Exercise 1

All tasks of this exercise use the interactive web simulator "Two Neurons"[1]. This program simulates two activation variables, informally called neurons, with external input, self-excitation, interaction, and noise, as defined by the equations

$$\tau_1 \dot{u}_1(t) = u_1(t) + h_1 + s_1(t) + c_{11} g(u_1(t)) + c_{12} g(u_2(t)) + q_1 \xi_1, \quad (1)$$
$$\tau_2 \dot{u}_2(t) = u_2(t) + h_2 + s_2(t) + c_{22} g(u_2(t)) + c_{21} g(u_1(t)) + q_2 \xi_2. \quad (2)$$

In the initial parameter setting most of these terms are set to 0 so the actual behavior of each neuron is that of a single dynamic activation variable without self-excitation and input. You will use this simulator to get a more practical understanding of the role of each parameter for the system as a whole.

The sliders on the right side of the simulator are used to modify the parameters of the dynamical system. The sliders on the top right modify the resting level h_1, the self-excitation strength c_{11}, the connection strength between neurons c_{12}, the variance of the noise q_1, the stimulus strength $s_1(t)$, and the time scale parameter τ. The slides on the bottom modify the analogous parameters for the second neuron. Note the naming convention for interactions between different activation variables or fields that is used through-out our work. The parameters for such interactions have a two-character index (e.g., c_{12}), with the first character specifying the target of the interaction (here, activation variable u_1) and the second character specifying its source (here, activation variable u_2).

The two right-most sets of axes show phase plots for the two activation variables. The red line shows the rate of change for different activation values, as specified by Eq. 1 and 2. The red dot indicates the current activation value and current rate of change of the activation variable, and attractor and repellor states in the dynamics are marked as squares and diamonds, respectively. The two sets of axes in the middle contain trajectory plots, showing the recent history of activation states for the two variables, with the

[1] `https://dynamicfieldtheory.org/examples/two_neurons.html`

present state indicated by the blue dot. The single set of axes on the left shows the trajectories of the two activation variables combined by plotting the activation of one variable against the activation of the other one, both for the current state (blue dot) and recent history (blue line).

Task 1: Single Dynamic Activation Variable with Input

Use the simulator to explore the dynamics of a single activation variable with variable input, as specified by

$$\tau_1 \dot{u}_1(t) = u_1(t) + h_1 + s_1(t) + q_1 \xi_1.$$

1. **Tracking:** Explore how the activation variable tracks a shifting input. Use the s_1 slider to set the input parameter to different values and observe how the zero-crossing of the phase plot of u_1 is shifted around. Observe how the state variable tracks the input by relaxing to the new attractor, both in the trajectory plot and the phase plot.

2. **Relaxation time:** Note how the state changes faster initially when the distance to the new attractor is larger, but the overall shape of the relaxation curve is always the same. Compare relaxation times for different values of τ: Set τ_2 to a value that is significantly different from the value of τ_1. Use the same resting level and non-zero stimulus for u_1 and u_2, then Reset both activation variables to observe the differences in relaxation time. Do this for several different parameter settings.

Task 2: Dynamics of a Single Activation Variable with Self-Excitation

Explore the dynamics of a single neuron with self-excitation, as specified by

$$\tau_1 \dot{u}_1(t) = u_1(t) + h_1 + s_1(t) + c_{11} g(u_1(t)) + q_1 \xi_1.$$

For this task, set the relaxation time parameters τ of both activation variables back to their initial values, $\tau_1 = \tau_2 = 20$, and set the resting levels back to $h_1 = h_2 = -5$. Start with a stimulus amplitude of zero.

1. **Detection:** Increase the self-excitation strength c_{11}, of the activation variable to a medium value and note the nonlinearity emerging in the

phase plot. Move the system through the detection instability by increasing the stimulus amplitude systematically. Move the system back through the reverse detection instability by decreasing the stimulus.

2. **Hysteresis:** Modify the self-excitation and stimulus to put the system u_1 into the bistable regime, then copy the parameter values to u_2 in order to create two identical systems. Demonstrate the hysteresis effect of this system by temporarily varying the stimulus of one system. After resetting the stimulus to the old value, the activation variables of these two identical systems should relax to different attractors.

3. **Perturbations:** Find parameter settings for a bistable system with moderate self-excitation, reset the system, and let it relax to the off-attractor. Subject the system to a random perturbation by temporarily adding a lot of noise to the system. Does the system stay in the off-state after the perturbation or switch to the on-state? Repeat this process several times and note the ratio of returns versus switches. How does this ratio change when you vary the self-excitation strength?

Task 3: Dynamics of Two Activation Variables with Mutual Inhibition

Explore the dynamics of two neurons with mutual inhibition, as specified by equations

$$\tau_1 \dot{u}_1(t) = u_1(t) + h_1 + s_1(t) + c_{11} g(u_1(t)) + c_{12} g(u_2(t)) + q_1 \xi_1,$$
$$\tau_2 \dot{u}_2(t) = u_2(t) + h_2 + s_2(t) + c_{22} g(u_2(t)) + c_{21} g(u_1(t)) + q_2 \xi_2.$$

1. **Bistability:** Set the interaction parameters of the system to mutual inhibition ($c_{12} = c_{21} = -10$). Add a stimulus to u_1, then to u_2. Remove the stimuli and reapply them in the opposite order. Note which attractor the system relaxes to in each case.

2. **Selection:** Add a small amount of noise to the system and give the same stimulus to both neurons. Reset the system several times to observe the selection decision the system makes. Change the relative strengths of the stimuli by a small amount and observe how the stronger stimulus is favored in the selection.

3. **Biases:** Reduce the inhibition of one neuron while keeping the other one invariant. How does this bias the selection decision and why?