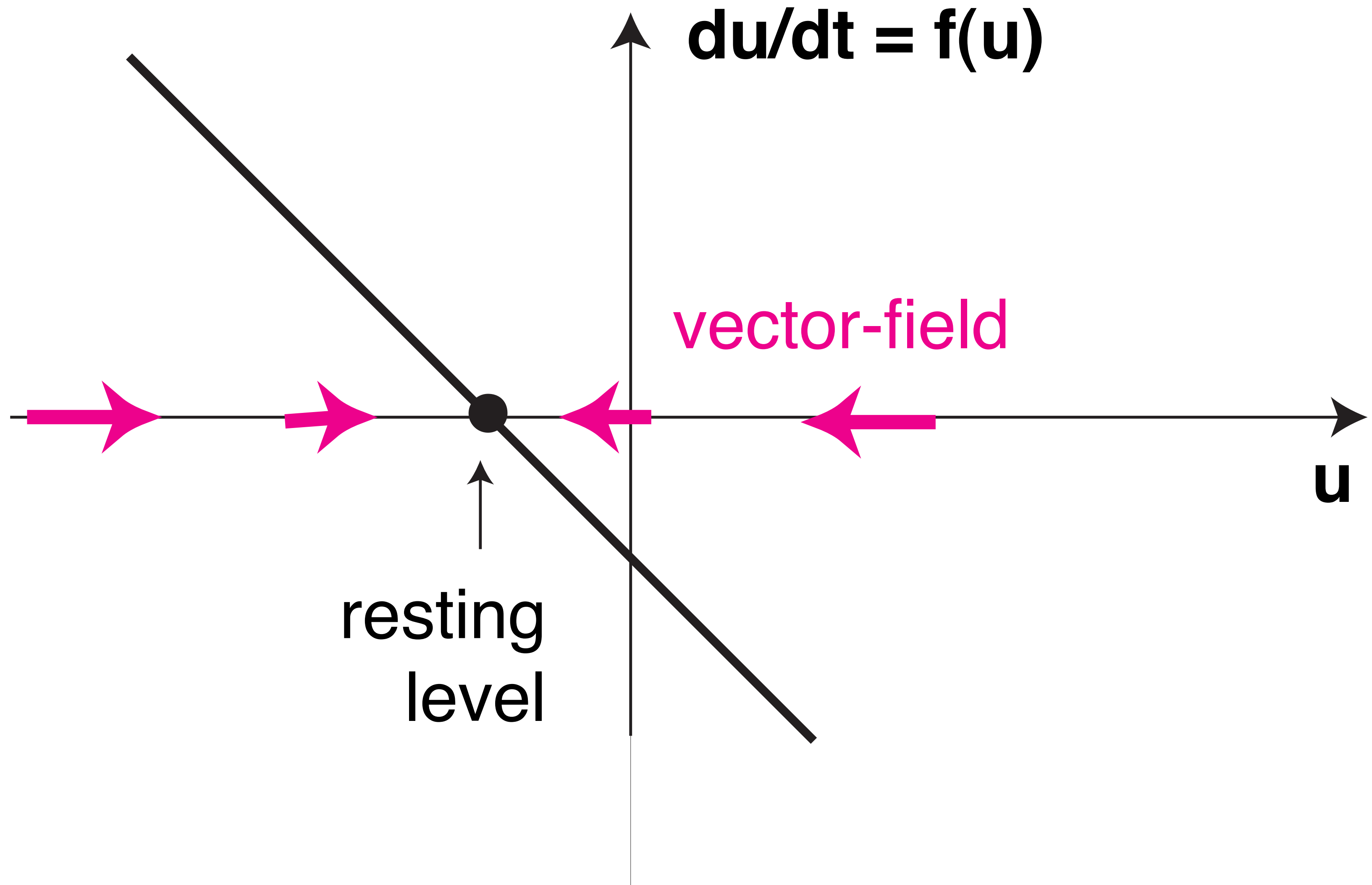
- In a dynamical system, the present predicts the future: given the initial level of activation u(0), the activation at time t: u(t) is uniquely determined



$$\frac{du(t)}{dt} = \dot{u}(t) = -u(t) + h \qquad (h < 0)$$

Neural dynamics

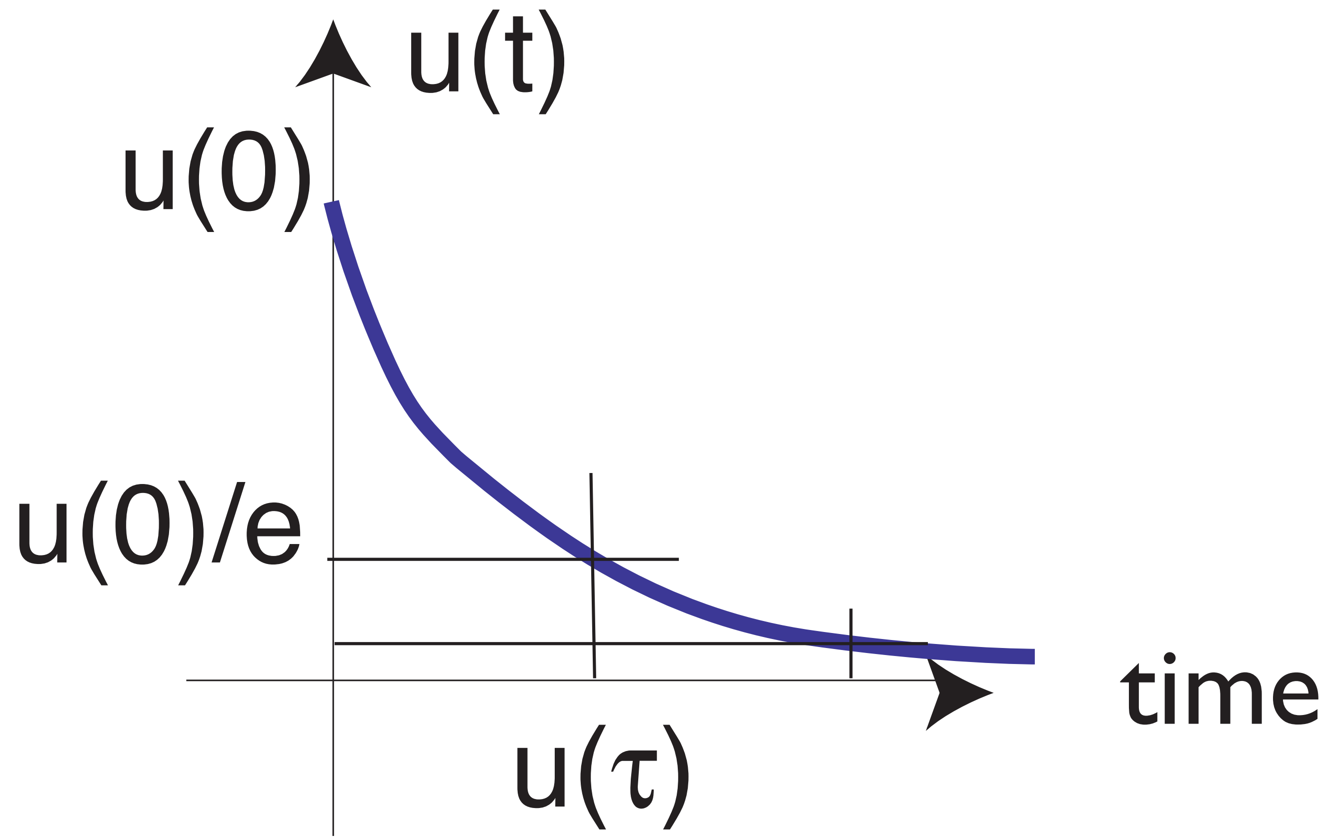
- stationary state=fixed point= constant solution
- stable fixed point: nearby solutions converge to the fixed point=attractor

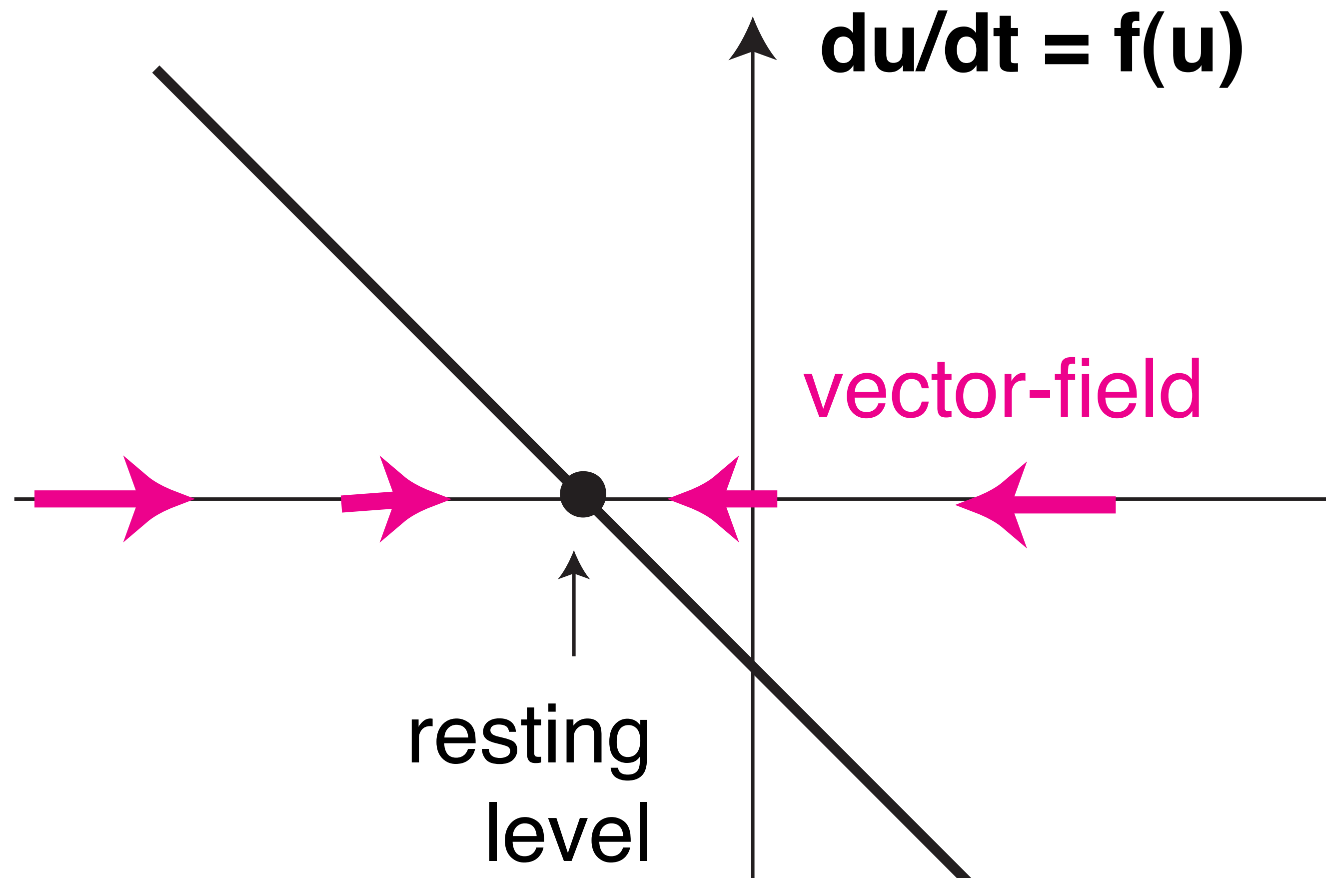


$$\frac{du(t)}{dt} = \dot{u}(t) = -u(t) + h \qquad (h < 0)$$

Neural dynamics

- exponential relaxation to fixed-point attractors
- => time scale

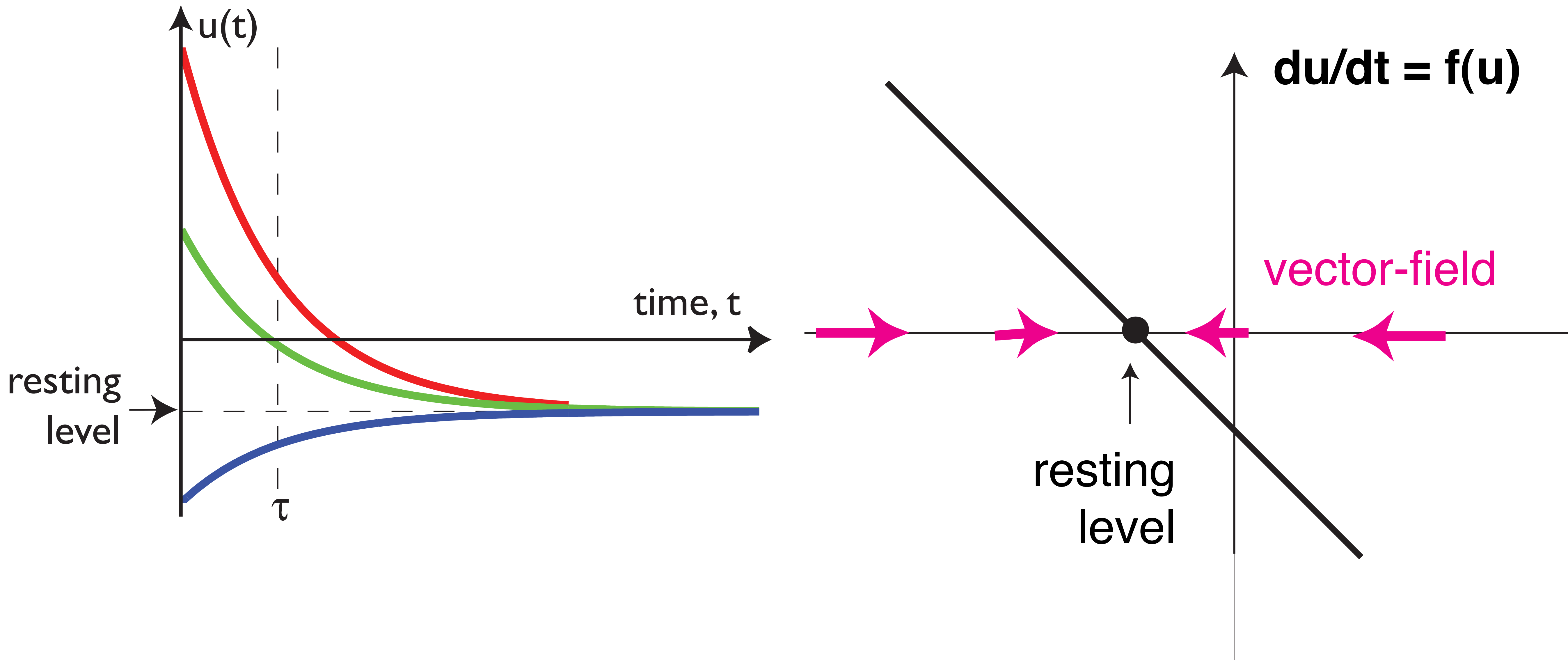




$$\tau \dot{u}(t) = -u(t) + h$$

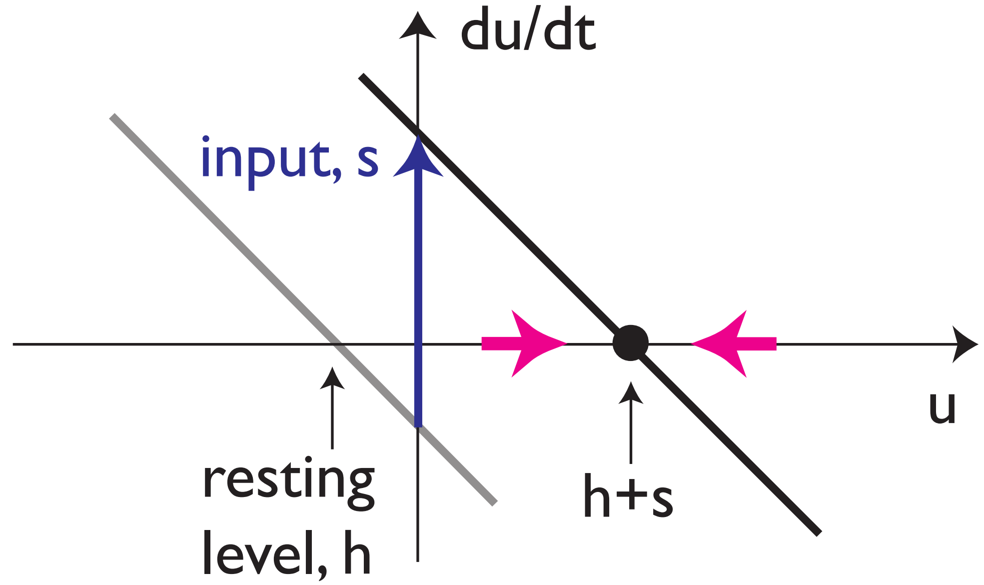
Neural dynamics

- attractor structures ensemble of solutions=flow

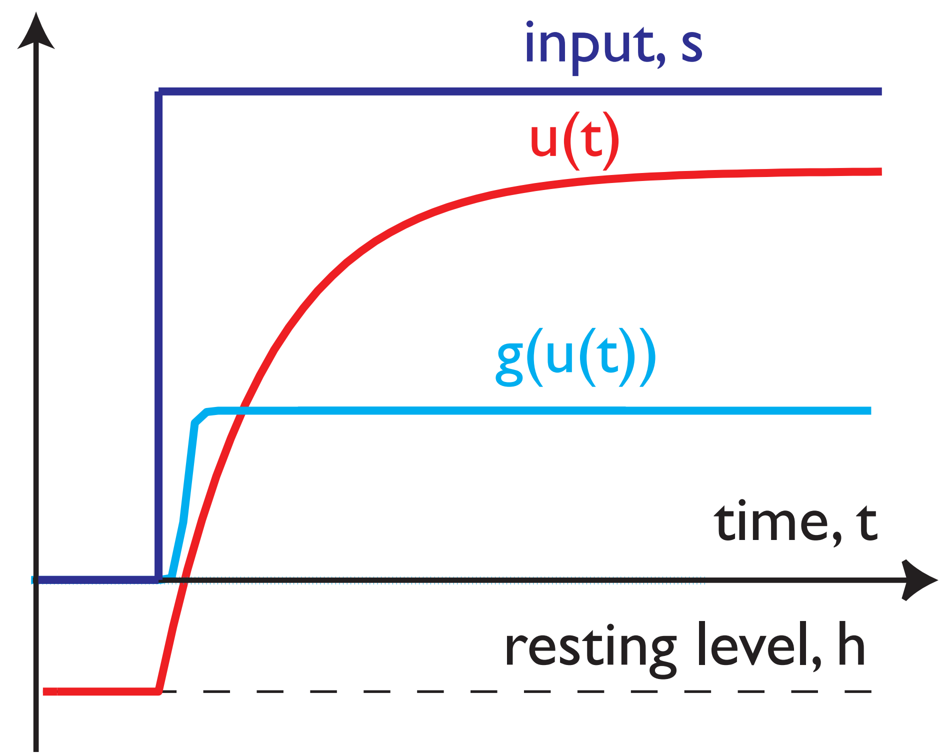


$$\tau \dot{u}(t) = -u(t) + h$$

Neuronal dynamics

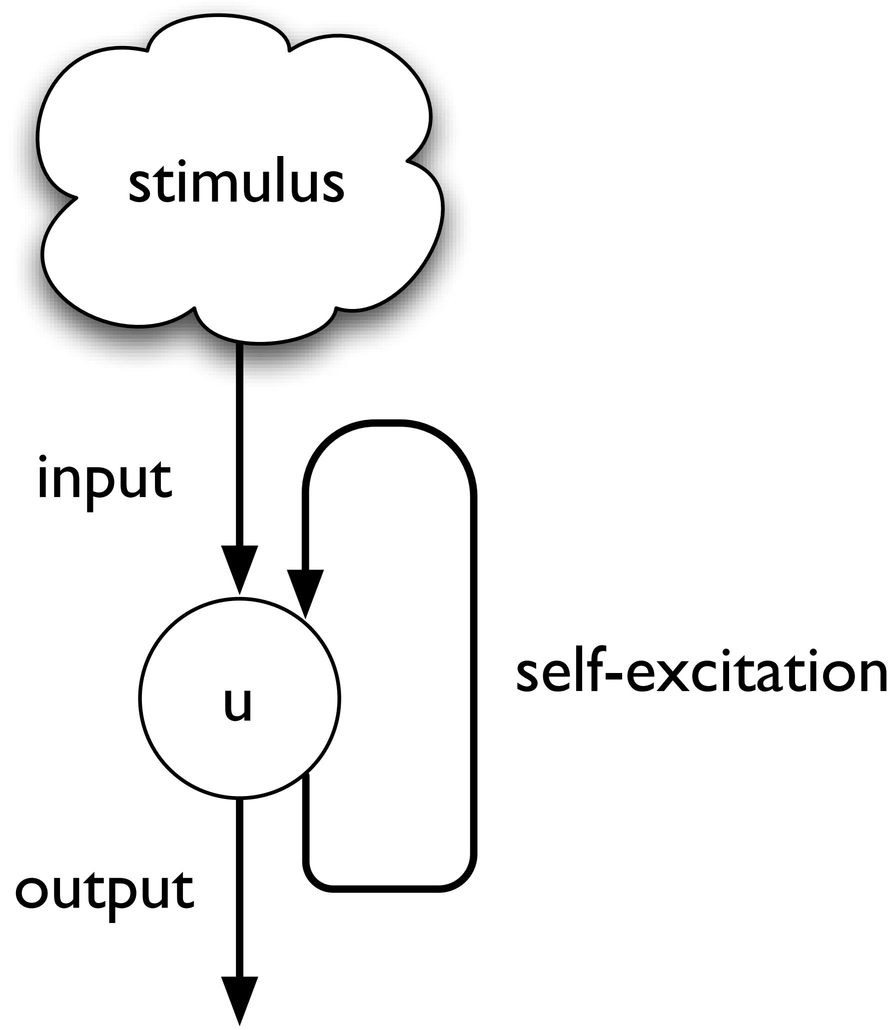


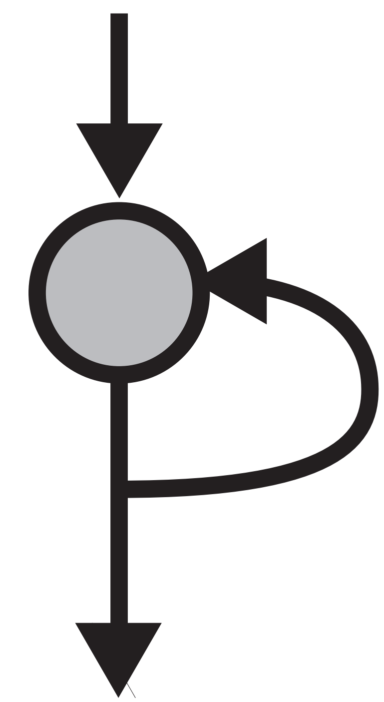
- inputs=contributions to the rate of change
  - positive: excitatory
  - negative: inhibitory
- => shifts the attractor
- activation tracks this shift (stability)



$$\tau \dot{u}(t) = -u(t) + h + \text{ inputs(t)}$$

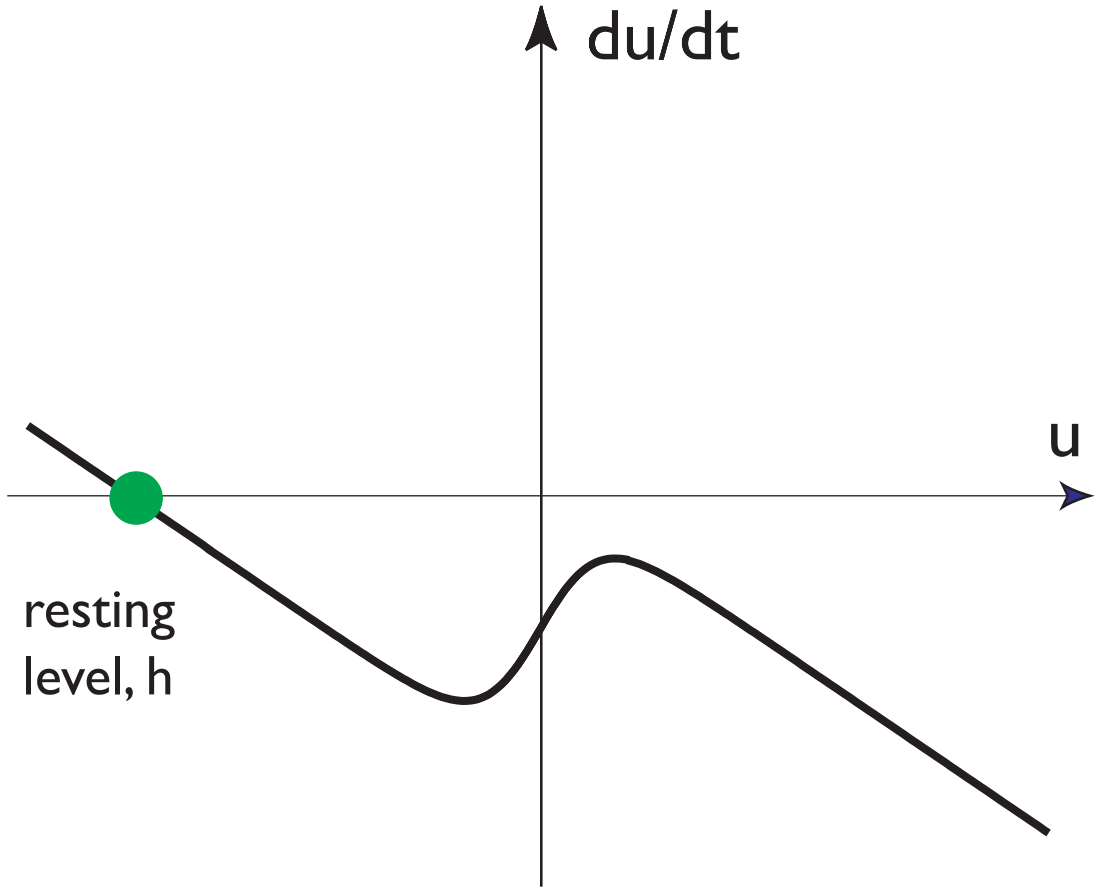
Neuronal dynamics with self-excitation

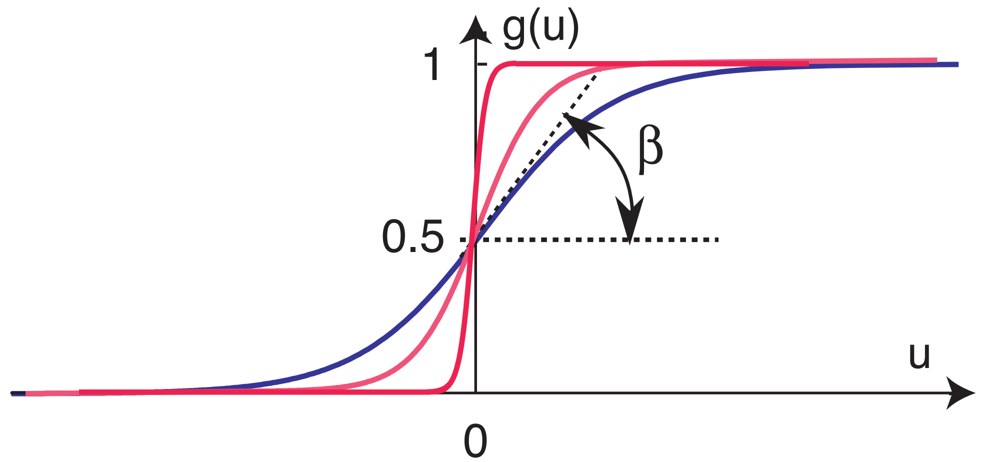




$$\tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t))$$

Neuronal dynamics with self-excitation

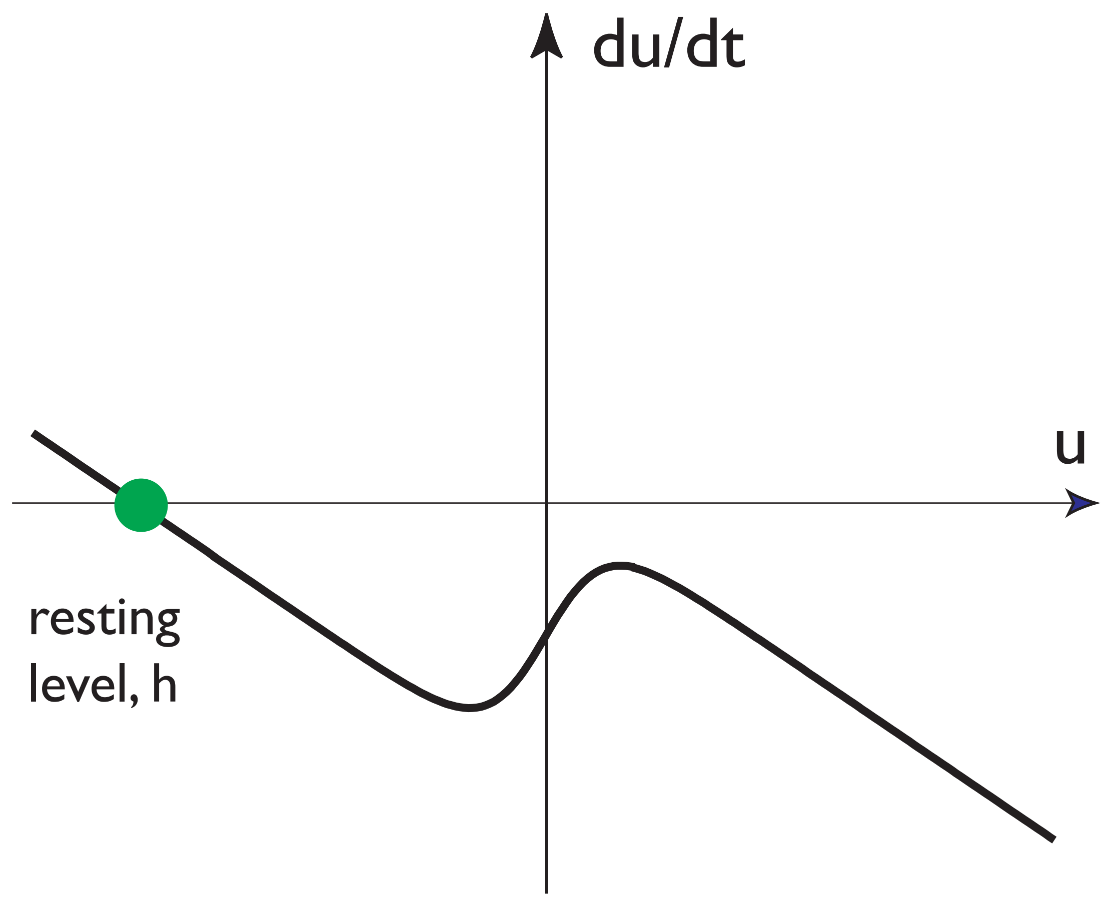




$$\tau\dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t))$$

Neuronal dynamics with self-excitation

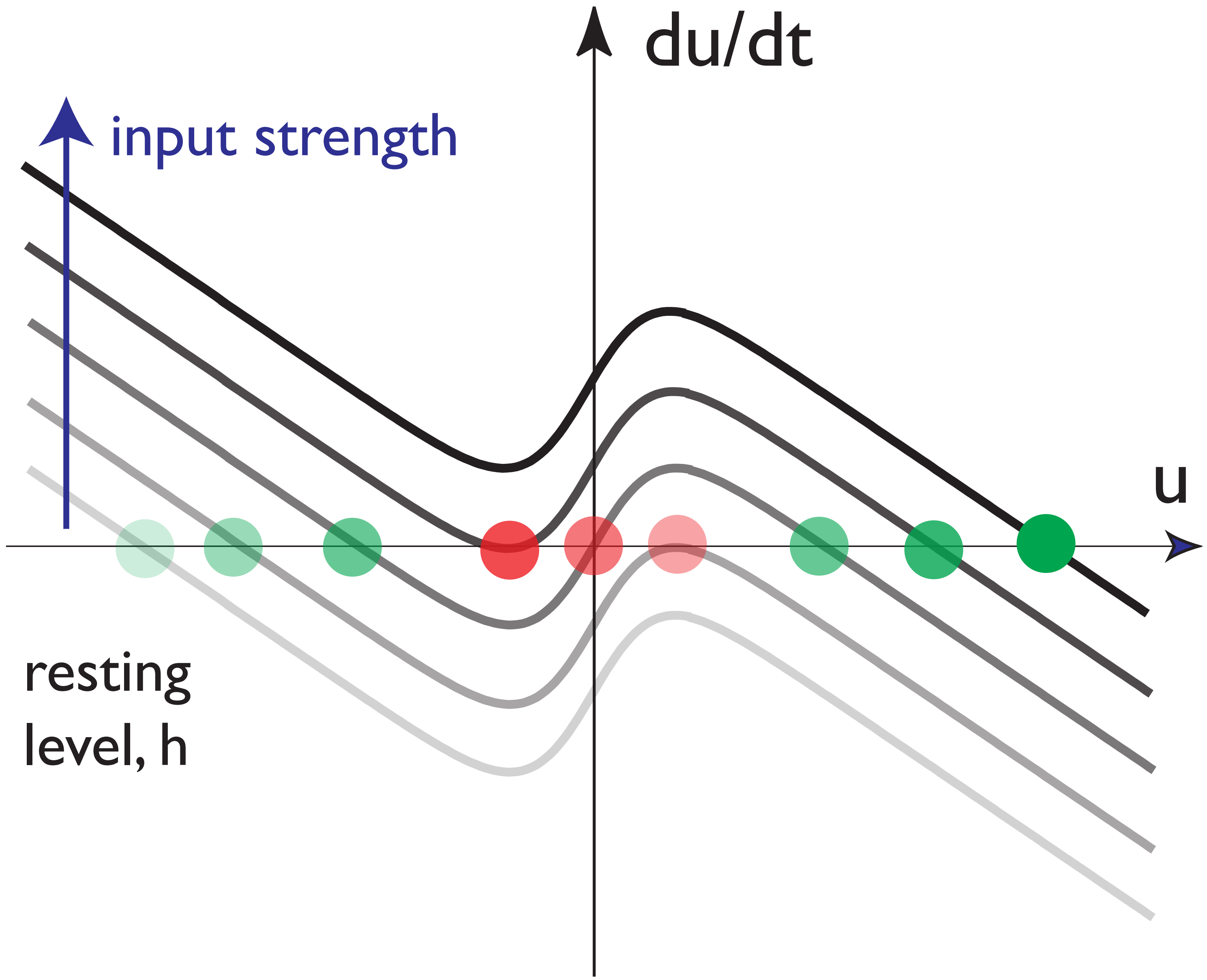
- => this is nonlinear dynamics!



$$\tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t))$$

Neuronal dynamics with self-excitation

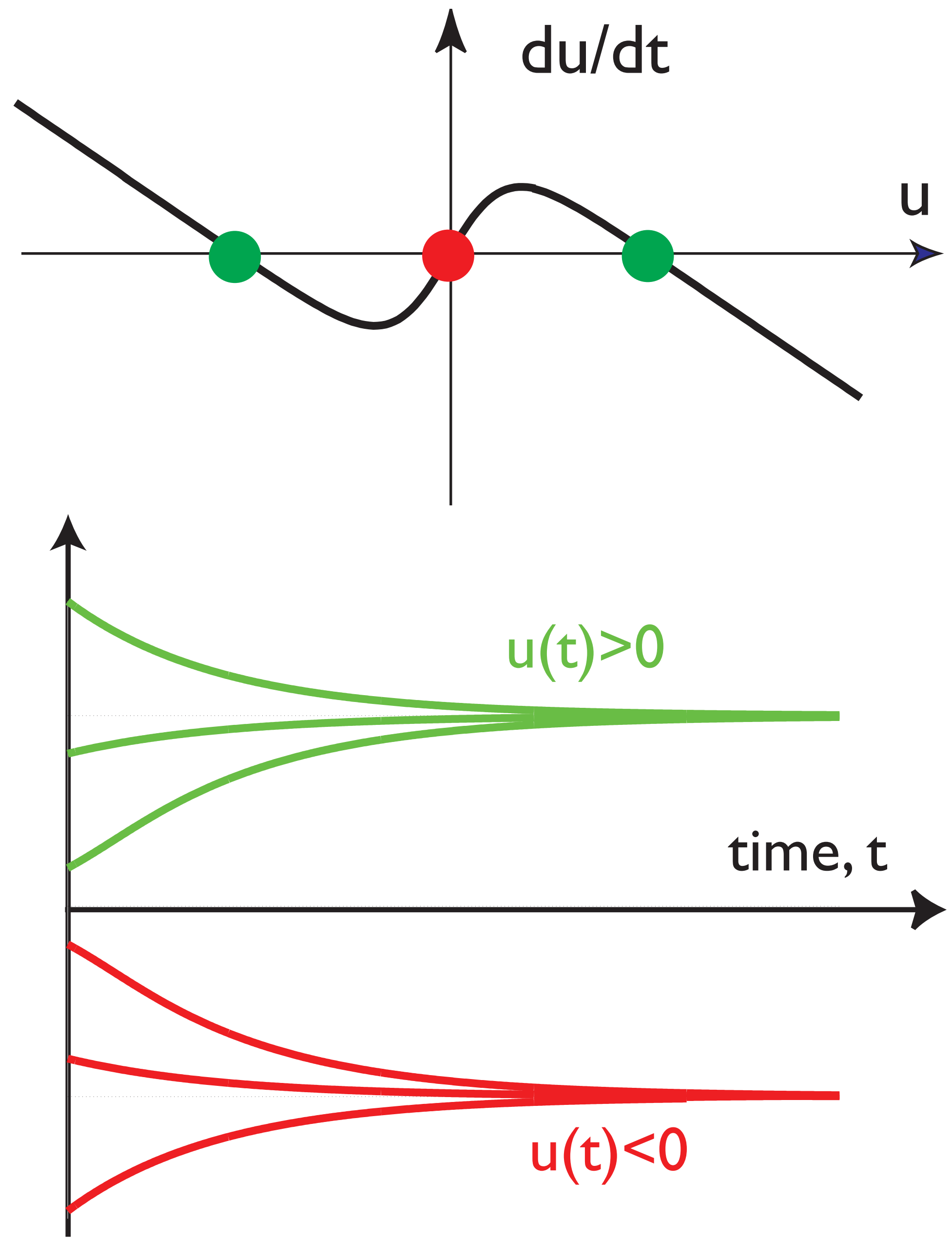
- stimulus input



$$\tau\dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t))$$

Neuronal dynamics with self-excitation

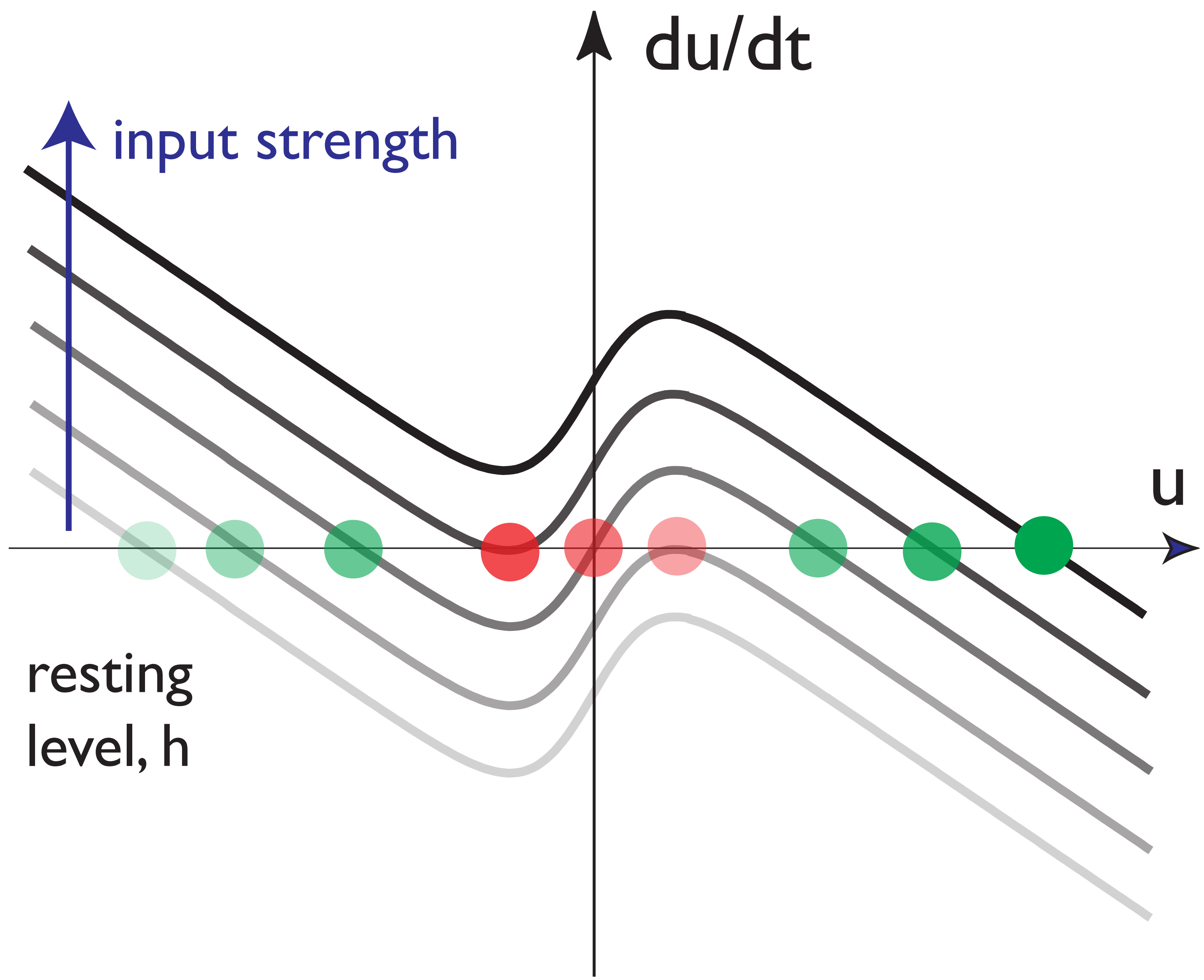
- at intermediate stimulus strength: bistable=> essential nonlinearity



$$\tau \dot{u}(t) = -u(t) + h + S(t) + c\sigma(u(t))$$

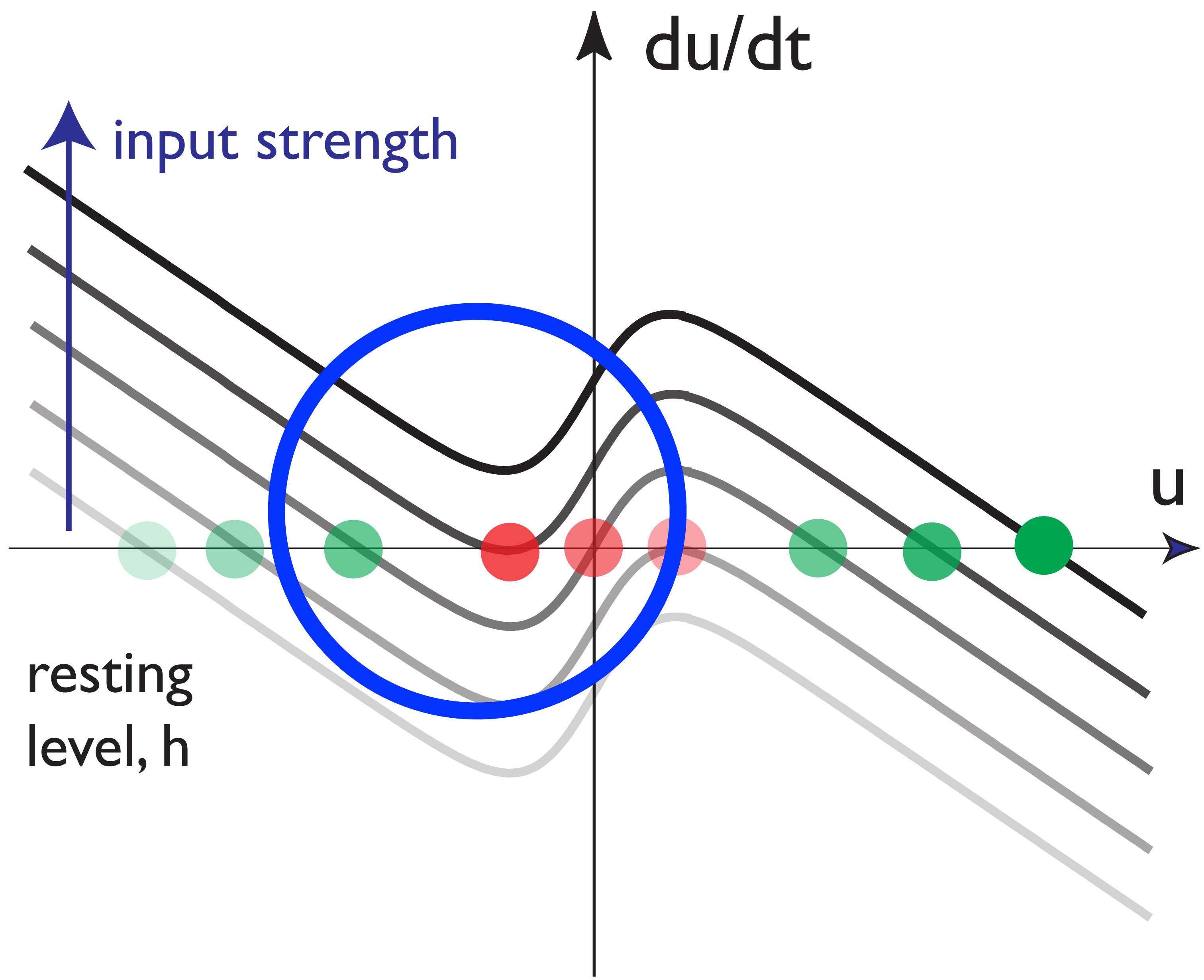
Neuronal dynamics with self-excitation

- with varying input strength system goes through two instabilities: the detection and the reverse detection instability



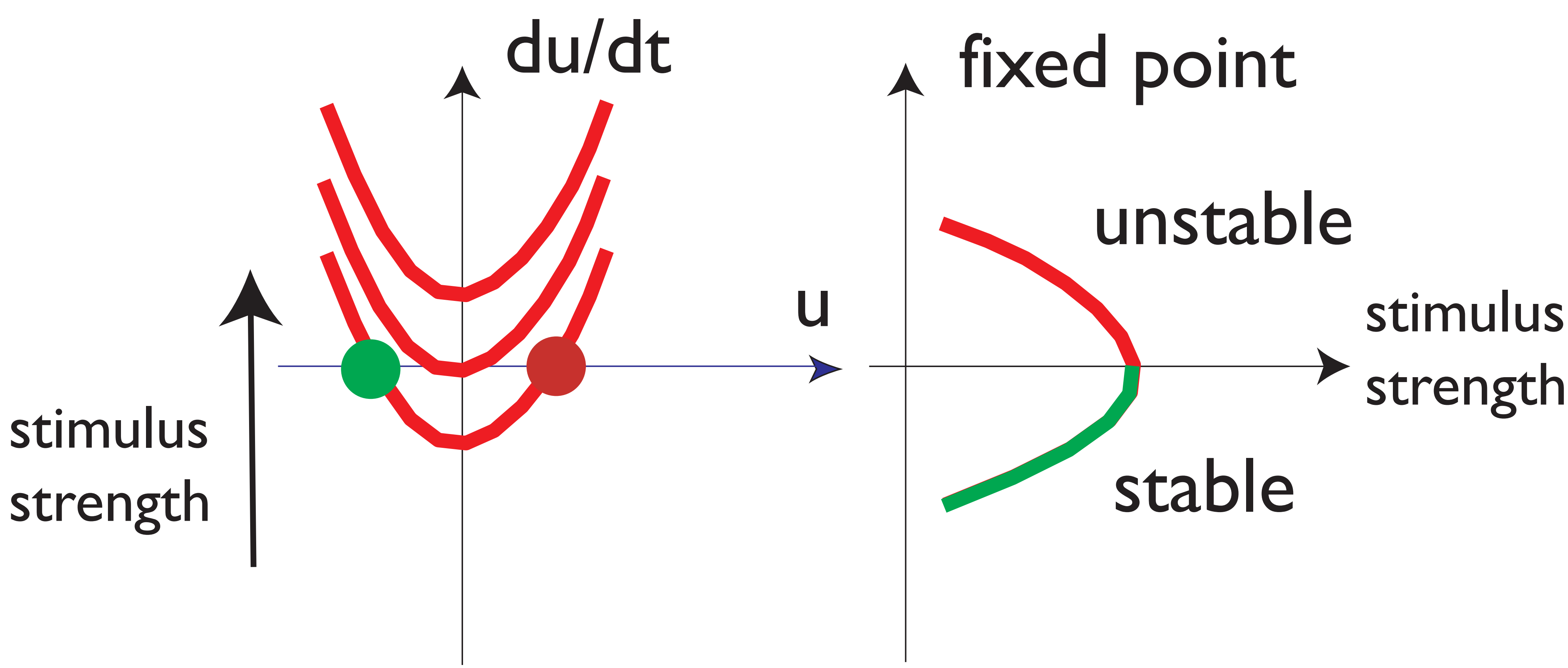
Neuronal dynamics with self-excitation

- with varying input strength system goes through two instabilities: the detection and the reverse detection instability



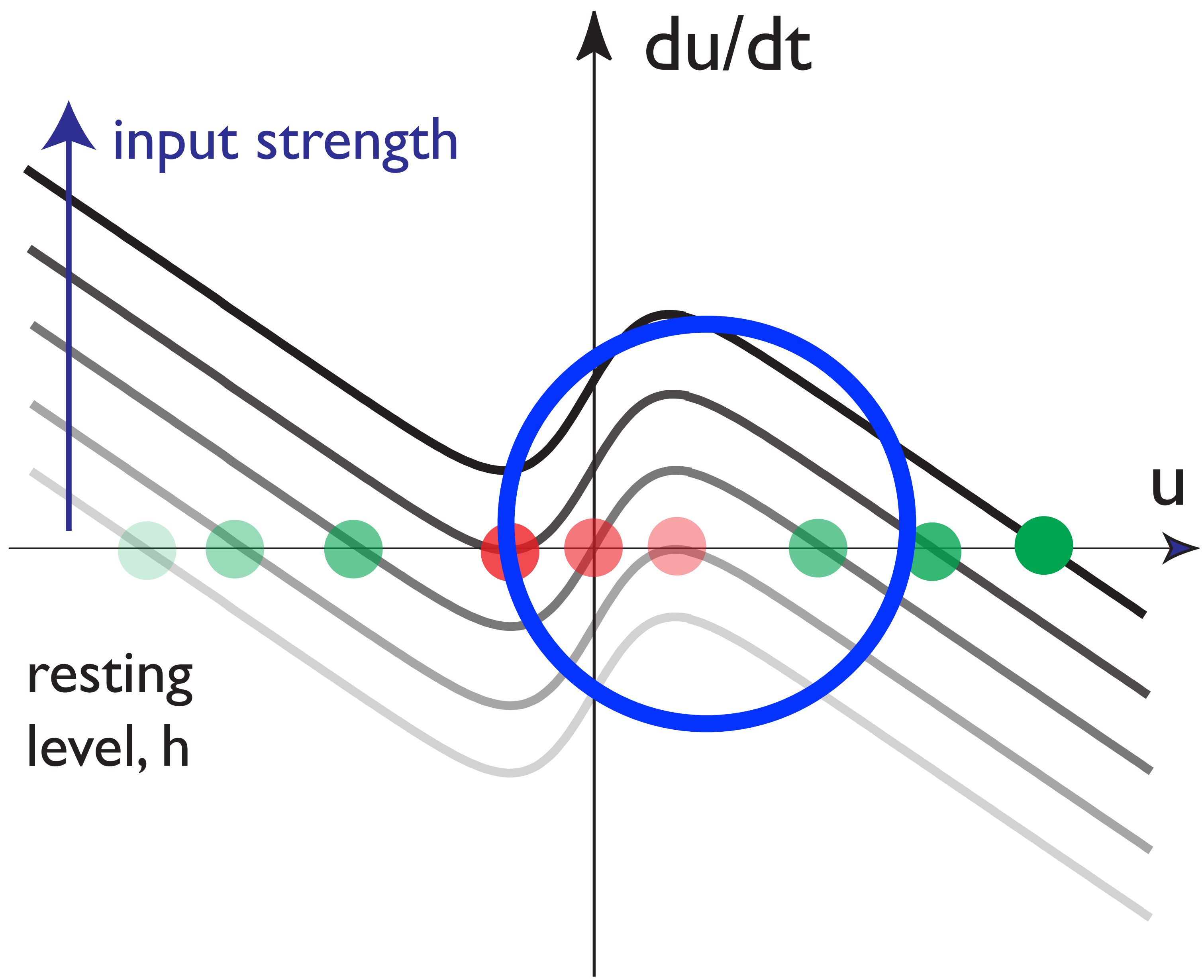
Neuronal dynamics with self-excitation

- detection instability



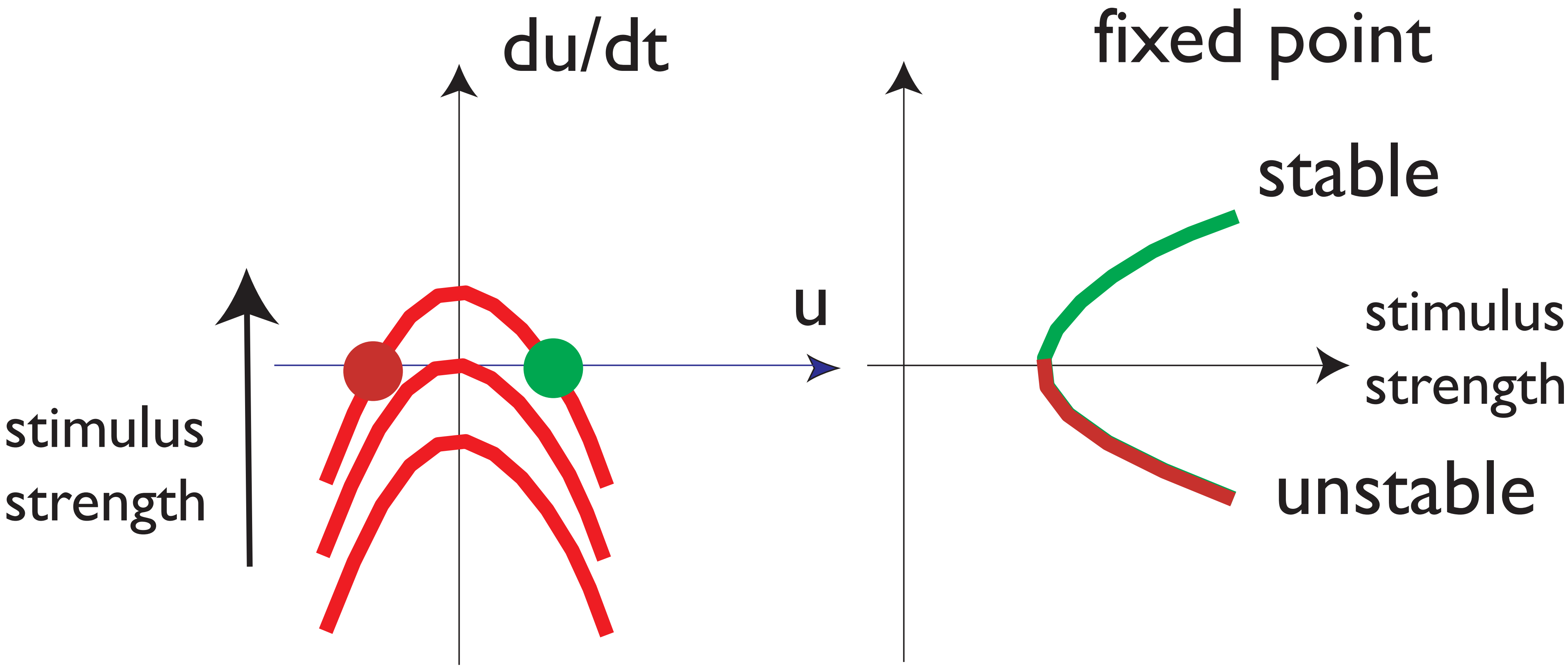
Neuronal dynamics with self-excitation

- with varying input strength system goes through two instabilities: the detection and the reverse detection instability



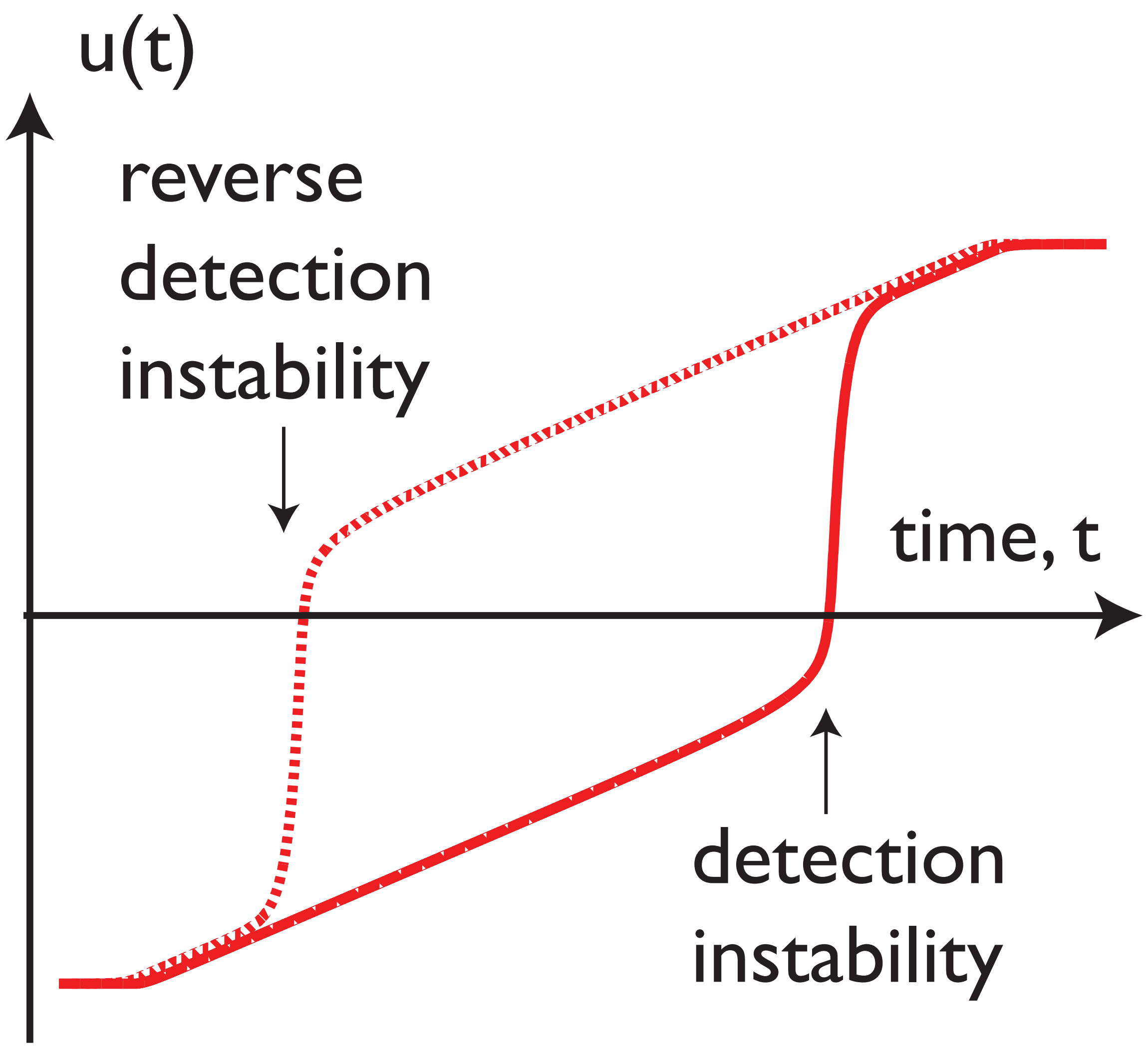
Neuronal dynamics with self-excitation

- reverse detection instability



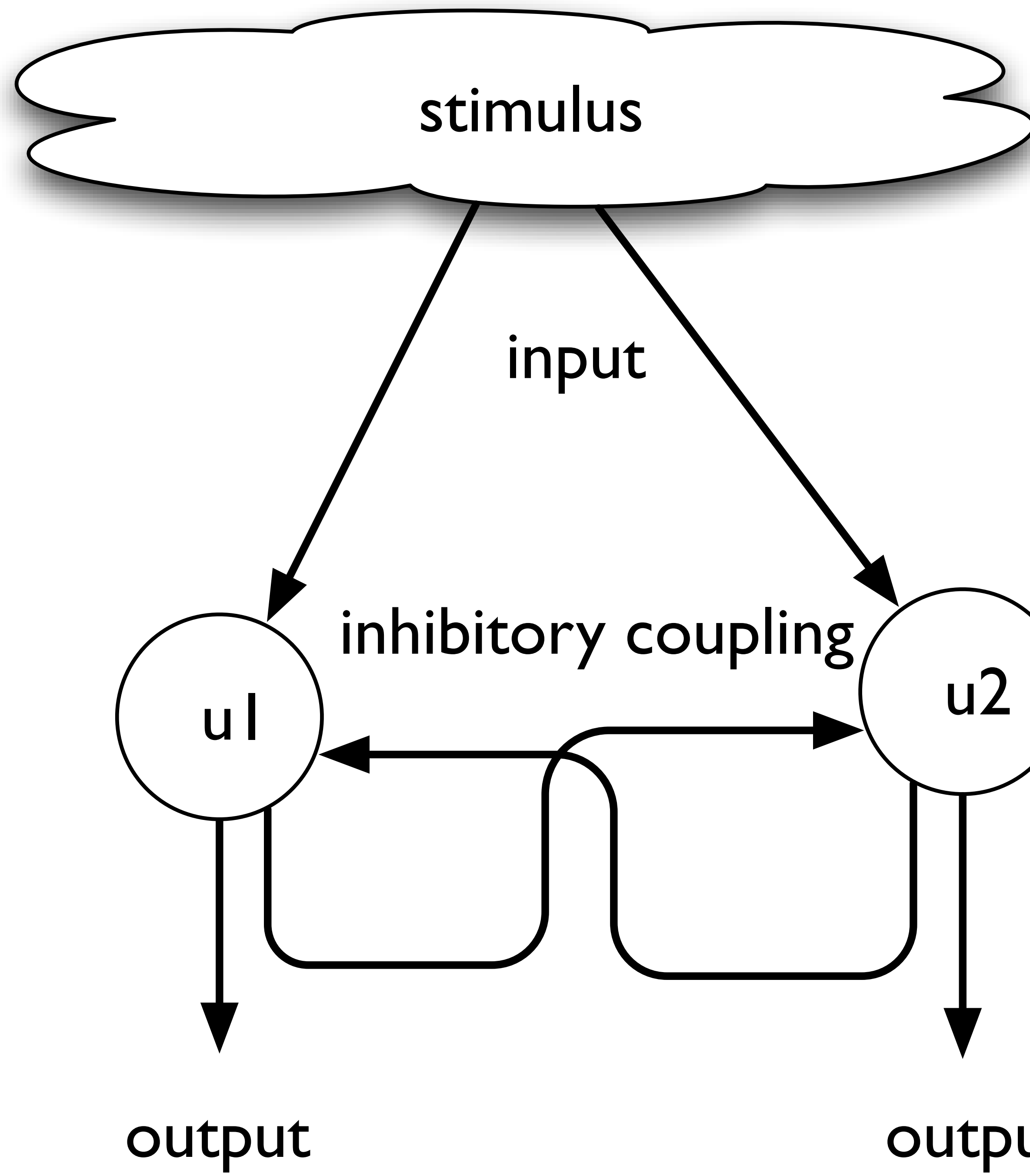
Neuronal dynamics with self-excitation

- signature of instabilities: hysteresis



- => simulation

Neuronal dynamics with competition



$$\tau \dot{u}_1(t) = -u_1(t) + h - \sigma(u_2(t)) + S_1$$
$$\tau \dot{u}_2(t) = -u_2(t) + h - \sigma(u_1(t)) + S_2$$

Neuronal dynamics with competition

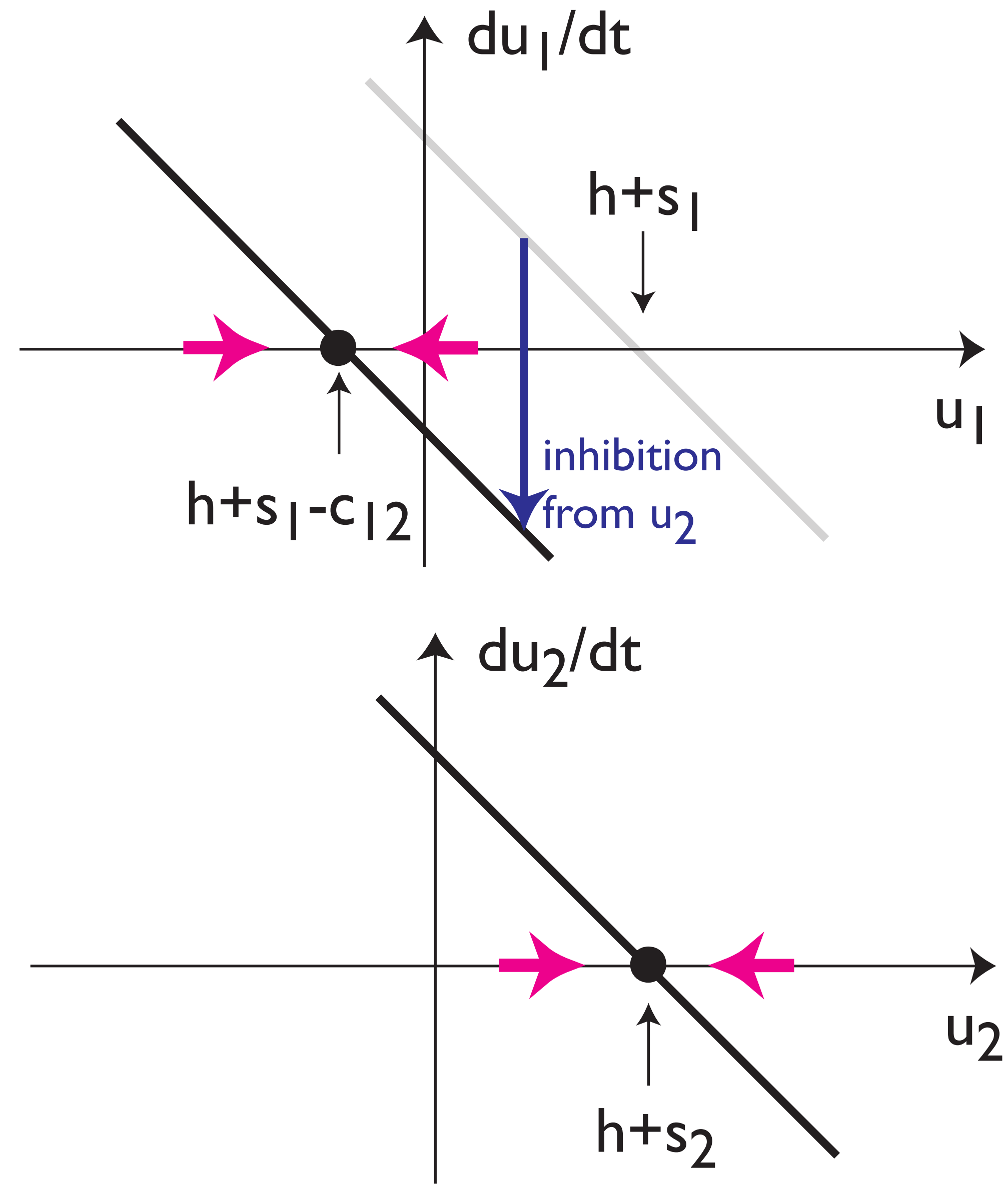
- interaction: the rate of change of activation at one site depends on the level of activation at the other site
- mutual inhibition

$$
\begin{aligned}
\tau \dot{u}_1(t) &= -u_1(t) + h - \sigma(u_2(t)) + S_1 \\
\tau \dot{u}_2(t) &= -u_2(t) + h - \sigma(u_1(t)) + S_2
\end{aligned}
$$

sigmoidal nonlinearity

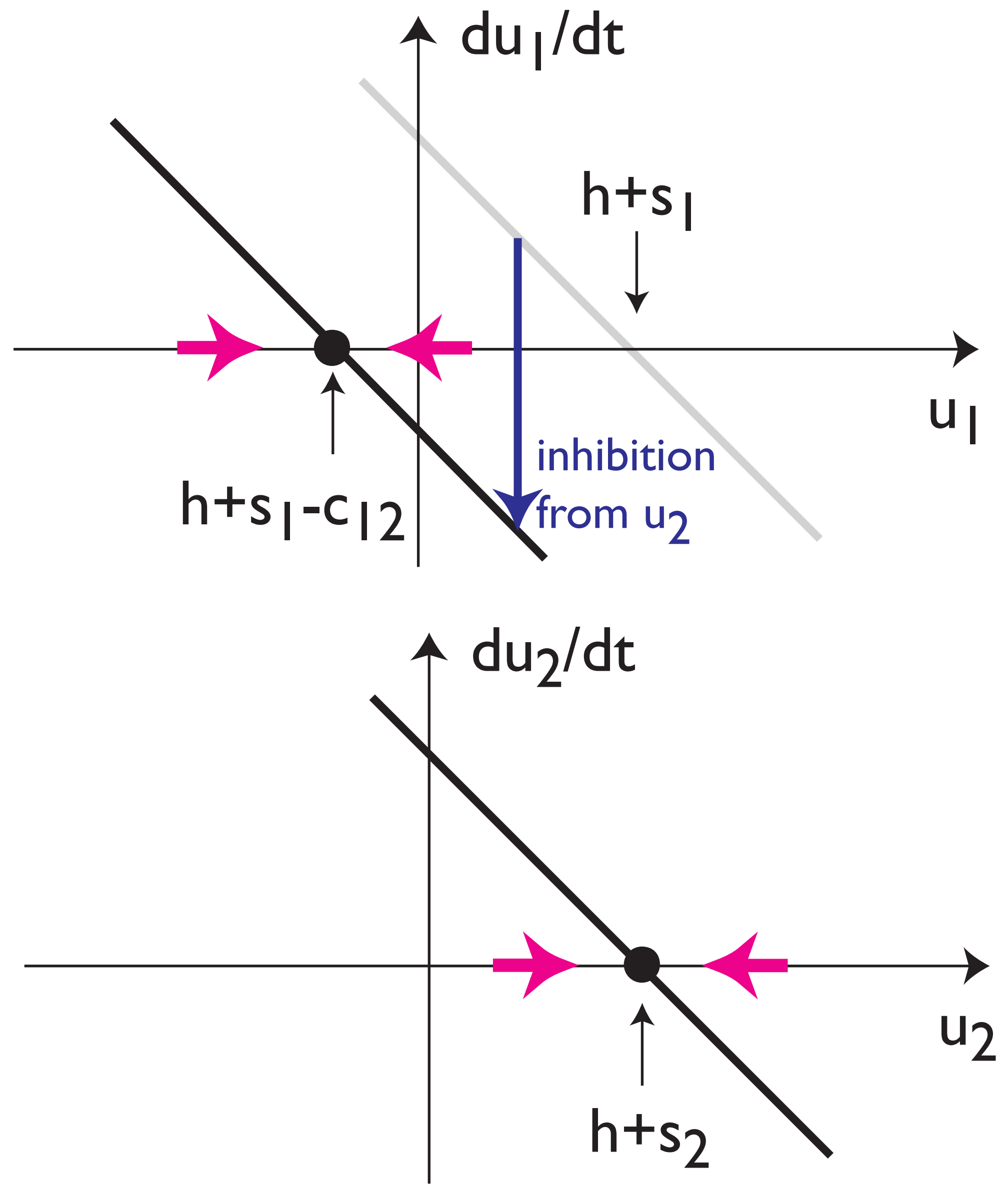
Neuronal dynamics with competition

- to visualize, assume that u_2 has been activated by input to positive level
- => then u_1 is suppressed

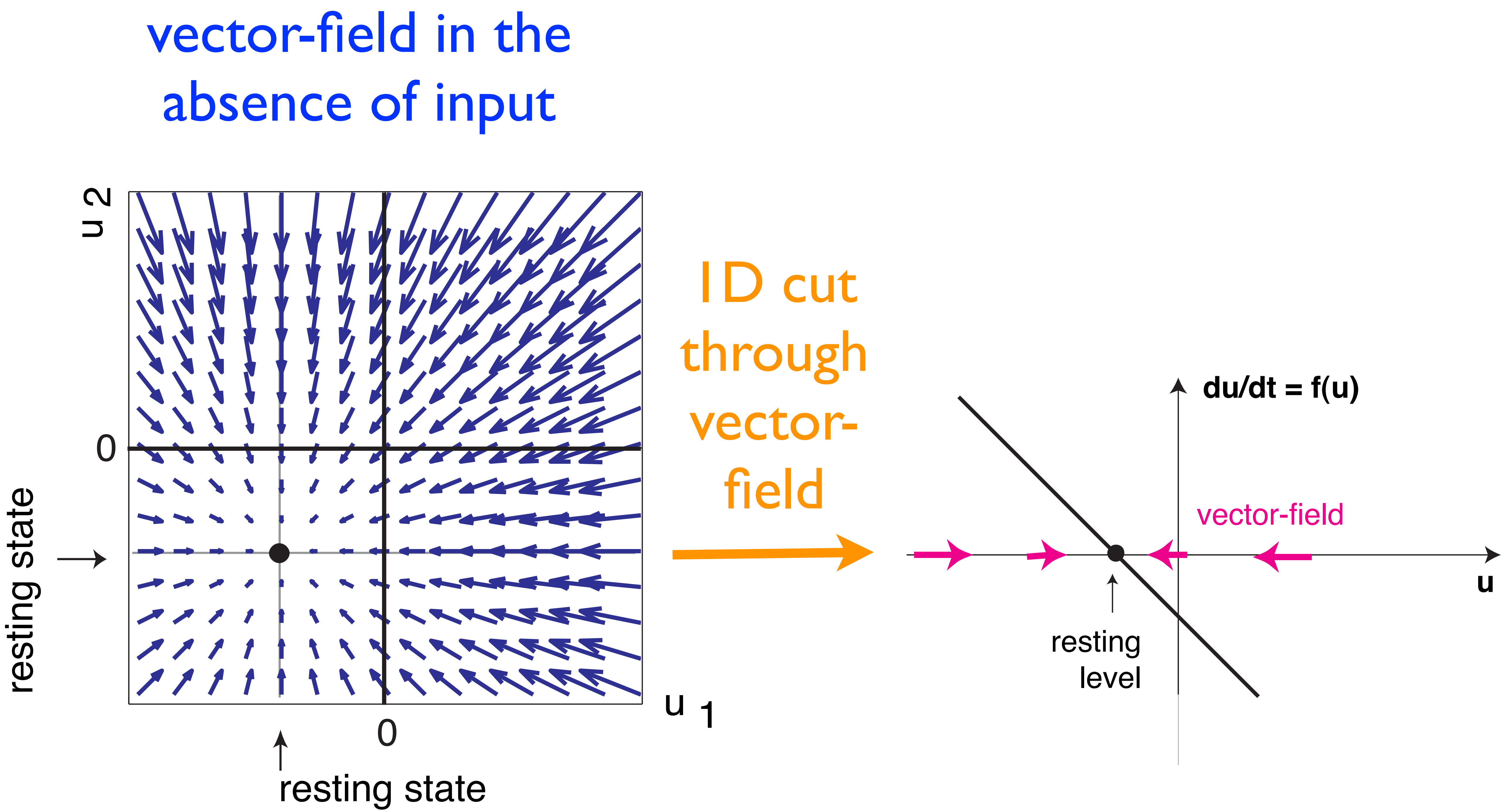


Neuronal dynamics with competition

- why would u_2 be positive before u_1 is? E.g., it grew faster than u_1 because its inputs are stronger/inputs match better
- => input advantage translates into time advantage which translates into competitive advantage

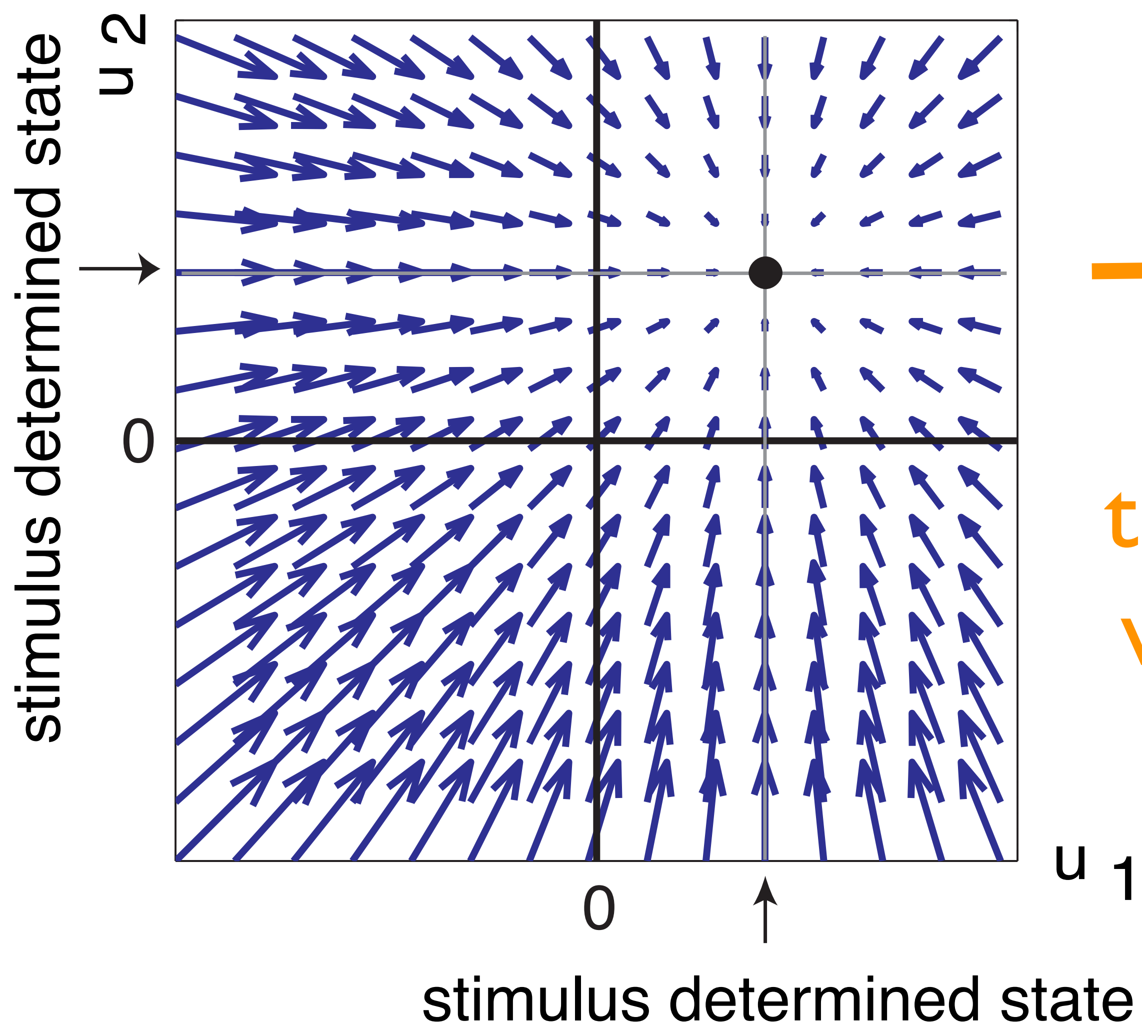


Neuronal dynamics with competition

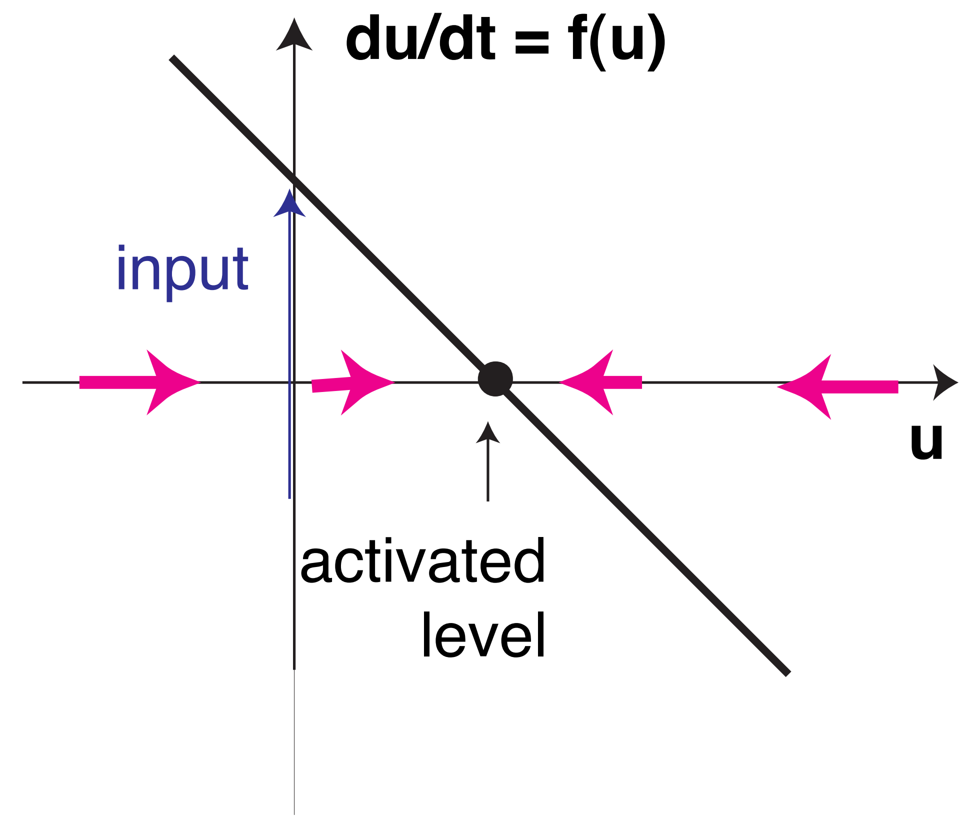


Neuronal dynamics with competition

vector-field (without interaction) when both neurons receive input



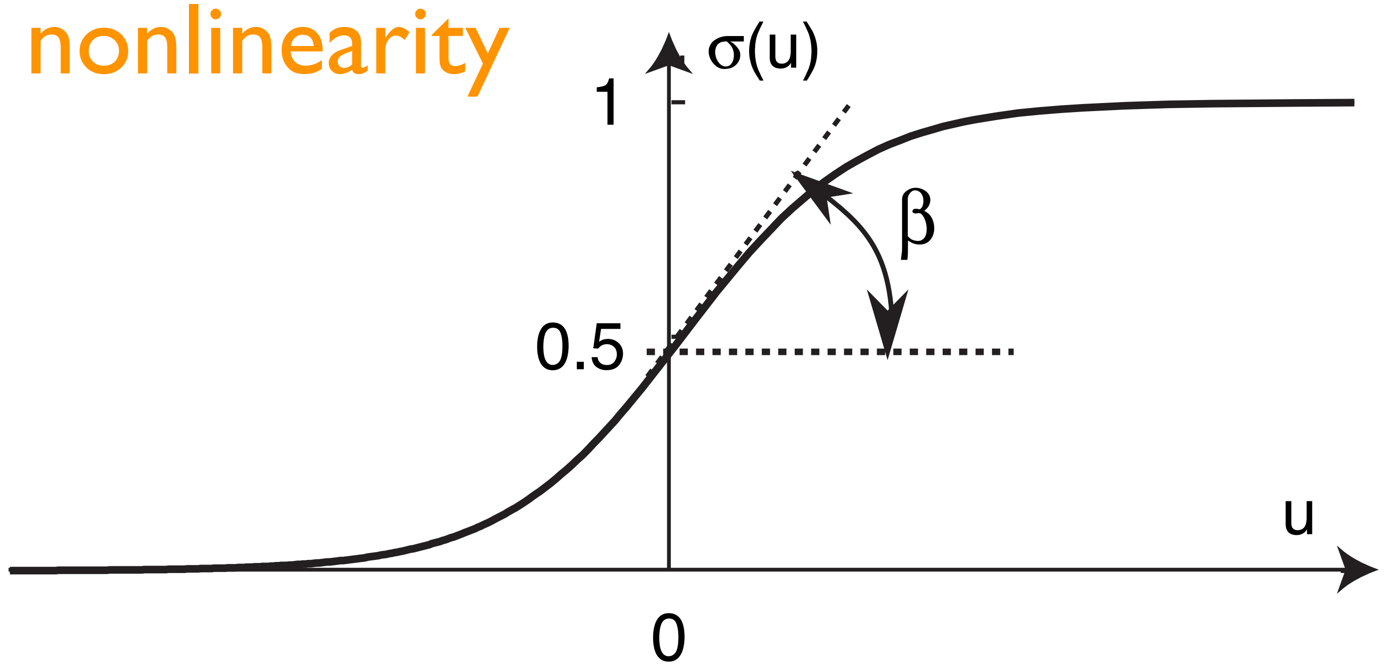
1D cut through vector-field



Neuronal dynamics with competition

- only activated neurons participate in interaction!

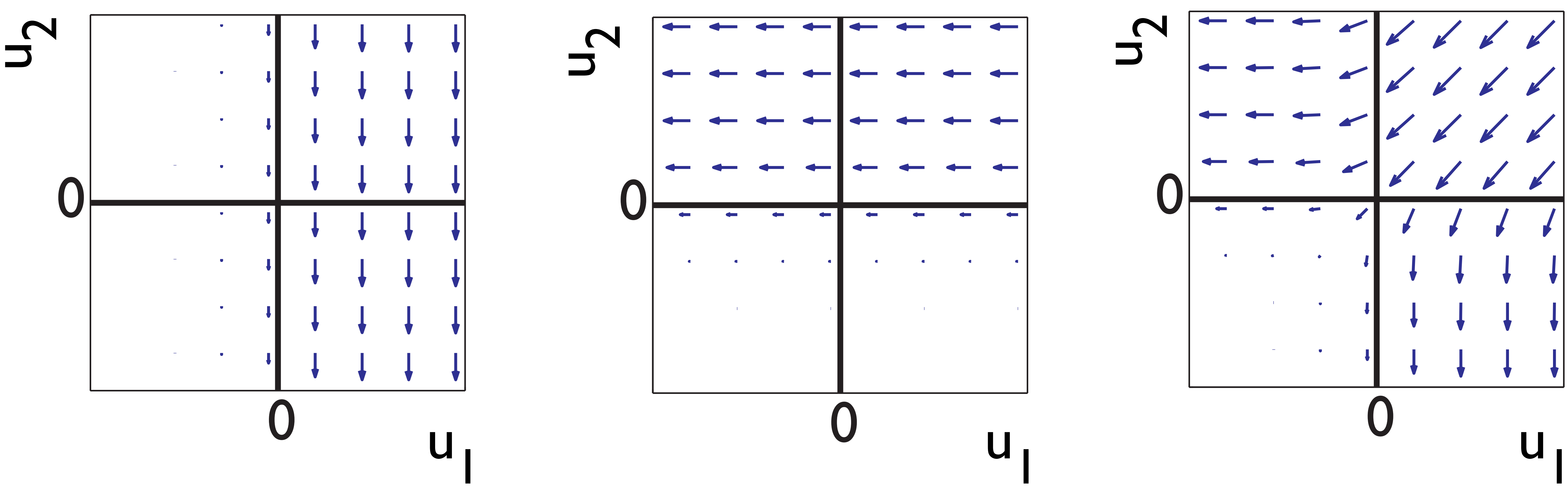
sigmoidal nonlinearity



Neuronal dynamics with competition

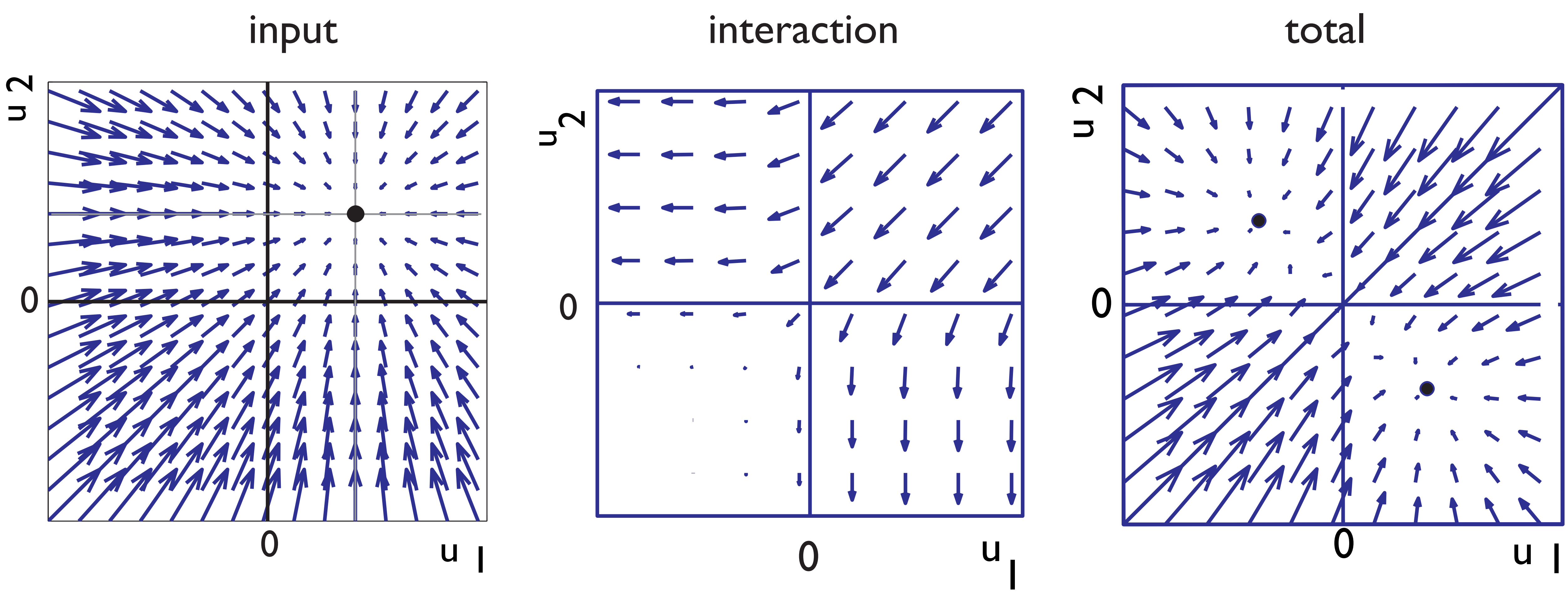
- vector-field of mutual inhibition

site 1 inhibits site 2 | site 2 inhibits site 1 | interaction combined



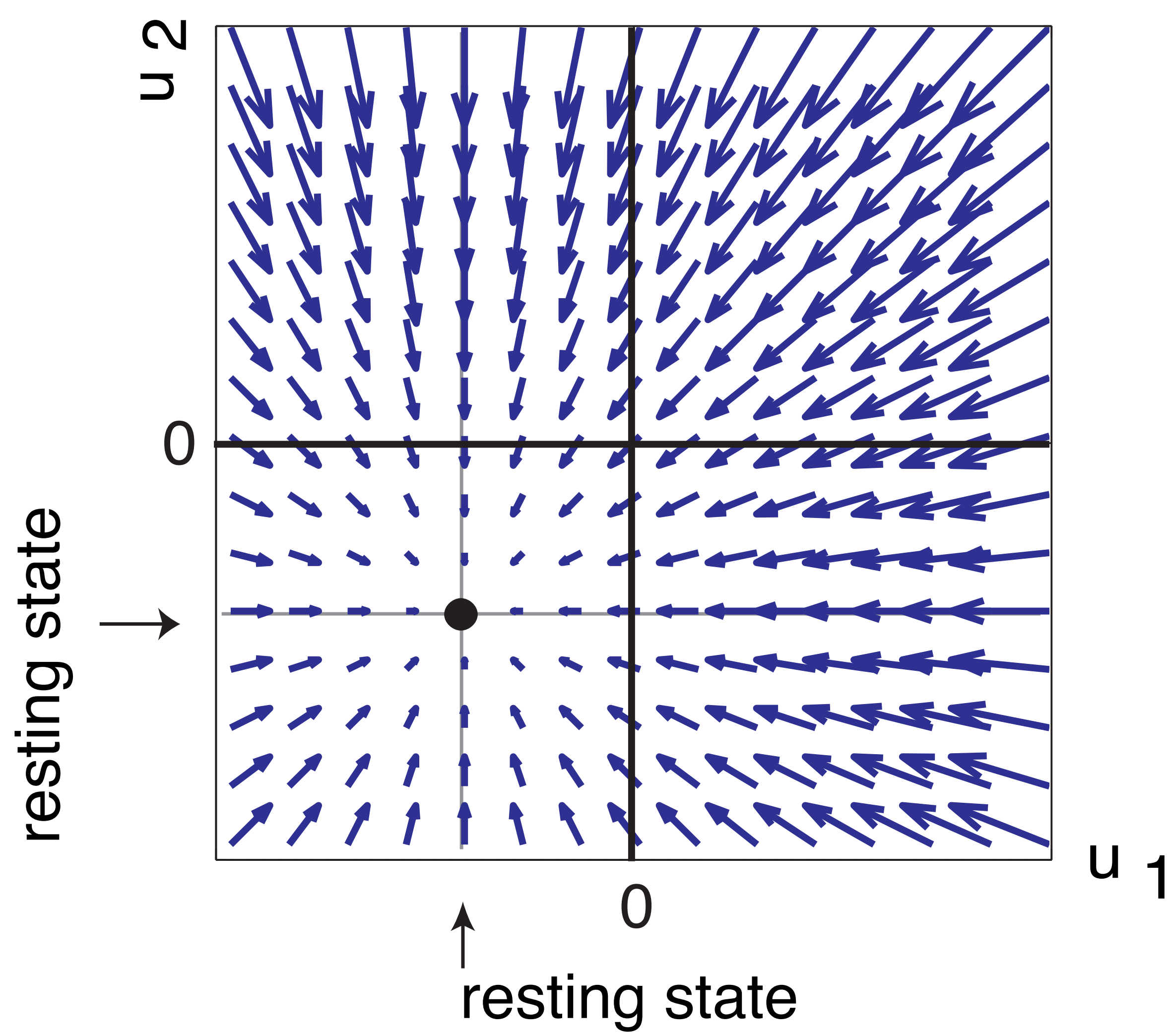
Neuronal dynamics with competition

vector-field with strong mutual inhibition: bistable

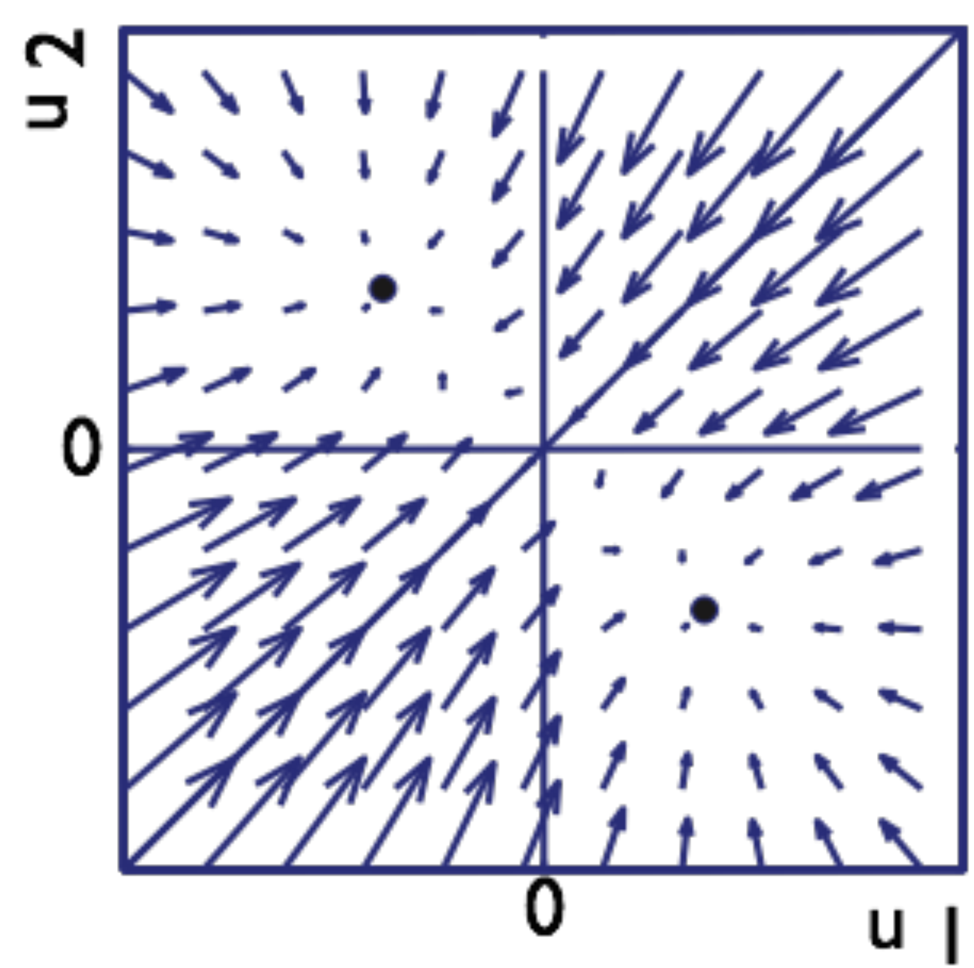


Neuronal dynamics with competition

before input is presented



after input is presented



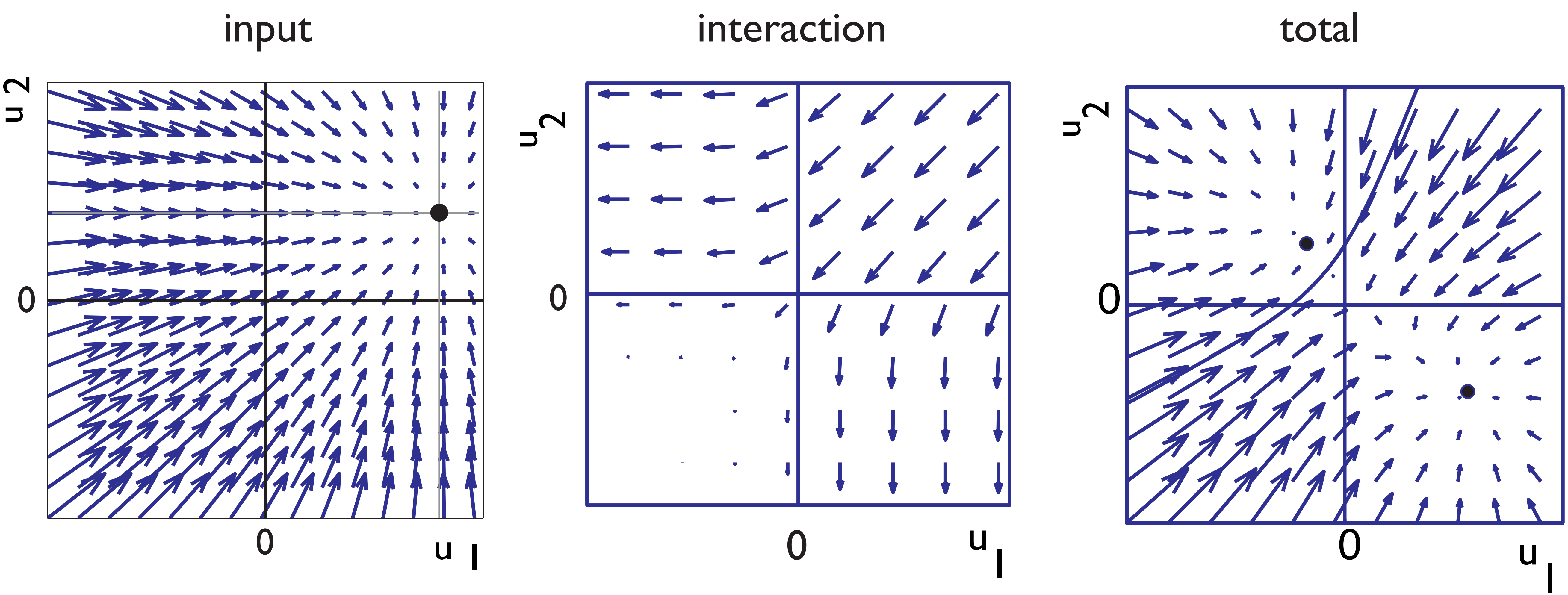
Neuronal dynamics with competition

=>biased competition

stronger input to site 1:

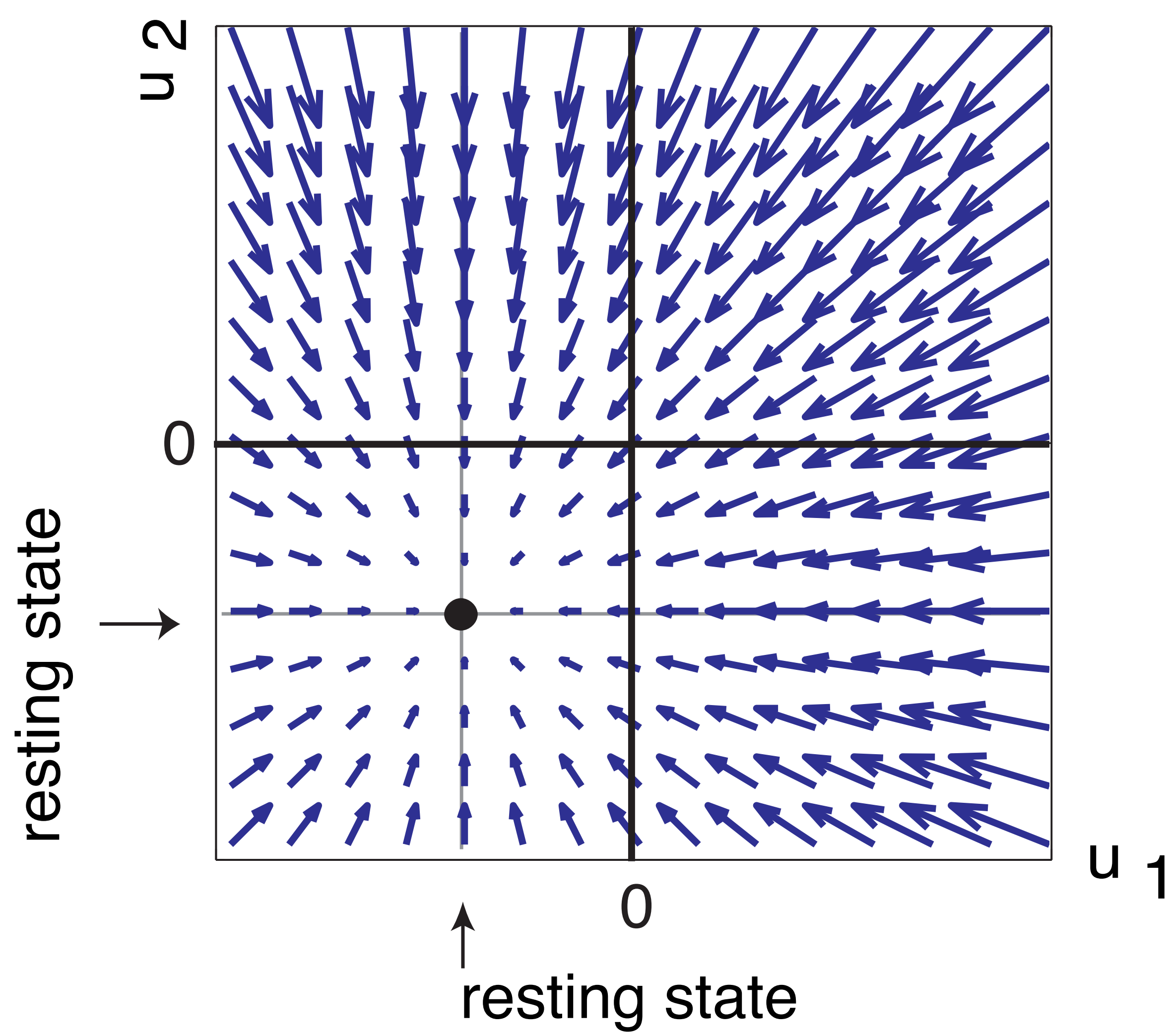
attractor with activated u_1 stronger,

attractor with activated u_2 weaker, may become unstable

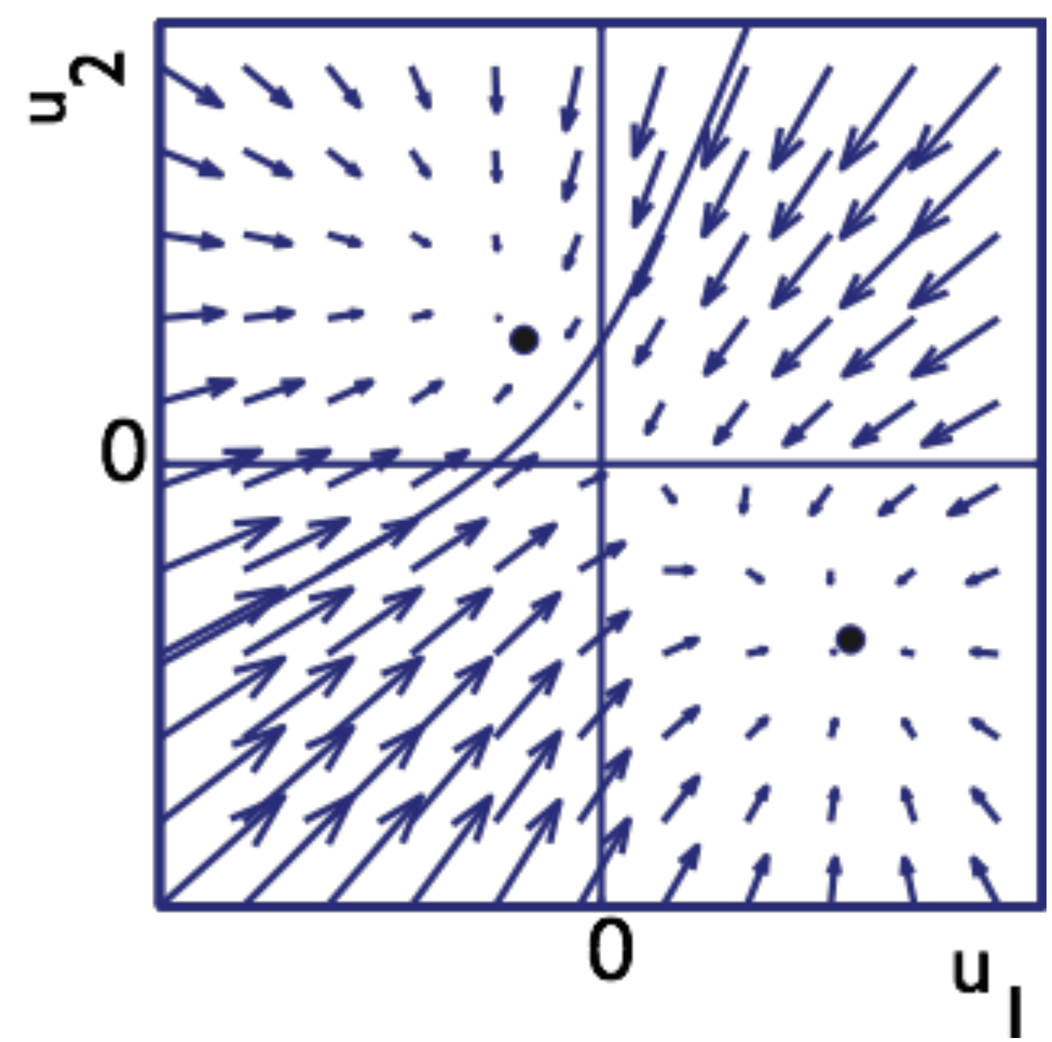


Neuronal dynamics with competition =>biased competition

before input is presented



after input is presented



- => simulation

Outlook

- Where do activation variables come from? How does an activation variable come to "stand" for a behavior or percept ?
- How do discrete activation variables reflect continuous behaviors?
- => DFT lecture