Exercise 2

The interactive web simulator "One Layer Field"[1] solves numerically the dynamic field Eq. 1 with added random noise, repeated here in full detail:

$$\tau \dot{u}(x,t) = u(x,t) + h + s(x,t) + \int k(x-x')\, g(u(x',t))\, dx' + q\xi(x,t), \quad (1)$$

where the sigmoidal function is given by

$$g(u) = \frac{1}{1+\exp(-\beta u)}. \quad (2)$$

The interaction kernel is given by

$$\begin{aligned} k(xx) = & \frac{c_{\text{exc}}}{\sqrt{2\pi}\sigma_{\text{exc}}} \exp\left[-\frac{(x-x')^2}{2\sigma_{\text{exc}}^2}\right] & (3) \\ & -\frac{c_{\text{inh}}}{\sqrt{2\pi}\sigma_{\text{inh}}} \exp\left[-\frac{(x-x')^2}{2\sigma_{\text{inh}}^2}\right] & (4) \\ & -c_{\text{glob}}. & (5) \end{aligned}$$

Note that in this formulation of the kernel, the amplitudes of the two Gaussian components are normalized, such that a change in the interaction widths σ does not change the total strength of the interaction. Localized input is supplied in the form

$$s(x,t) = \sum_i a_i \exp\left[-\frac{(x-p_i)^2}{2w_i^2}\right]. \quad (6)$$

Sliders at the bottom of the graphical user interface provided by the program enable one to control the widths, w_{si}, locations p_{si}, and amplitudes a_{si}, of three such inputs ($i = 1, 2, 3$) . Sliders are also available to vary the parameters $h, q, c_{\text{exc}}, c_{\text{inh}}$, and c_{glob}. Predefined sets of parameter values can be loaded by making a selection in the drop-down menu on the bottom right.

[1] `https://dynamicfieldtheory.org/examples/one_layer_field.html`

The state of the field is shown in the top set of axes. The blue line shows the current distribution of activation, $u(x, t)$. The green line is the input shifted by the resting level, $h + s(x, t)$, and the red line shows the field output (sigmoidal function of the field activation) at each position, $g(u(x, t))$, scaled up by a factor of 10 for better visibility. In the bottom set of axes, the shape of the interaction kernel is displayed. Note that the kernel is plotted over distances in the feature dimension, with zero at the center of the plot. This interaction pattern is then applied homogenously for all positions in the field. The goal of this exercise is to explore and reproduce different instabilities of the dynamics.

Task 1: Detection Instability

This task works best with the predefined parameter set "stabilized". Start out with the field in the resting state (the default) and introduce a localized input by increasing one of the stimulus amplitudes. For small input strengths, observe how the field (blue line) tracks the changing input (green line); this is the subthreshold solution. When activation first reaches zero from below, the field output at that location rises (red line). Observe how at this point very small changes in input strength lead to a new solution, the self-stabilized peak, which has more activation at its peak than input (blue line exceeds green line).

1. Show that, up to the detection instability, the system is bistable, by lowering input again to a level at which you previously saw the subthreshold solution. You can reset the field to the initial condition by pressing the Reset button. You will find that from the resting level the field converges to the subthreshold solution again.

2. While a self-stabilized peak stands in the field, move the inducing input laterally with the slider that changes the location of the input function. If you do this slowly enough, the peak will track input. If you do this too fast, the peak disappears at the old location in a reverse detection instability and reappears at the new location in a detection instability.

3. After having induced a peak again by increasing localized input, observe the reverse detection instability by lowering the input strength gradually. Close to where activation reaches zero from above you may observe the collapse of the self-stabilized peak and a quick relaxation to the subthreshold solution.

Task 2: Memory Instability

Vary the resting level h, increasing it step-wise. At each level, induce a peak as in the first task and then try to destabilize it through the reverse detection instability by returning localized input strength to zero. At a critical value of the resting level, you will find that the peak decays slowly, then not at all after you have returned the localized input strength to zero. This is the memory instability, leading to a regime in which peaks can be sustained without localized input.

1. You can load a convenient parameter set within the memory regime by selecting the predefined parameter set "memory". Induce a peak, remove localized input, then reintroduce this input in a location close to the sustained peak. In which way is the peak updated?

2. Do the same, but now reintroduce input at a location far from the sustained peak. What happens?

Task 3: Selection

Choose the predefined parameter set "selection". Provide two localized inputs by increasing two stimulus amplitudes to intermediate values (between 6 and 8). Observe how only the location first receiving input develops a peak.

1. Increase input strength at the second location until you observe the selection instability.

2. Return that input strength to the original values. Show that the system is bistable.

3. Do the symmetric exercise, increasing input strength at the first location.

4. Adjust two input strengths to be exactly the same, making sure that there is some random noise in the field ($q > 0$). Use the Reset button to restart the field from the resting level. Observe how one of the two locations with input is selected. Repeat several times and convince yourself that selection is stochastic.

Task 4: Boost-Induced Detection

Supply small subthreshold input that is not sufficient to induce peaks at three locations. Then slowly increase the resting level until a detection instability is triggered somewhere in the field. Observe how a peak is generated at one of the three locations that have small input. Try to see how small you can make that localized input and still observe the peak at one of the three locations. You can do this with or without noise.