

Dynamic Field Theory: embodied cognition

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Neural dynamics

■ Neuro-physics

■ $\tau \dot{u} = -u + h + \text{inputs}$ ~integrate and fire...

■ spiking mechanism replaced by the sigmoid threshold function in population picture

■ attractor dynamics

■ $-u$ term is the source of the stability of neural states

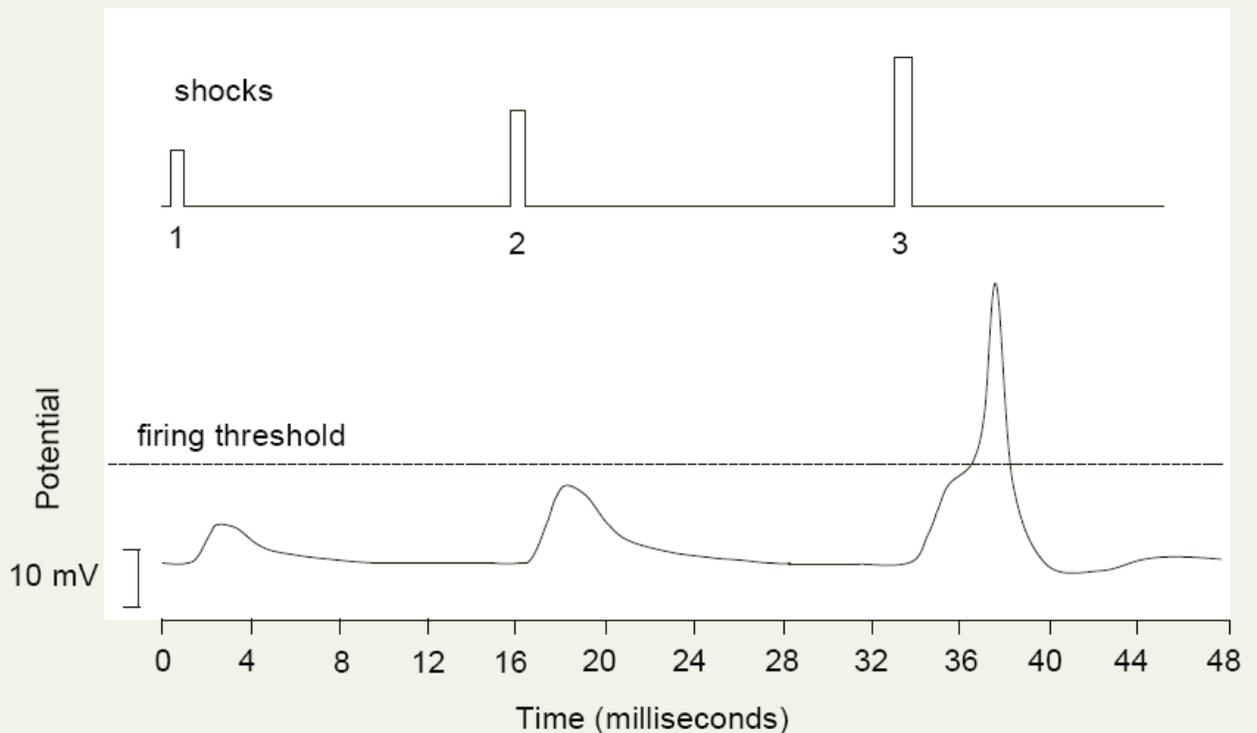
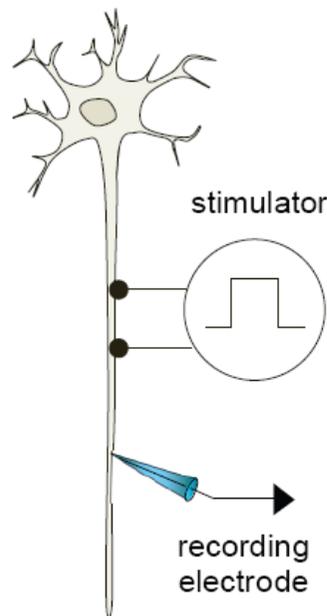
■ this dynamics as a low-pass filter of input

Neuro-physics

- membrane potential, $u(t)$, evolves as a dynamical system

$$\tau \dot{u}(t) = -u(t) + h + \text{input}(t)$$

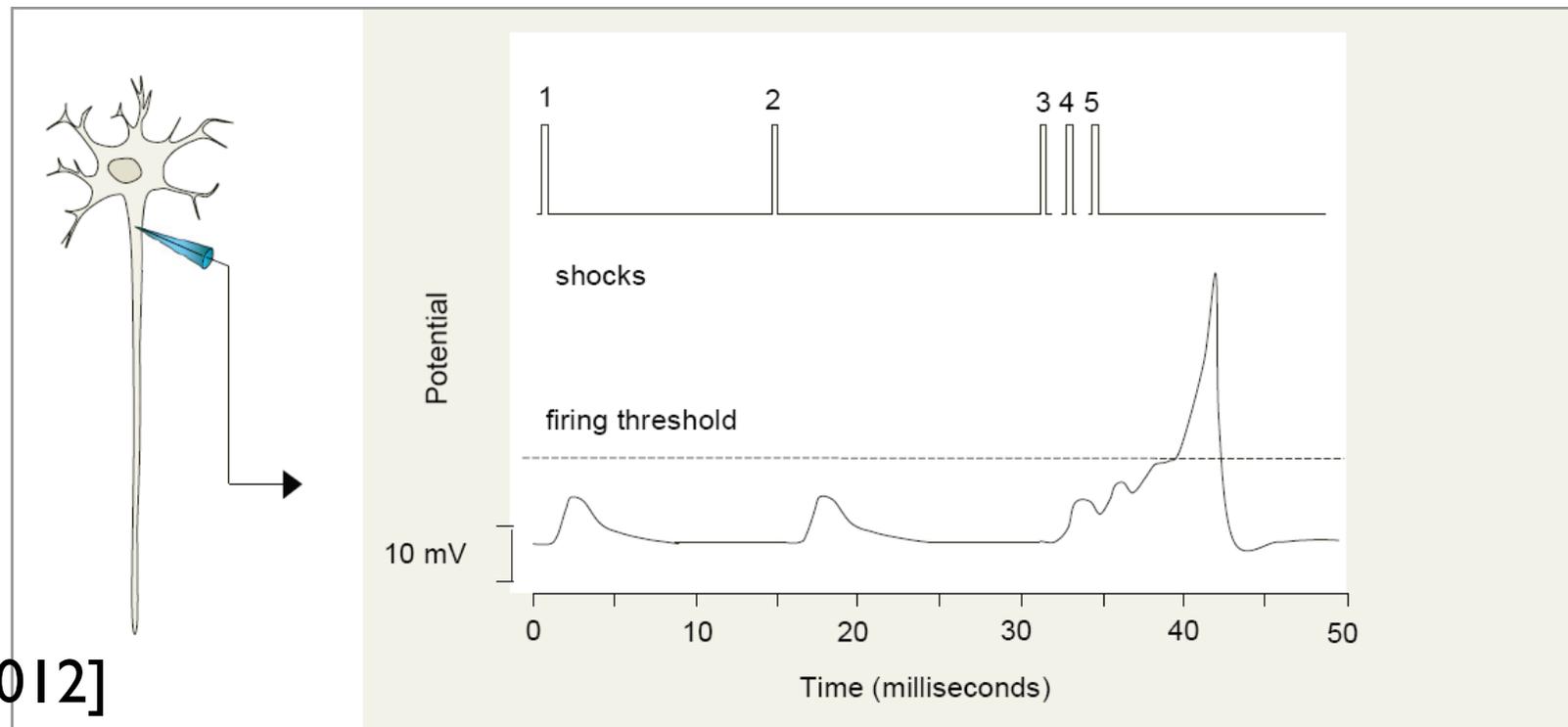
- time scale, $\tau \approx 10$ ms



[from: Tresilian, 2012]

Neuro-physics

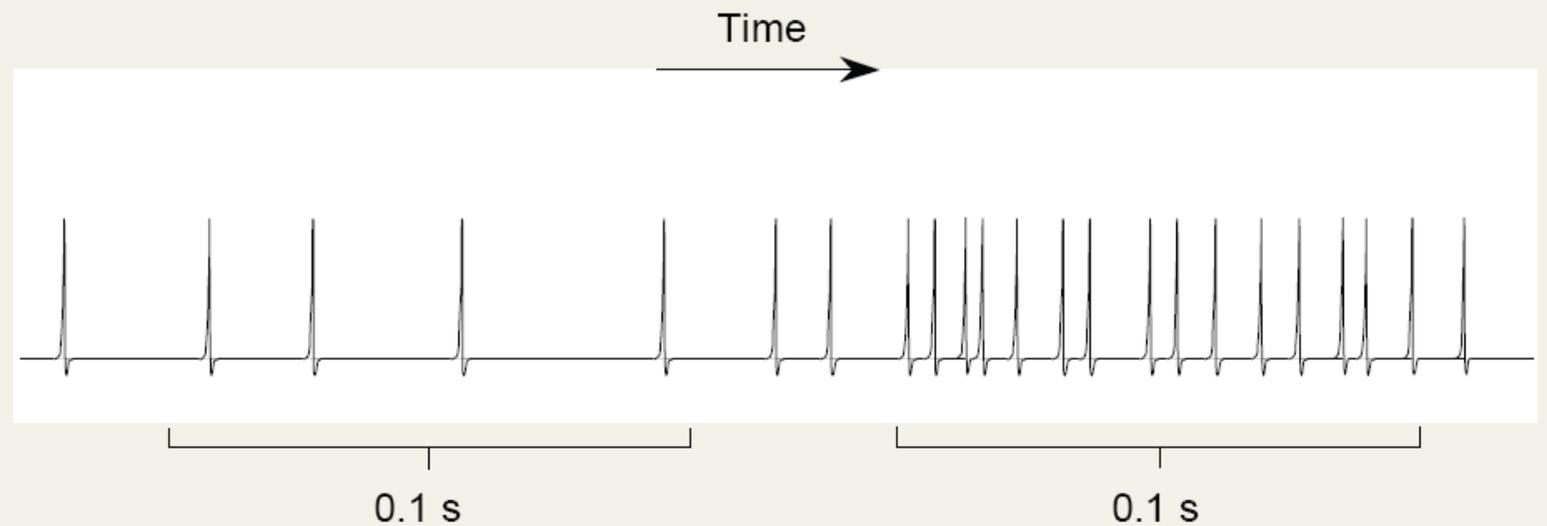
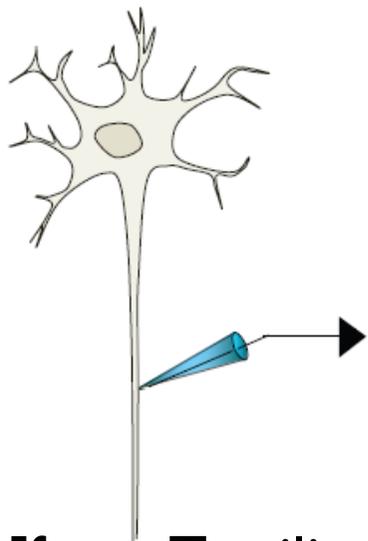
- spikes when membrane potential exceeds threshold.... and only spikes are transmitted to downstream neurons



[from: Tresilian, 2012]

Neuro-physics

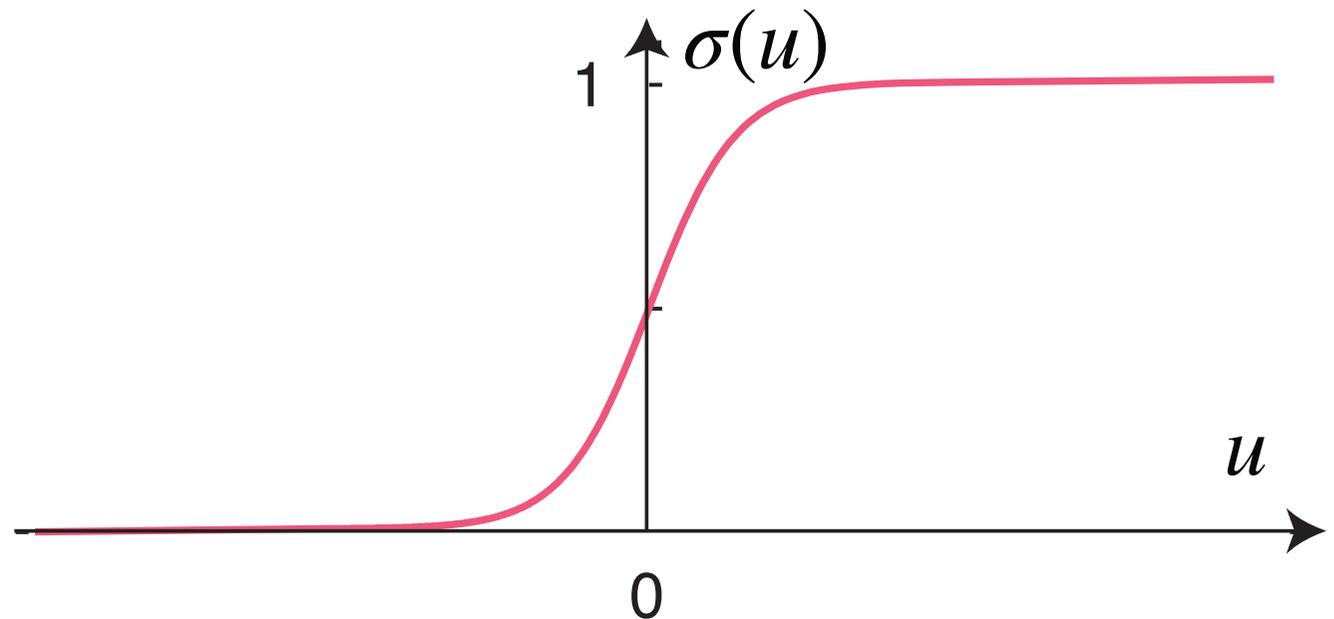
- firing rate reflects level of input...



[from: Tresilian, 2012]

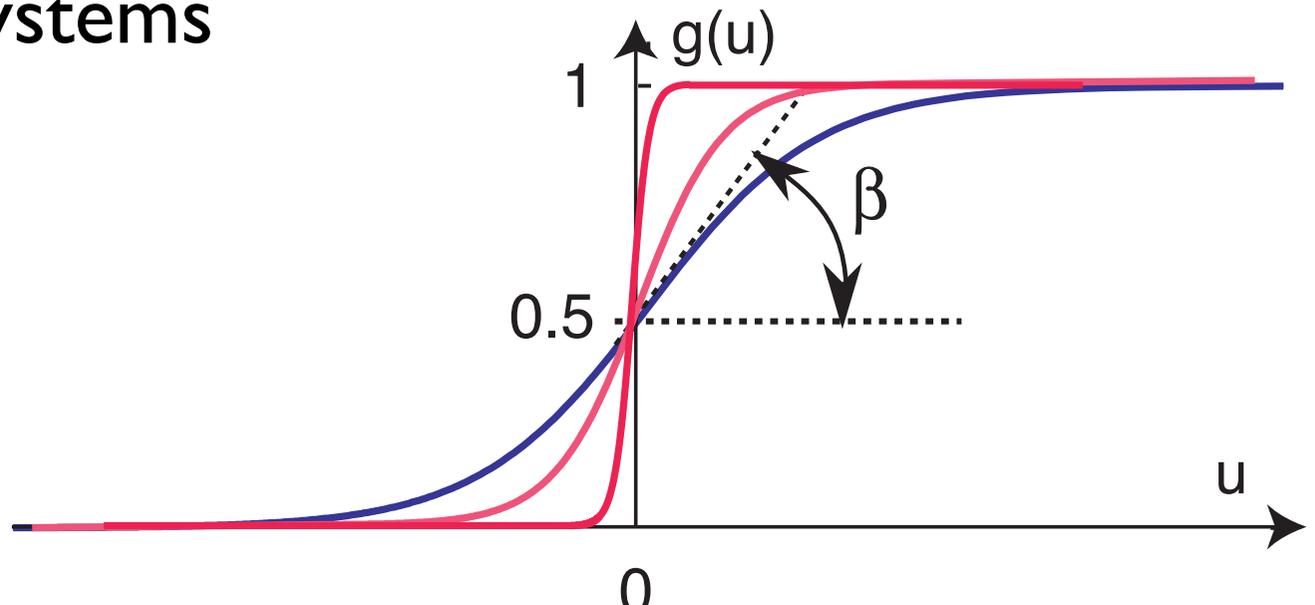
Neural dynamics

- spiking mechanism replaced by a threshold function
- that captures the effective transmission of spikes in populations



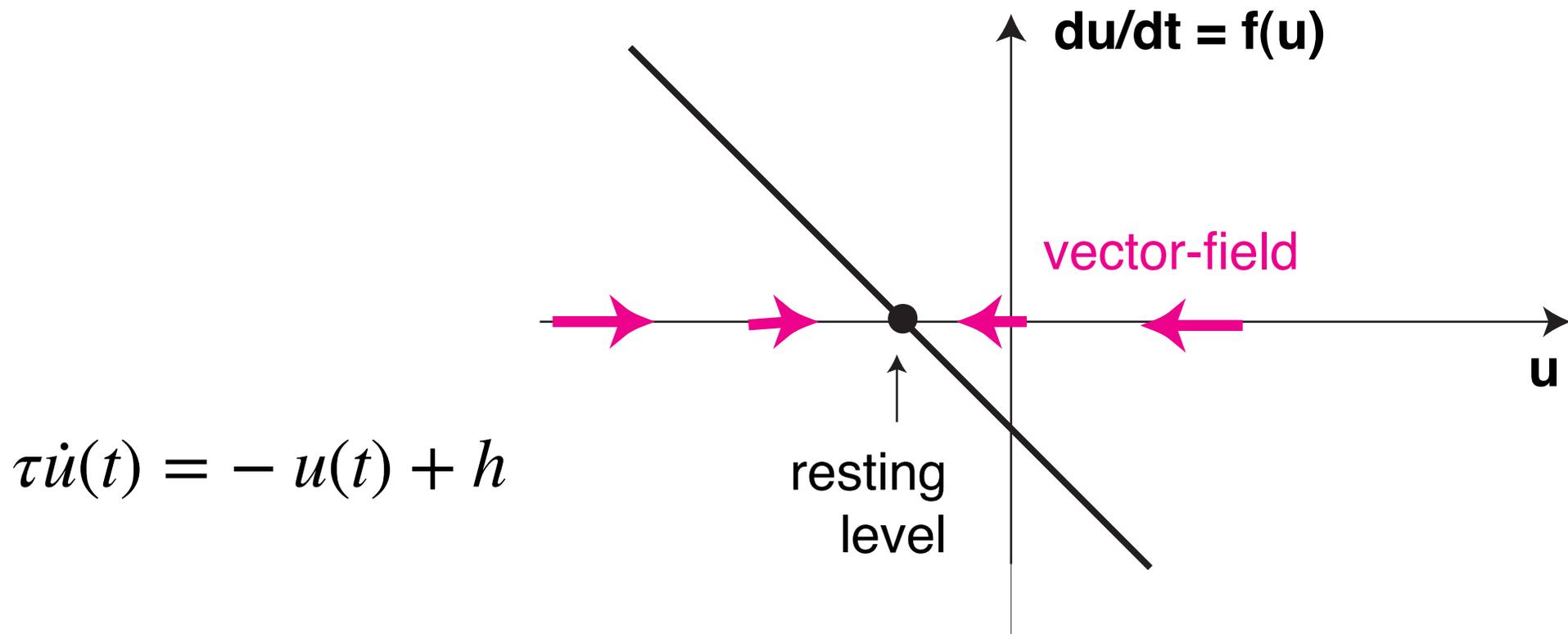
Neural dynamics

- activation as a real number with threshold at zero, abstracting from biophysical details ~ **population level membrane potential**
- low levels of activation: not transmitted to downstream systems (including motor systems)
- high levels of activation: transmitted to downstream systems



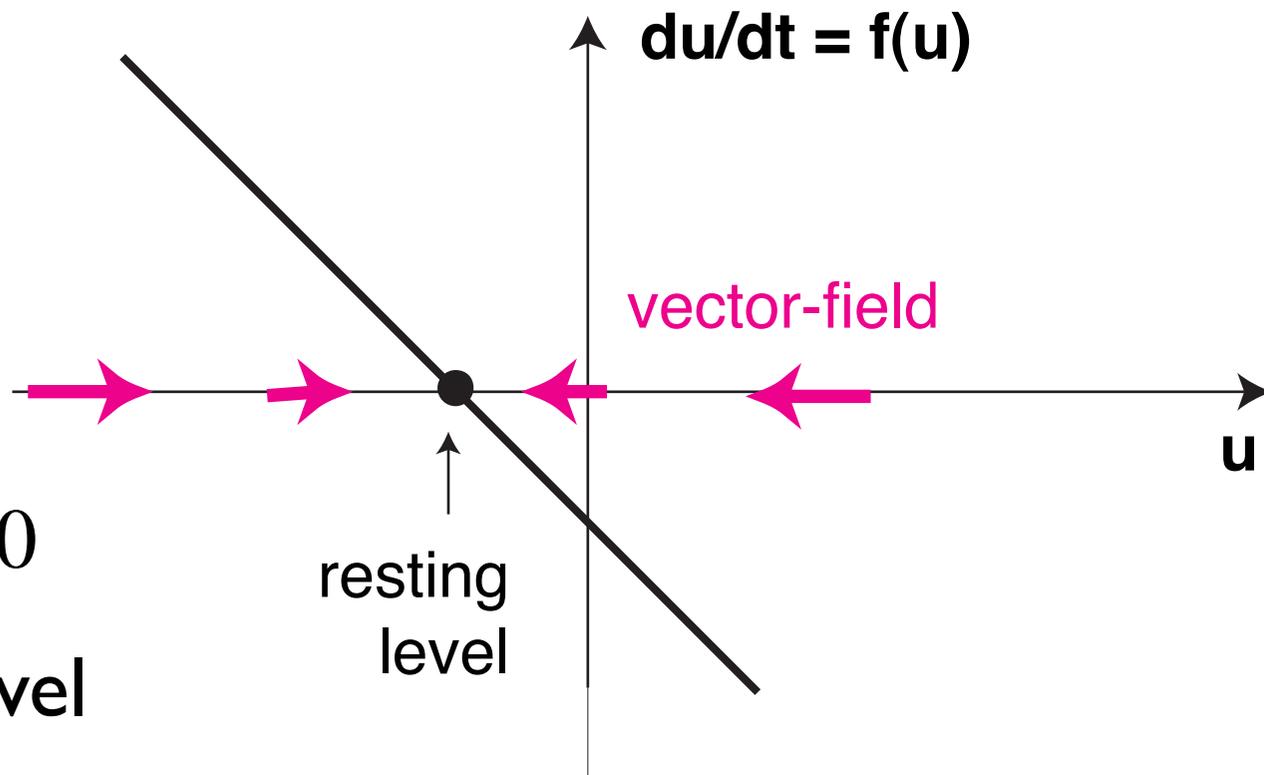
Neural dynamics

- dynamical system: the present (activation) state predicts the future evolution of the state
- => given an initial level of activation, $u(0)$, the time course of activation, $u(t)$, for $t > 0$ is uniquely determined



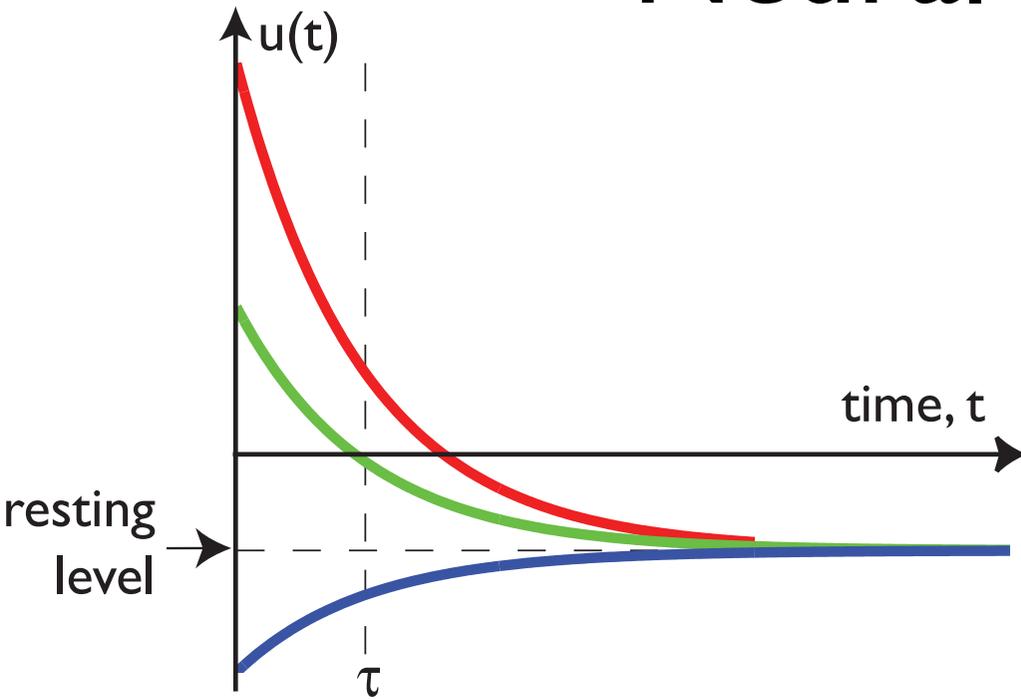
Neural dynamics

- **fixed point** = constant solution (stationary state)
- **stable fixed point = attractor**: nearby solutions converge to the fixed point

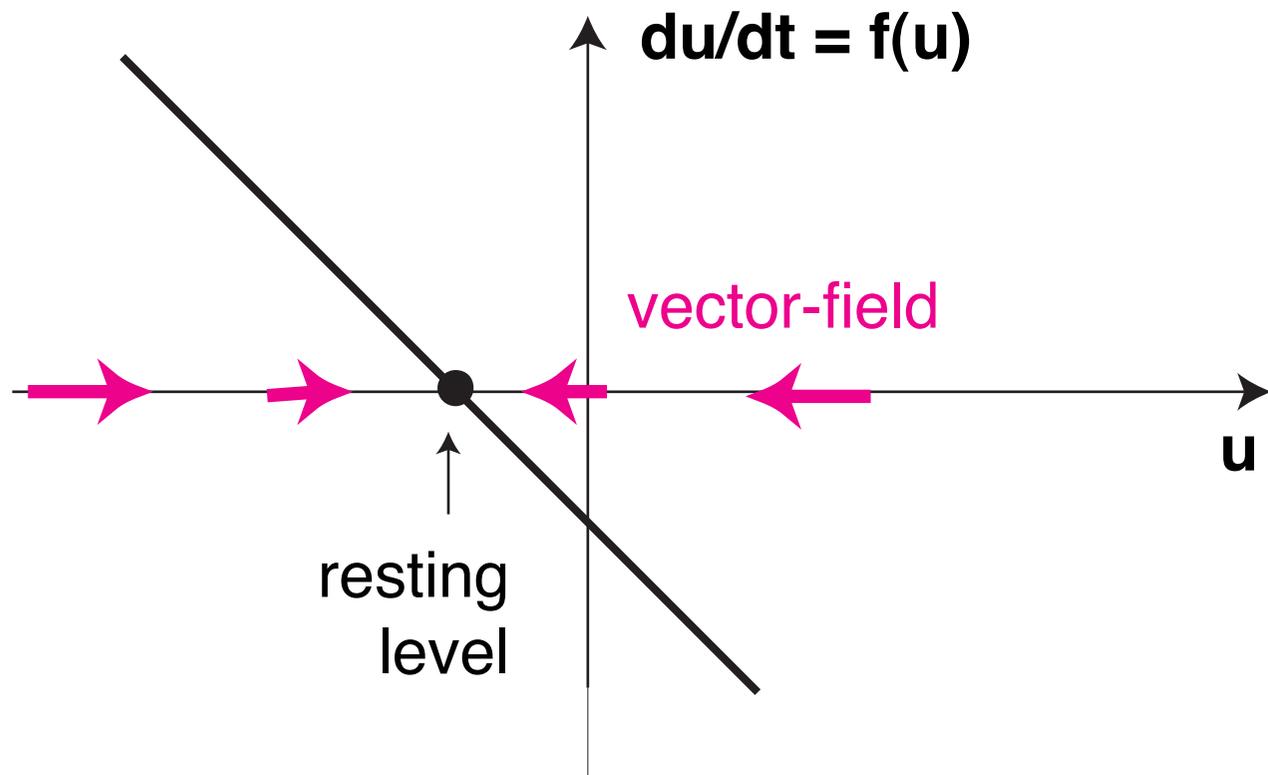


$$\tau \dot{u}_{\text{fp}} = -u_{\text{fp}} + h = 0$$
$$\Rightarrow u_{\text{fp}} = h \quad \text{resting level}$$

Neural dynamics



■ attractors structure the ensemble of solutions (from all initial conditions) = flow



$$\tau \dot{u}(t) = -u(t) + h$$

Neuronal dynamics

■ inputs are contributions to the rate of change of activation

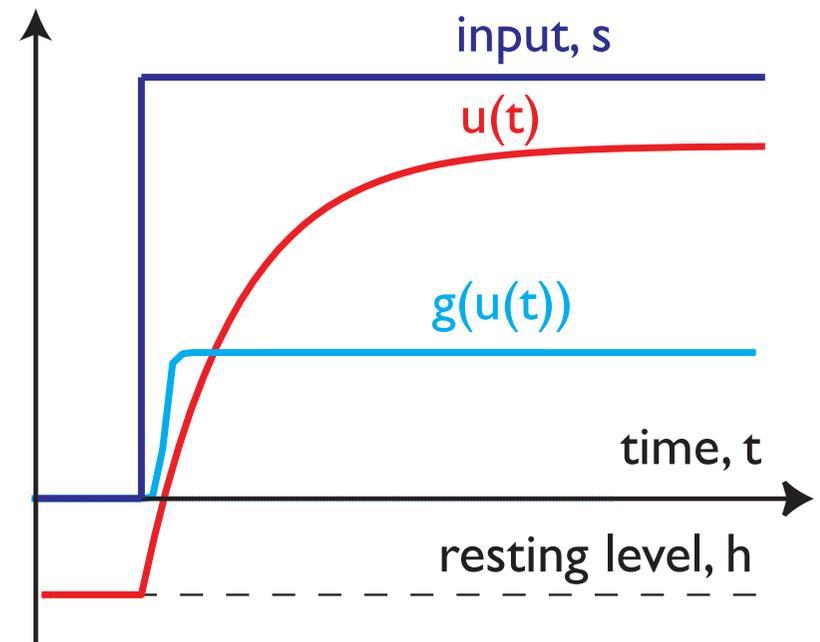
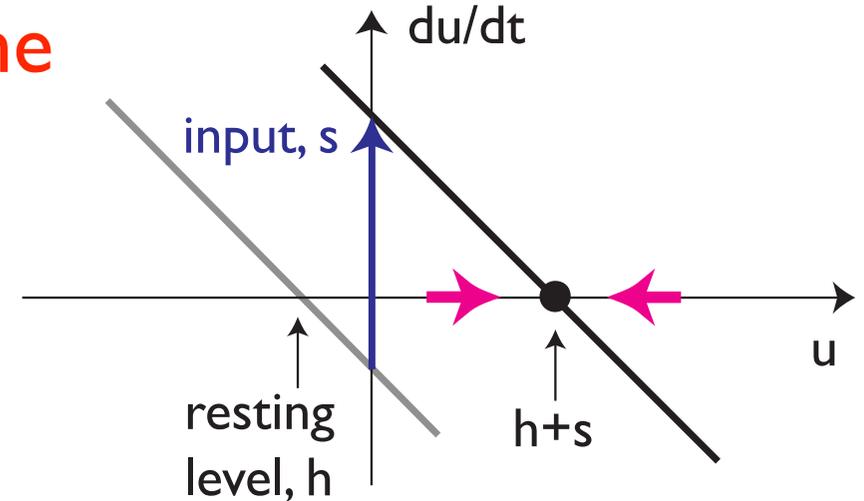
■ positive: excitatory

■ negative: inhibitory

$$\tau \dot{u}(t) = -u(t) + h + s(t)$$

■ => input shifts the attractor

■ => activation tracks this shift due to stability

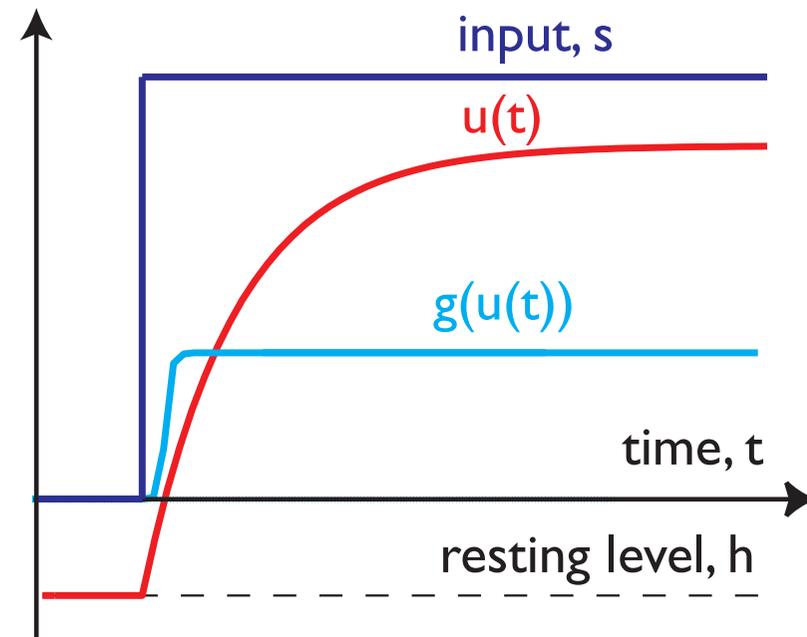


Neuronal dynamics

- transmitted to down-stream neurons/motor systems: $\sigma(u(t))$
- [we use $\sigma(u)$ and $g(u)$ interchangeably in some papers/the DFT book]

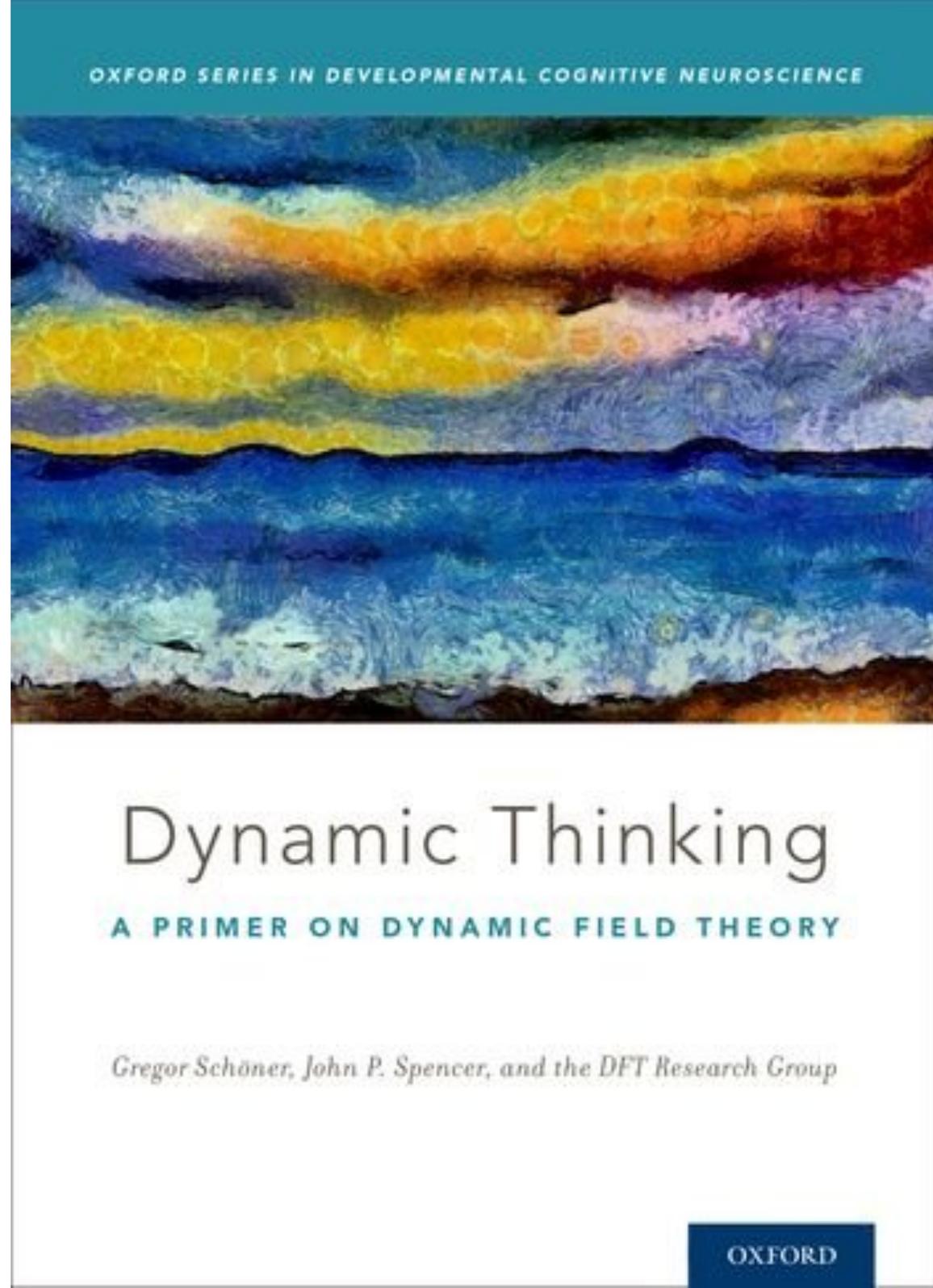
- \Rightarrow the “input-driven solution” of the neural dynamics low-pass filters time varying input

$$\tau \dot{u}(t) = -u(t) + h + s(t)$$



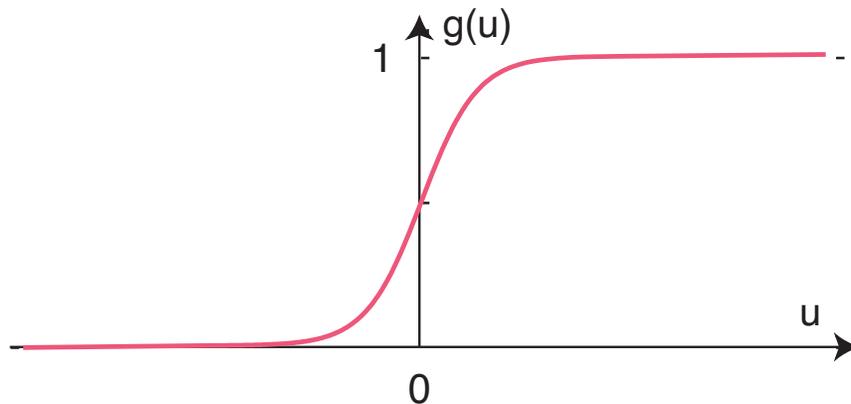
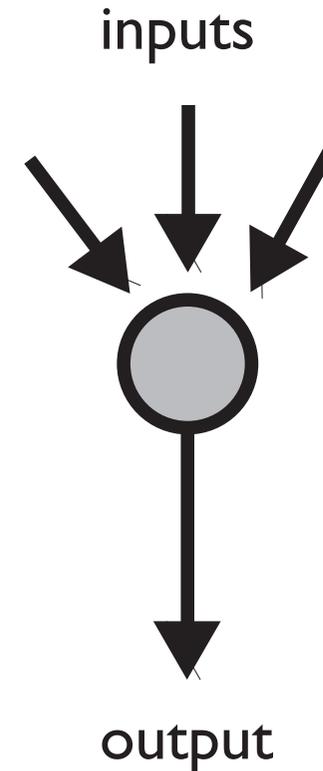
=> simulation

■ dynamicfieldtheory.org



Connectionism: similar abstraction

- neurons sum input activations and pass them through a sigmoidal threshold function
- some connectionist models neglect the low-pass filtering/ time delaying properties of the neural membrane dynamics



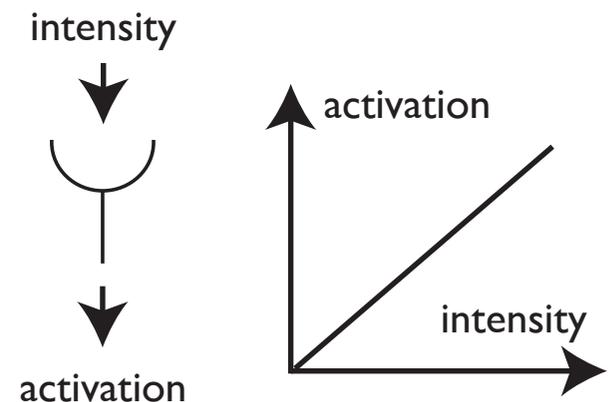
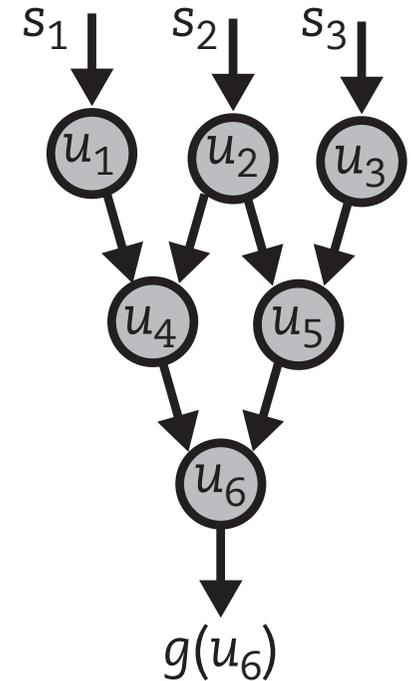
$$\text{output} = g \left(\sum (\text{inputs}) \right)$$

Neural field

- defined by pattern of forward connectivity to sensory/motor surfaces
 - as described by tuning curves/receptive fields
 - analogous to forward NN ...
- neglect sampling by discrete neurons => neural fields
- notion of feature spaces that are represented in neural fields

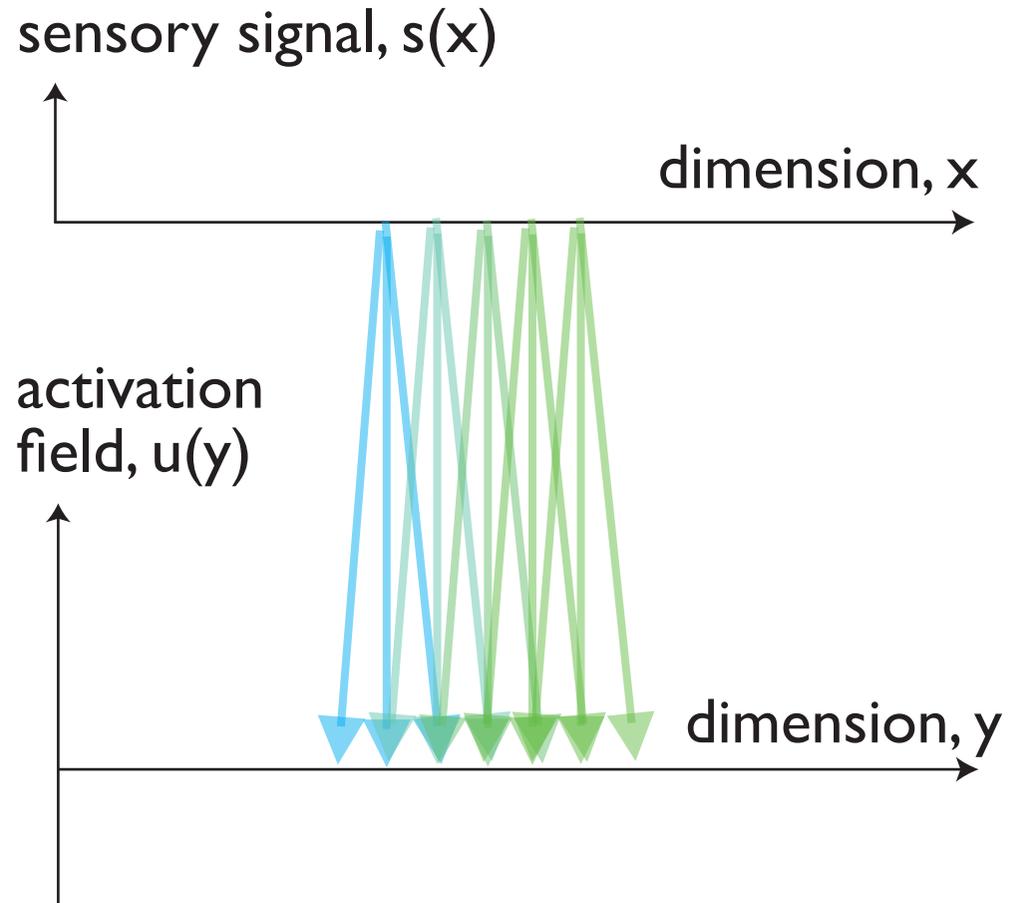
Neural dynamic networks

- in networks neural activation variables, the forward connectivity determines “what a neuron stands for”
- = **space code** (or labelled line code)
- in **rate code**, the activation level “stands for” something, e.g. a sensed intensity
- generic neural networks combine both codes



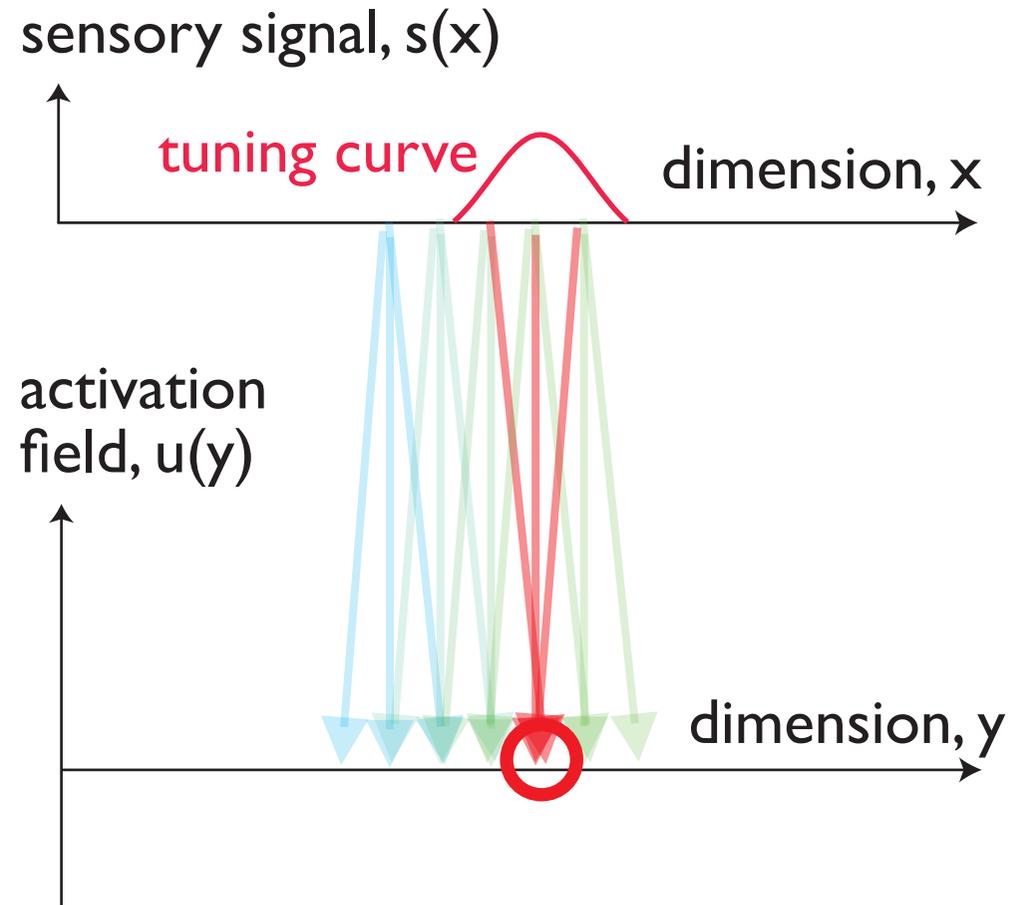
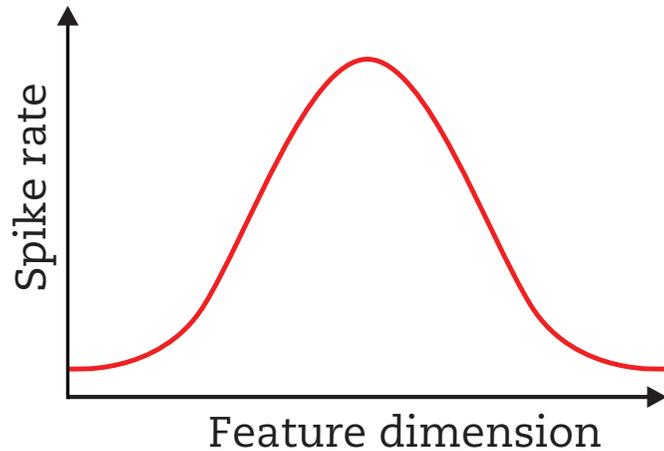
Neural fields

- forward connectivity from the sensory surface extracts perceptual feature dimension



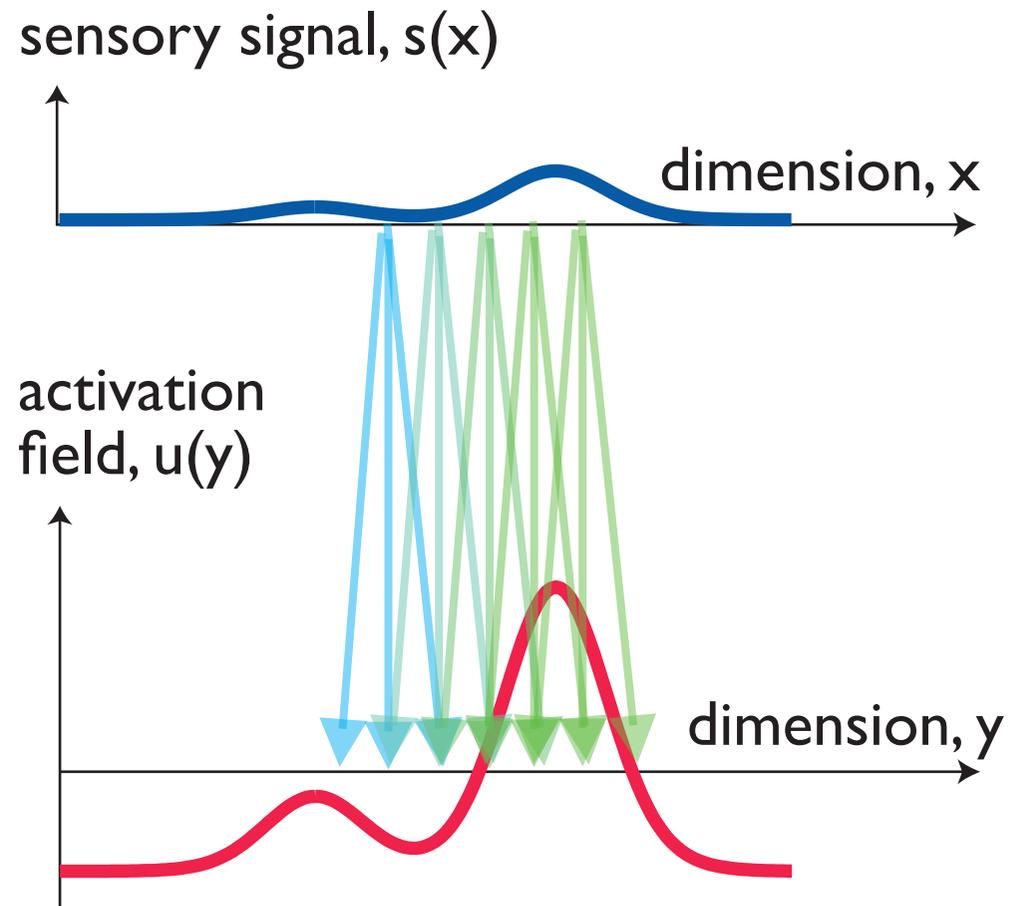
Neural fields

- forward connectivity predicts/models tuning curves



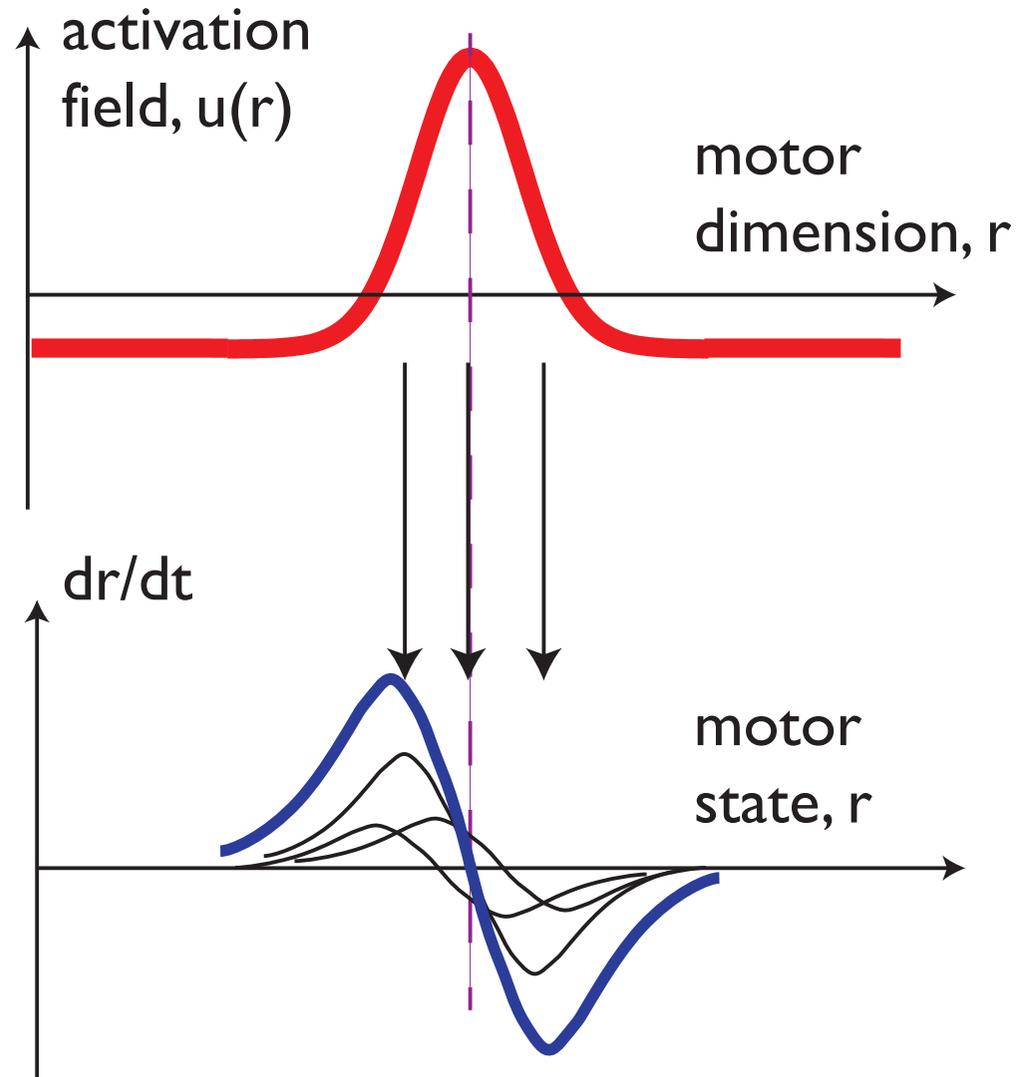
Neural fields

- forward connectivity thus generates a map from sensory surface to feature dimension
- neglect the sampling by individual neurons => activation fields



Neural fields

- analogous notion for forward connectivity to motor surfaces...
- (actually involves behavioral dynamics)
- (e.g., through neural oscillators and peripheral reflex loops)

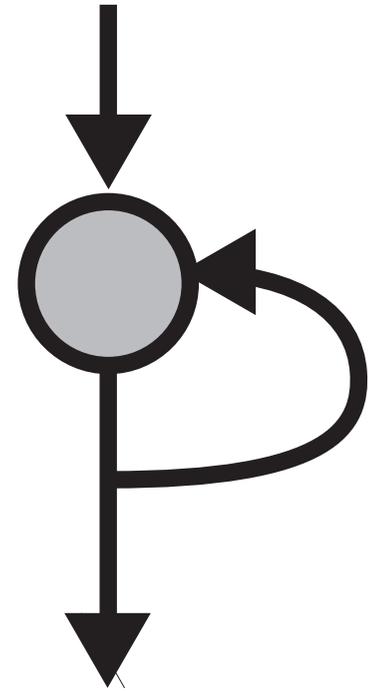


Neural dynamics: state

- neural activation that is not entirely determined by input...but depends on the activation state
- this originates from recurrent connectivity (“interaction” or “coupling”) that is organized to keep activation states stable
- detection instability
- selection/competition
- => dynamic regimes/instabilities

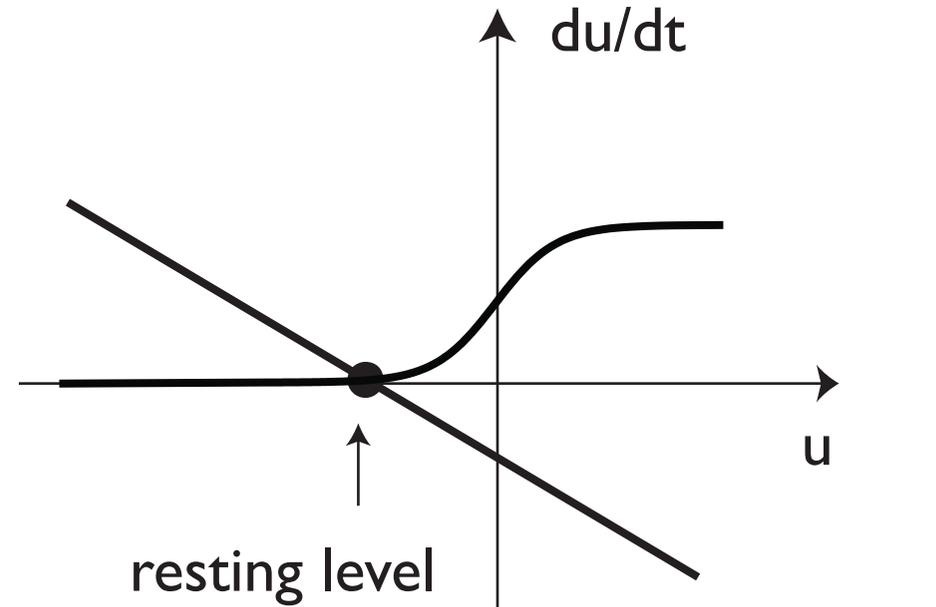
Neuronal dynamics with self-excitation

- single activation variable with self-excitation
- (representing a small population with excitatory coupling)

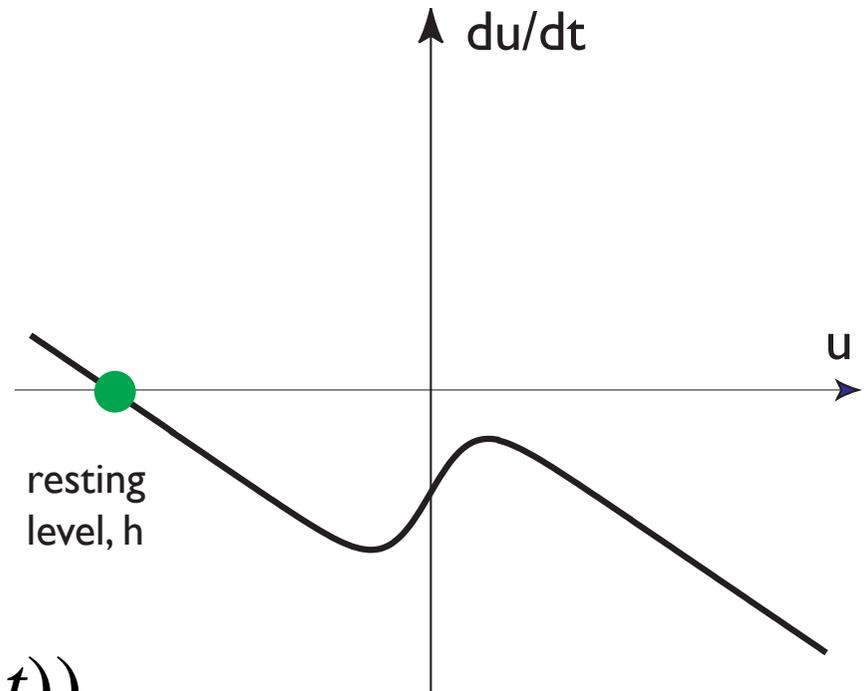


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t))$$

Neuronal dynamics with self-excitation

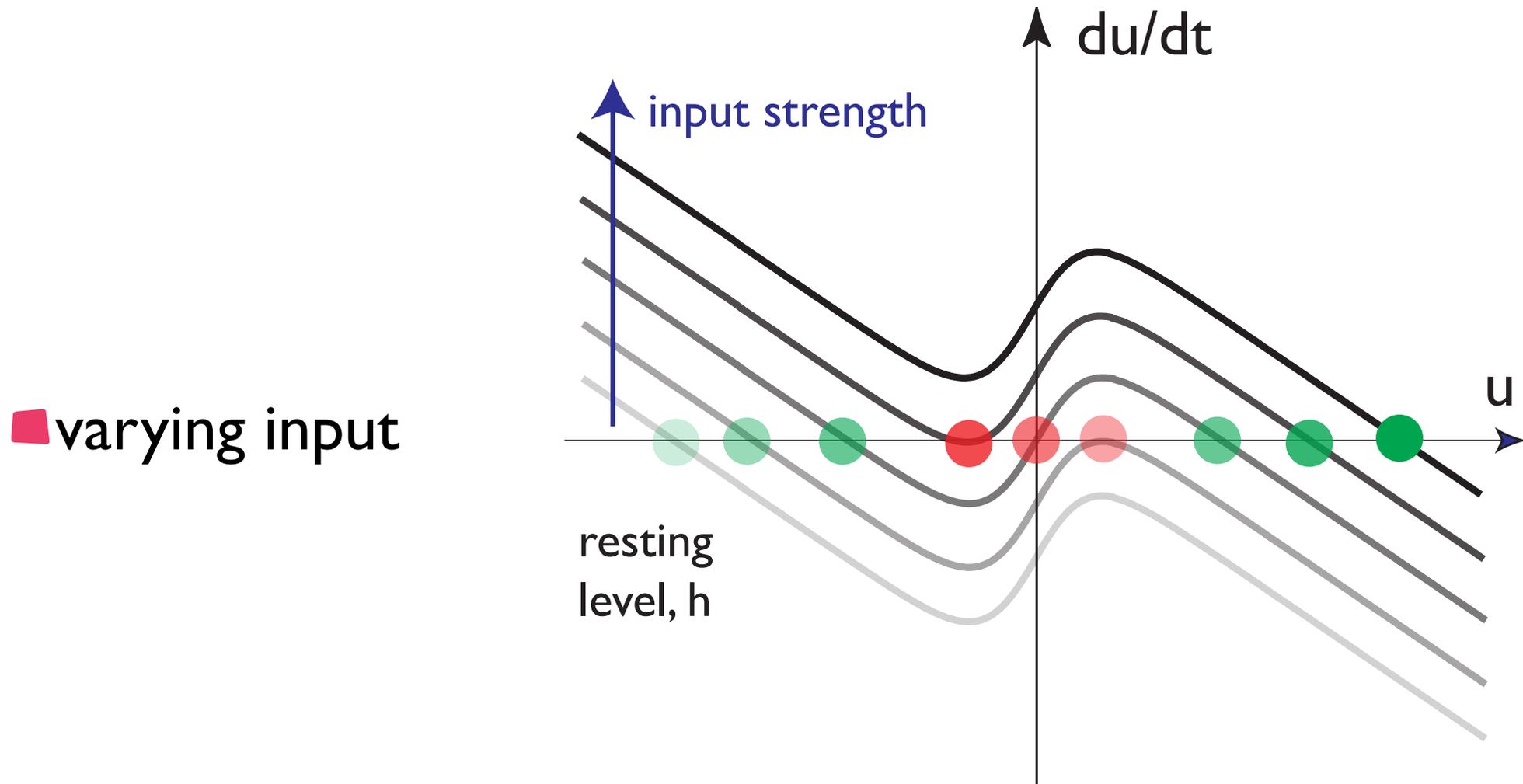


■ => nonlinear dynamics!



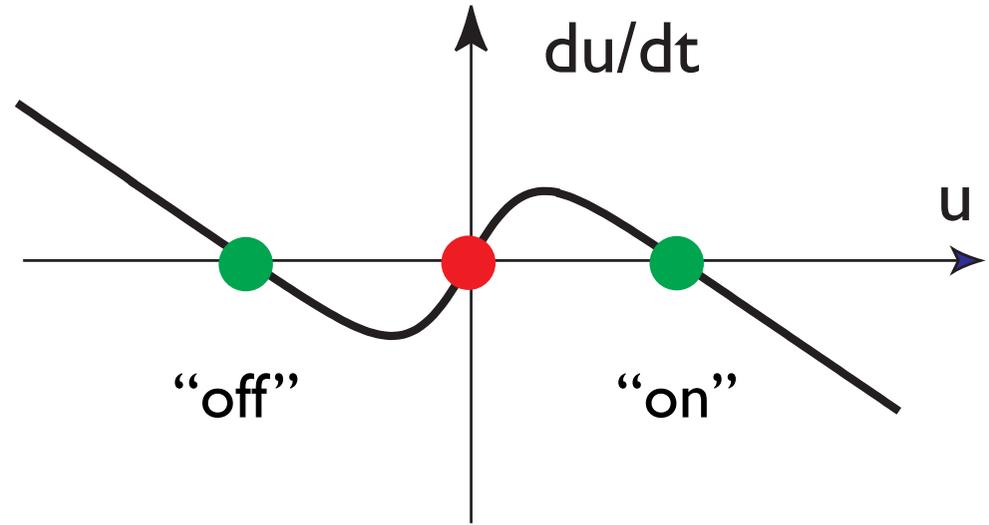
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Neuronal dynamics with self-excitation

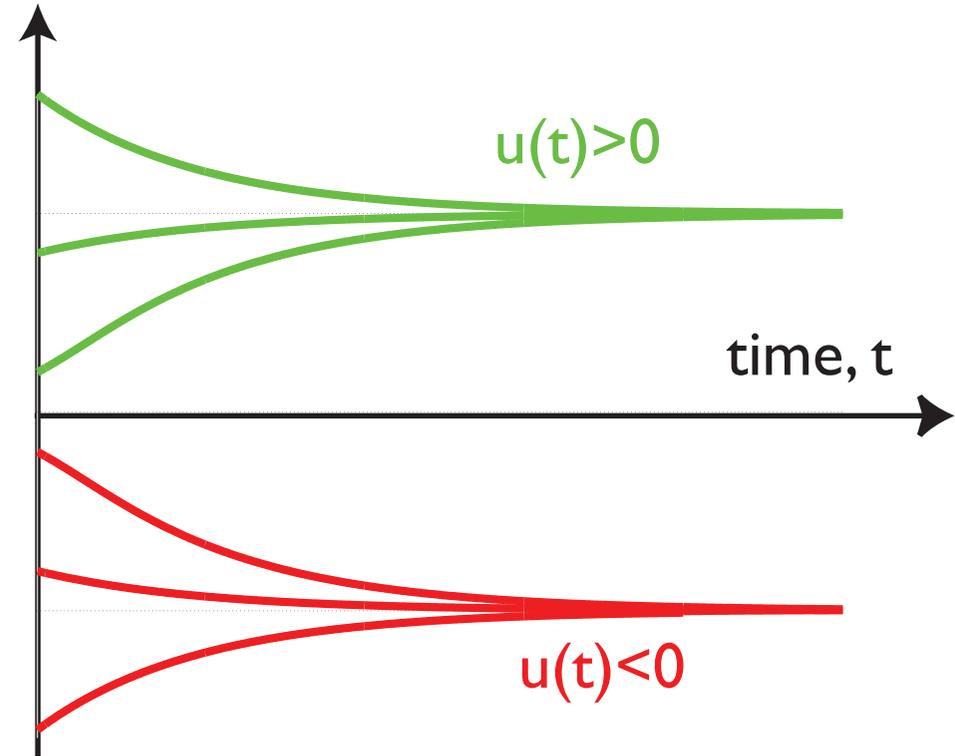


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Neuronal dynamics with self-excitation



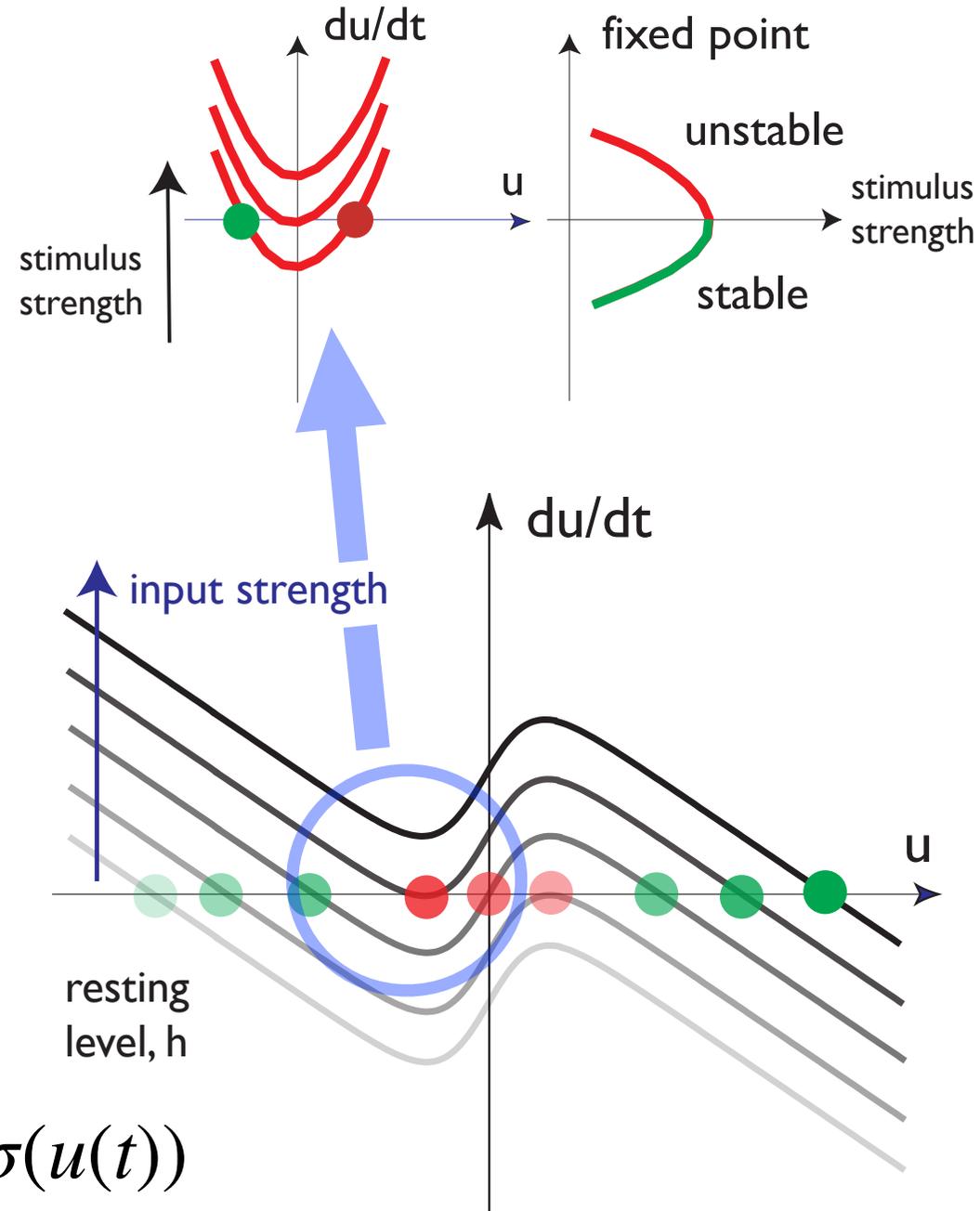
- for some inputs: bistable dynamics
- "on" vs "off" state



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t))$$

Neuronal dynamics with self-excitation

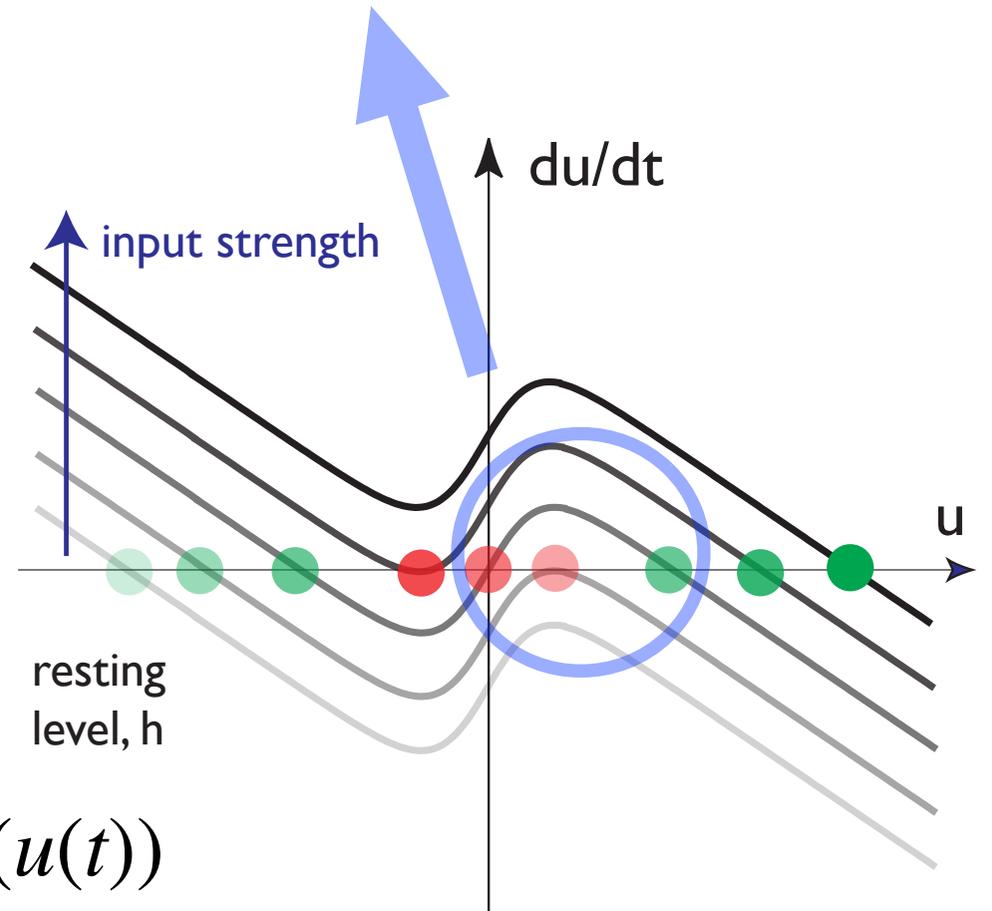
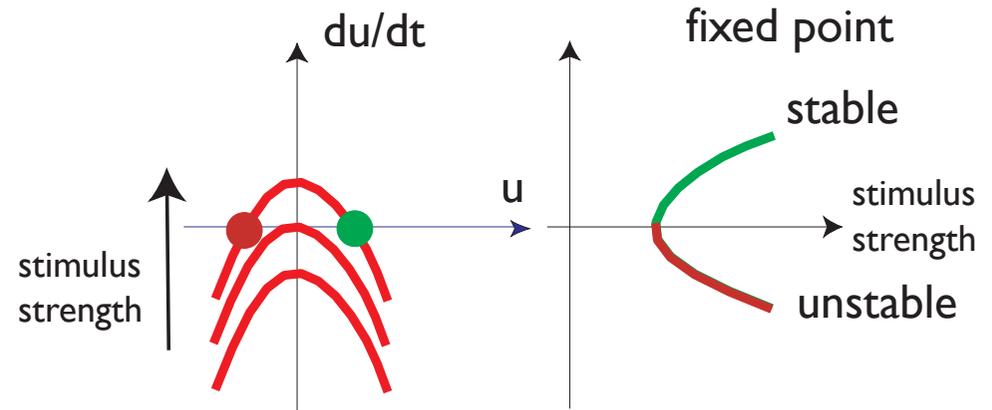
■ increasing input strength
=> **detection instability**



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t))$$

Neuronal dynamics with self-excitation

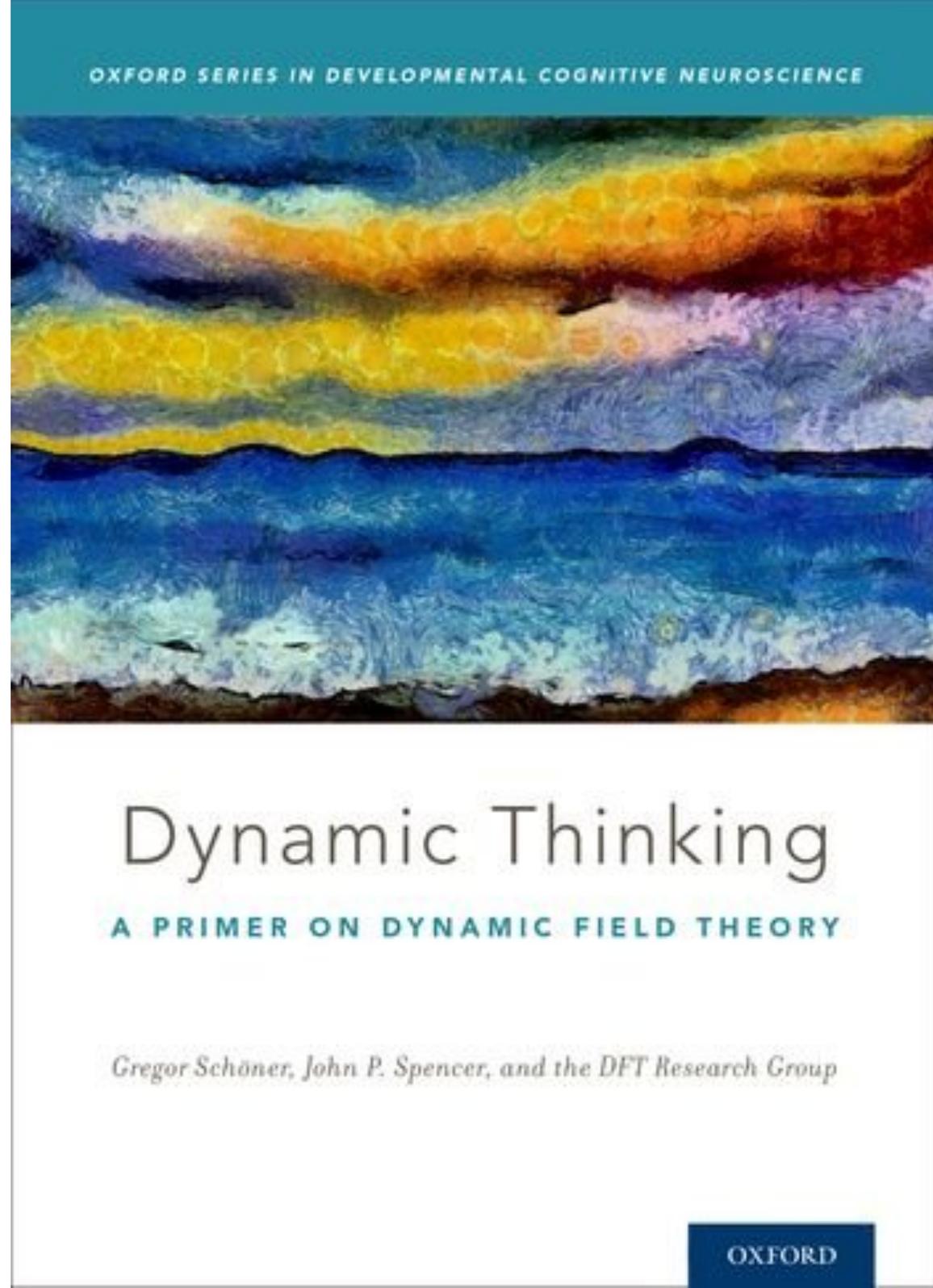
- decreasing input strength => **reverse detection instability**



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t))$$

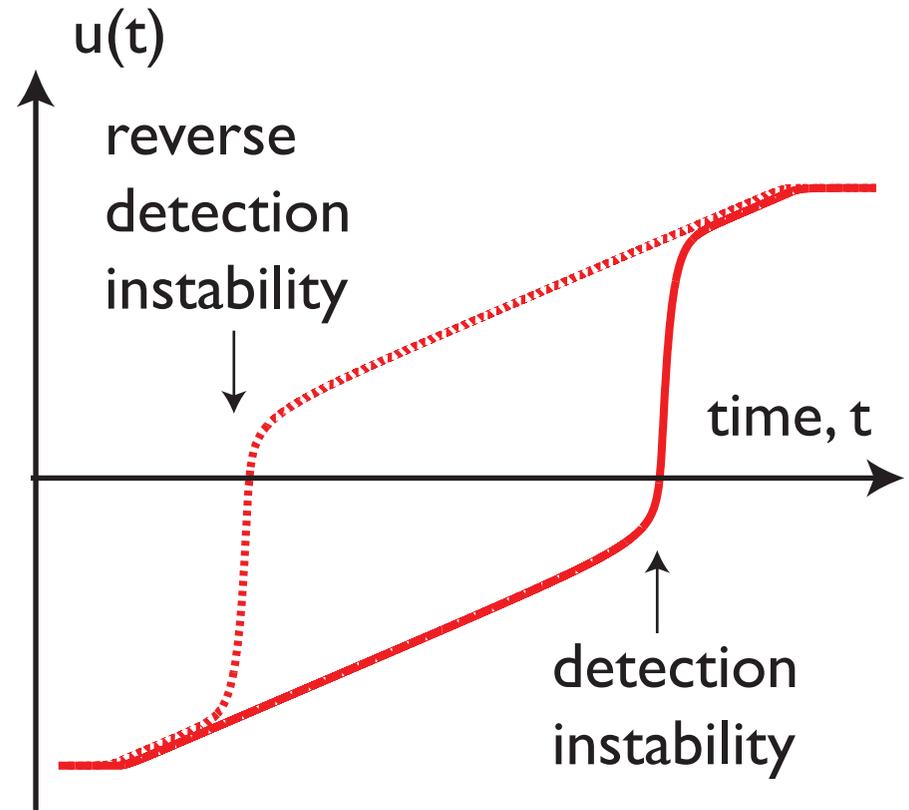
=> simulation

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Neuronal dynamics with self-excitation

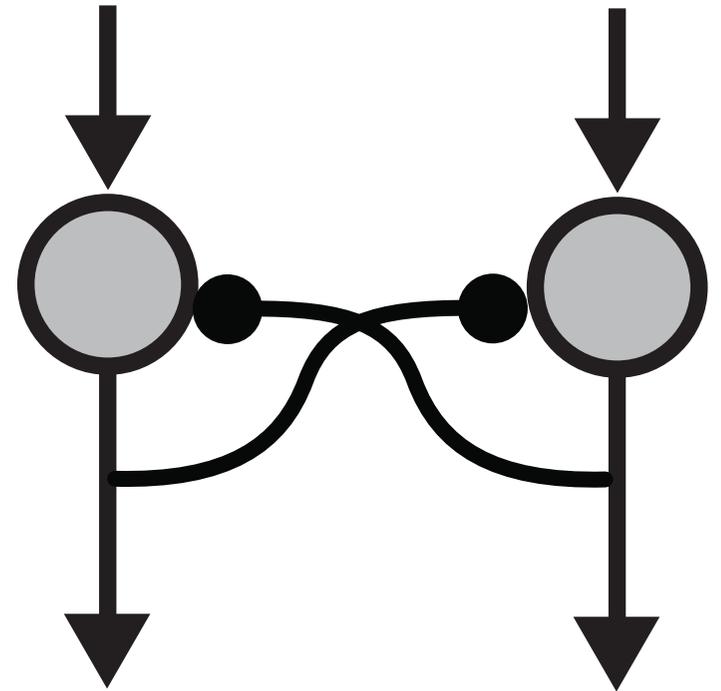
- the detection and its reverse => create **discrete events** from time-continuous changes



$$\tau \dot{u}(t) = -u(t) + h + s(t) + c \sigma(u(t))$$

Neuronal dynamics with competition

- two activation variables with reciprocal inhibitory connection
- (representing two small populations with inhibitory connections)

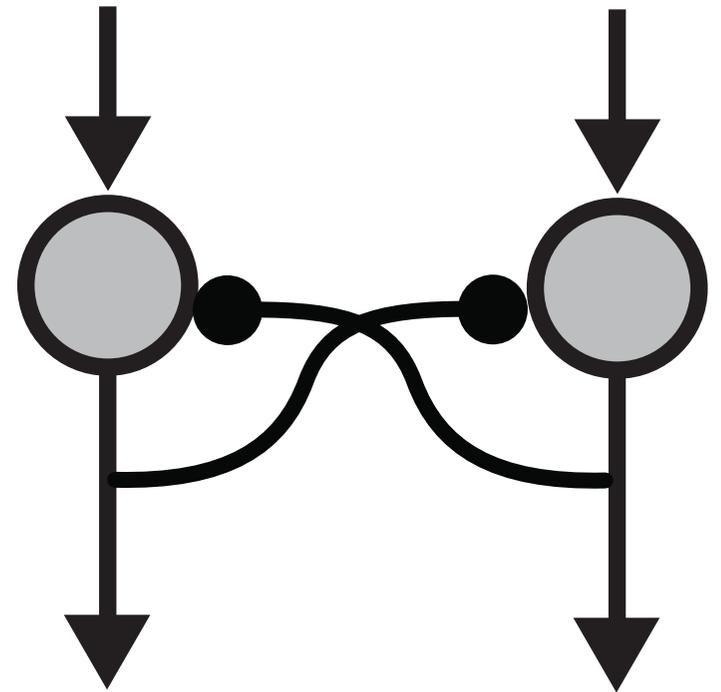


$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12} \sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21} \sigma(u_1(t))$$

Neuronal dynamics with competition

- **Coupling/interaction:** the rate of change of one activation variable depends on the level of activation of the other activation variable



coupling/interaction

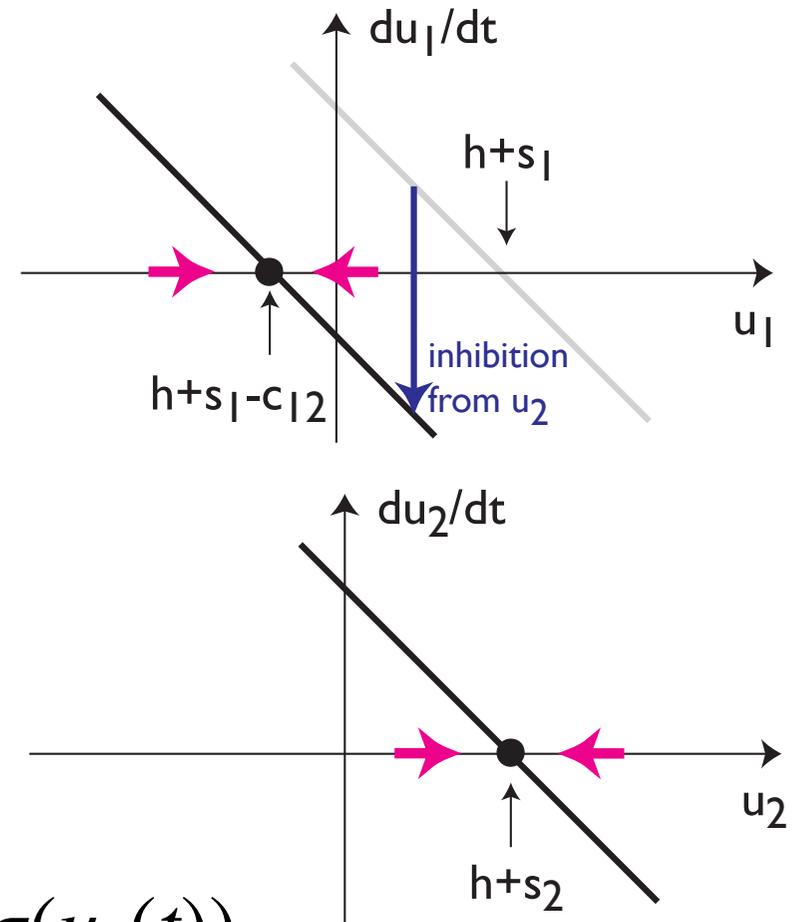


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Neuronal dynamics with competition

- assume $u_2 > 0 \Rightarrow u_2$ inhibits u_1
- \Rightarrow attractor for $u_1 < 0$
- $\Rightarrow u_1$ does not inhibit u_2

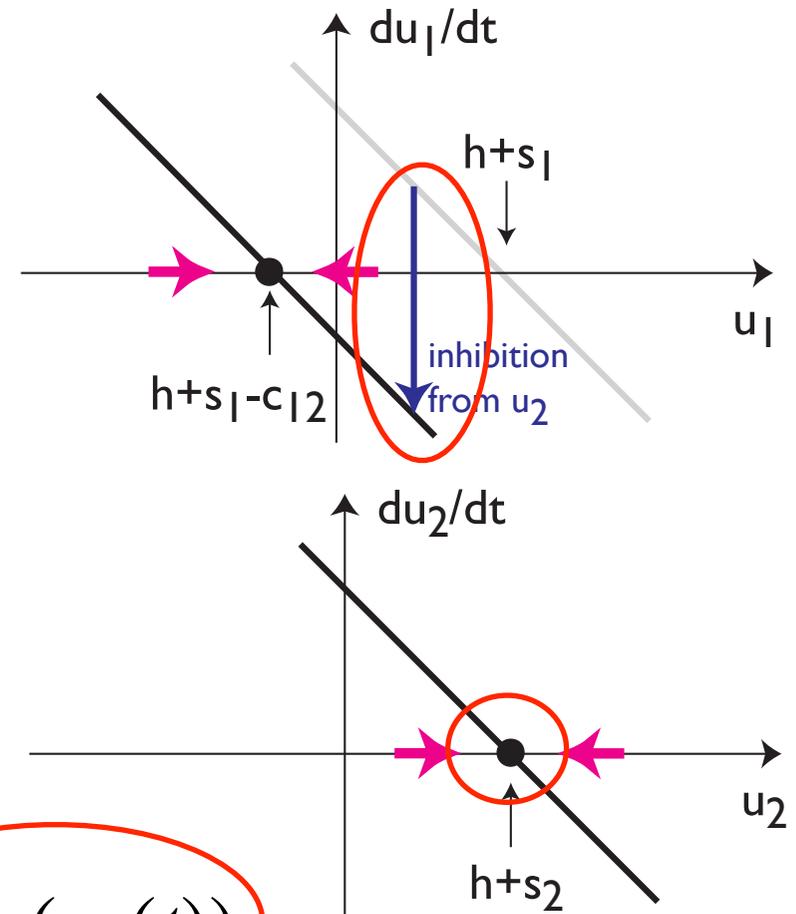


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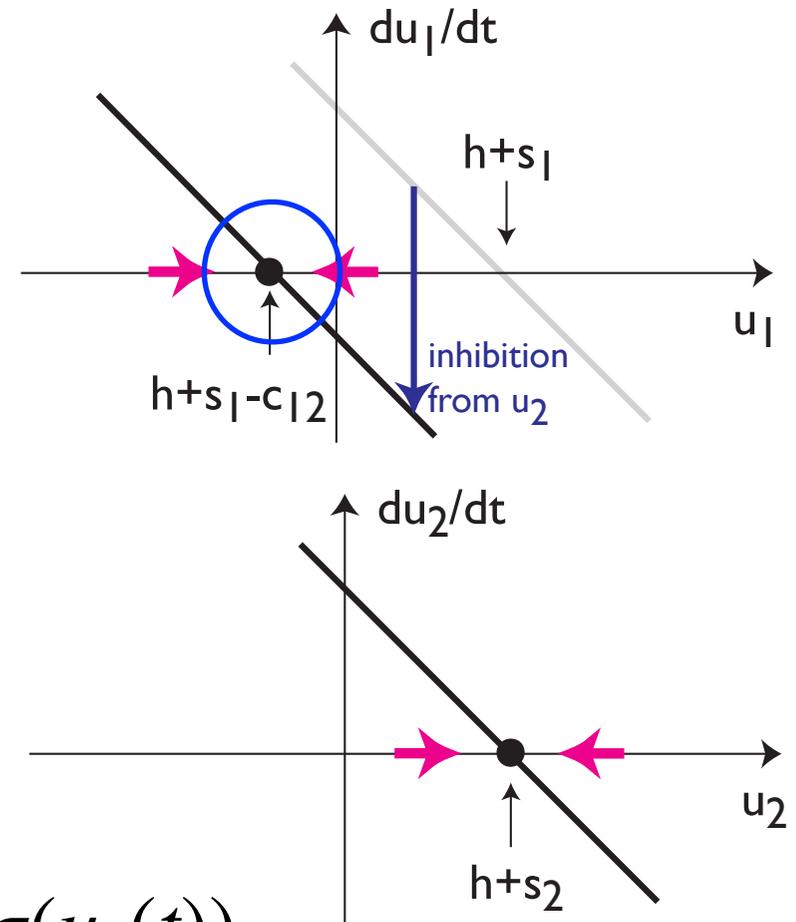


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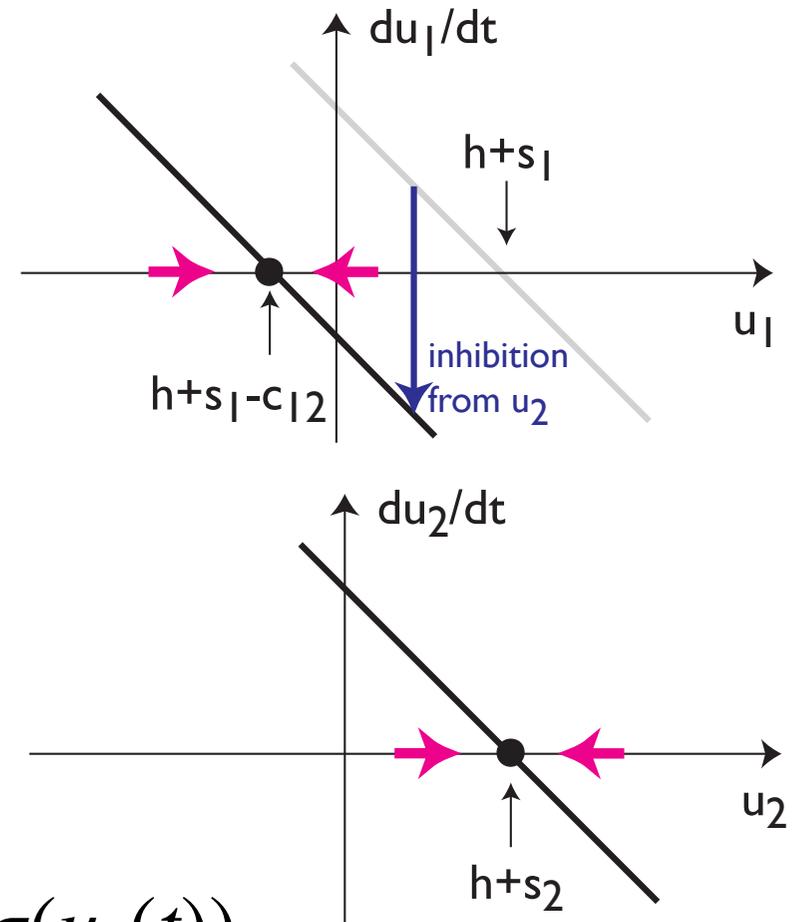


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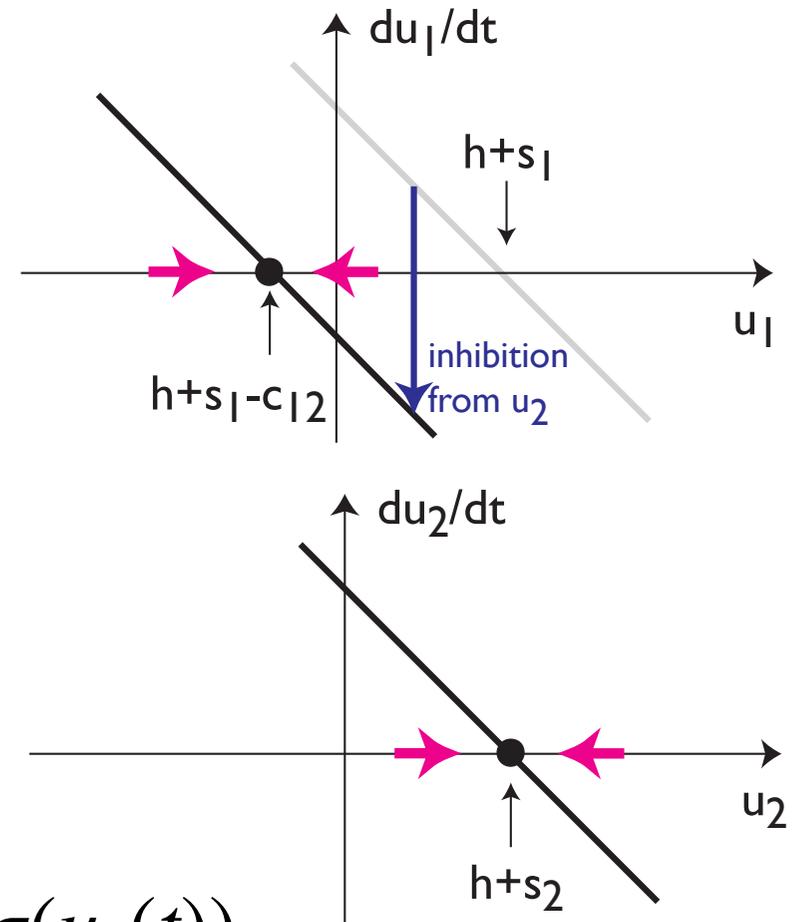


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$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21} \sigma(u_1(t))$$

Neuronal dynamics with competition

- $u_2 > 0$ and $u_1 < 0$
- symmetry: $u_2 < 0$ and $u_1 > 0$
- \Rightarrow competition/selection

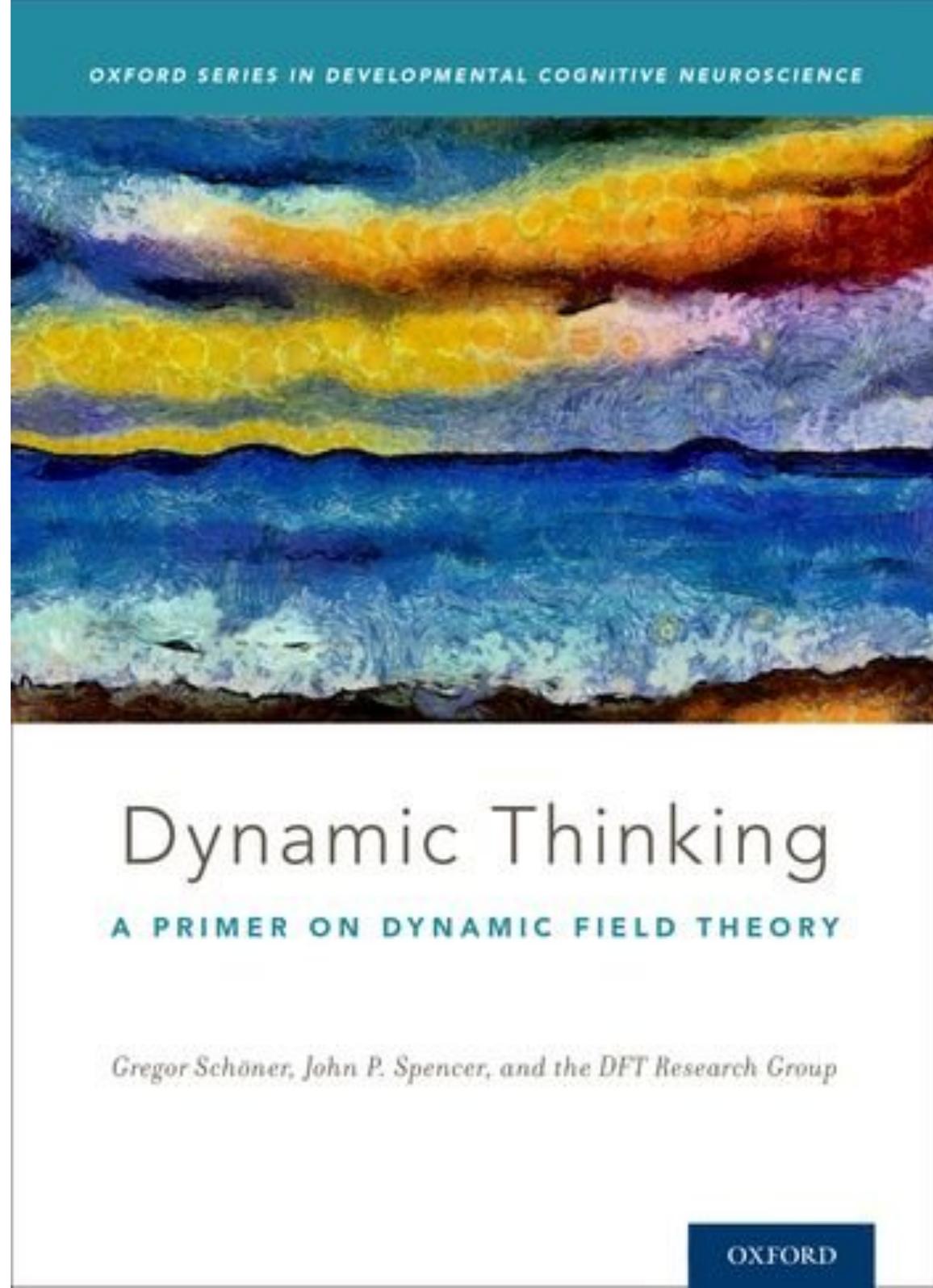


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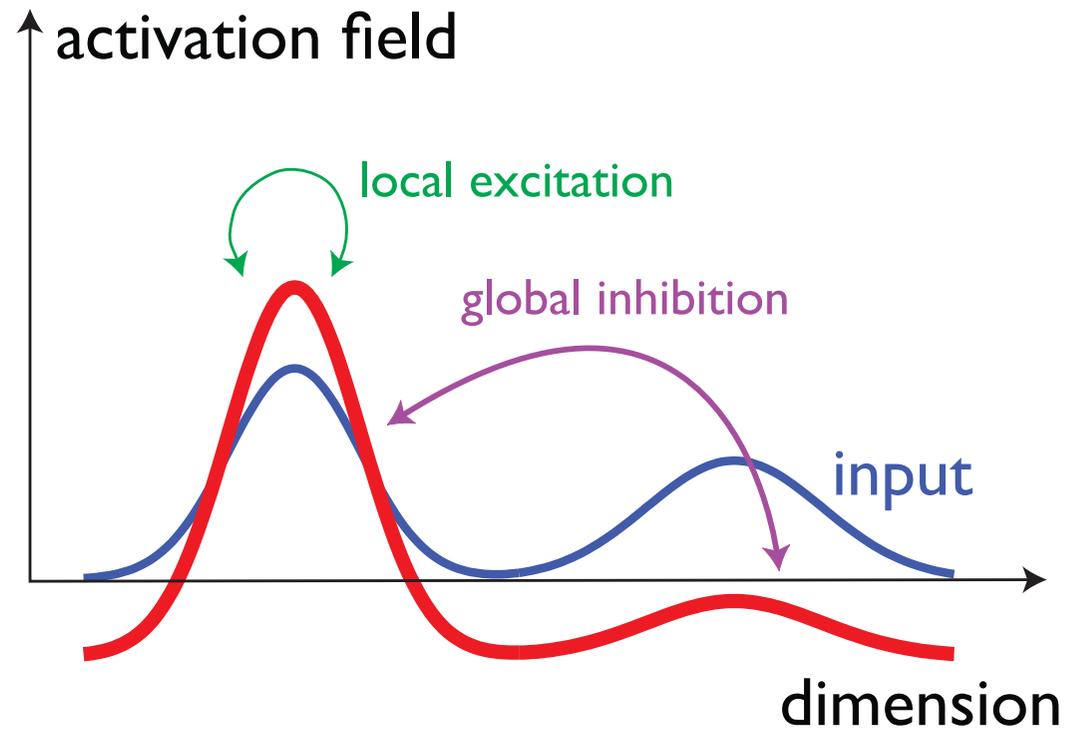
=> simulation

■ dynamicfieldtheory.org

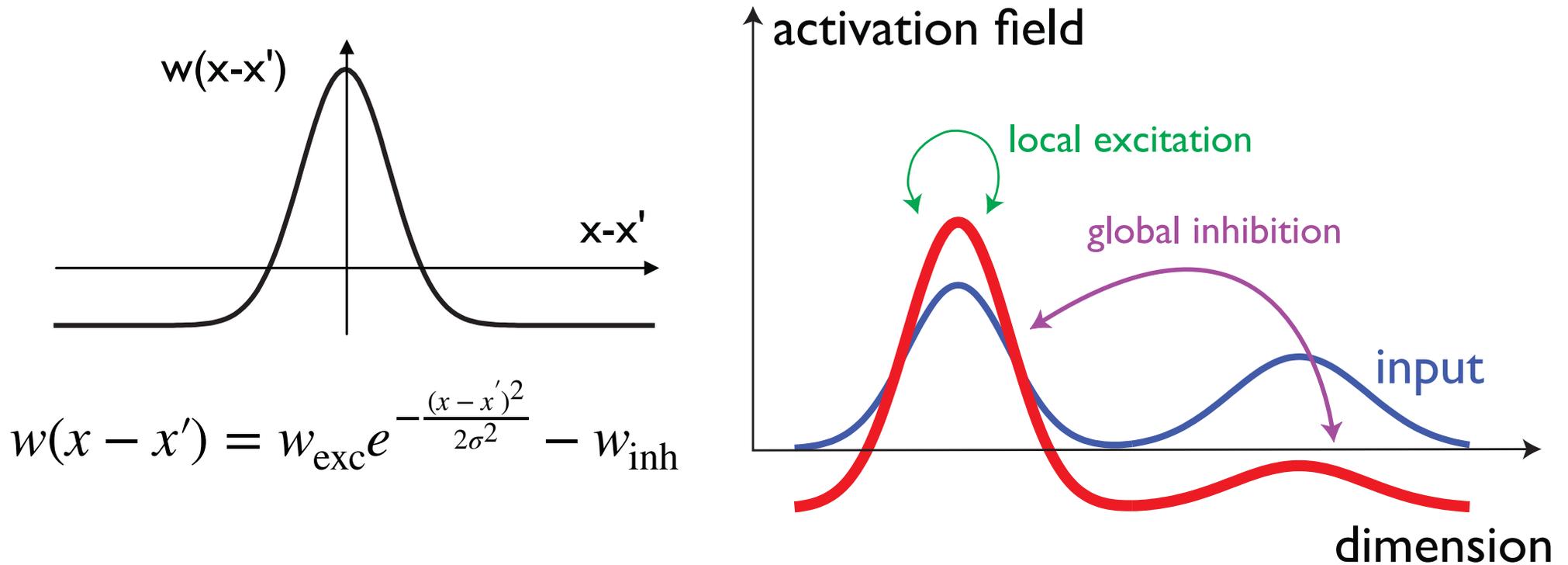


Neural dynamics of fields

- combine detection with selection
- => local excitation/
global inhibition

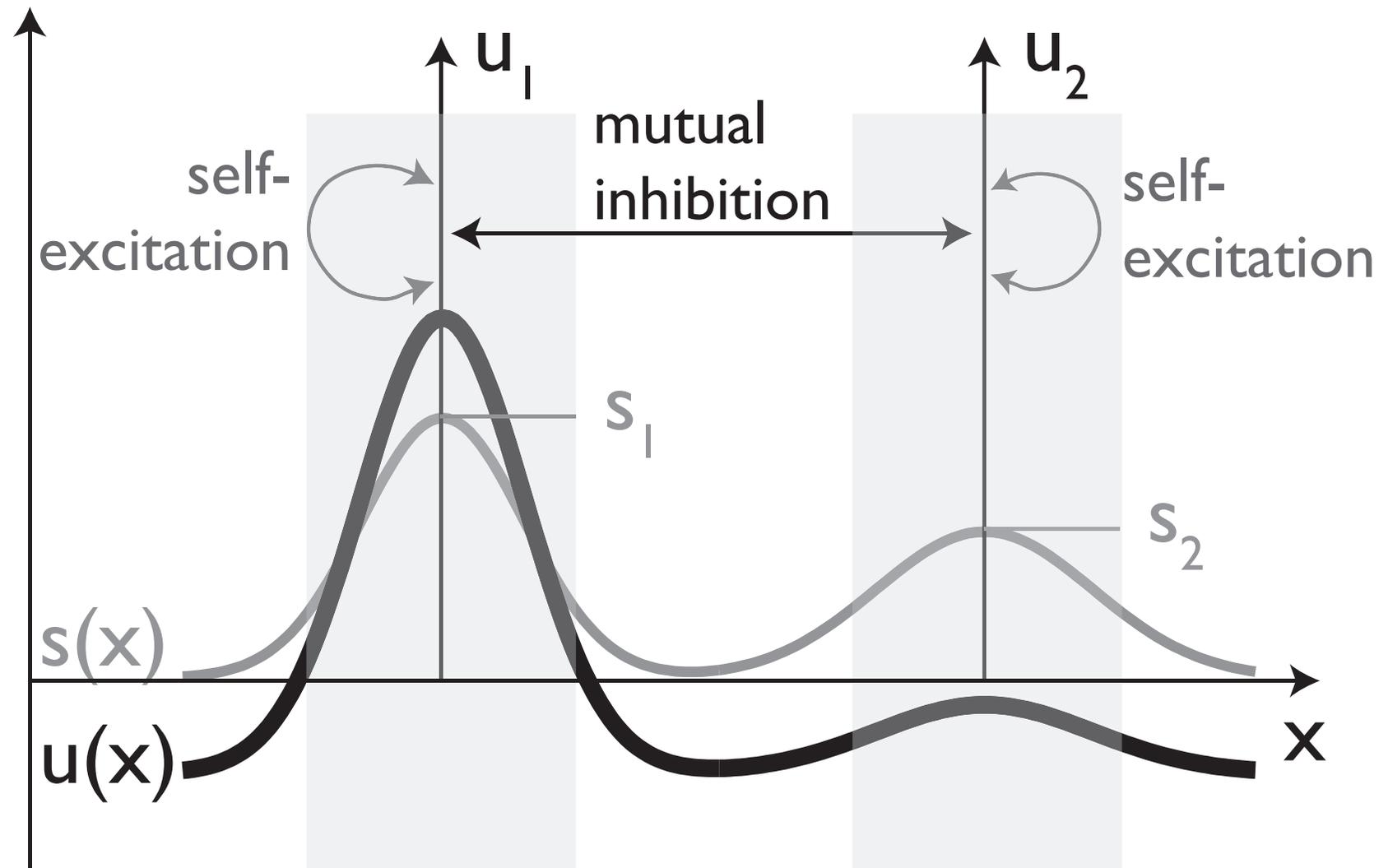


Neural dynamics of fields



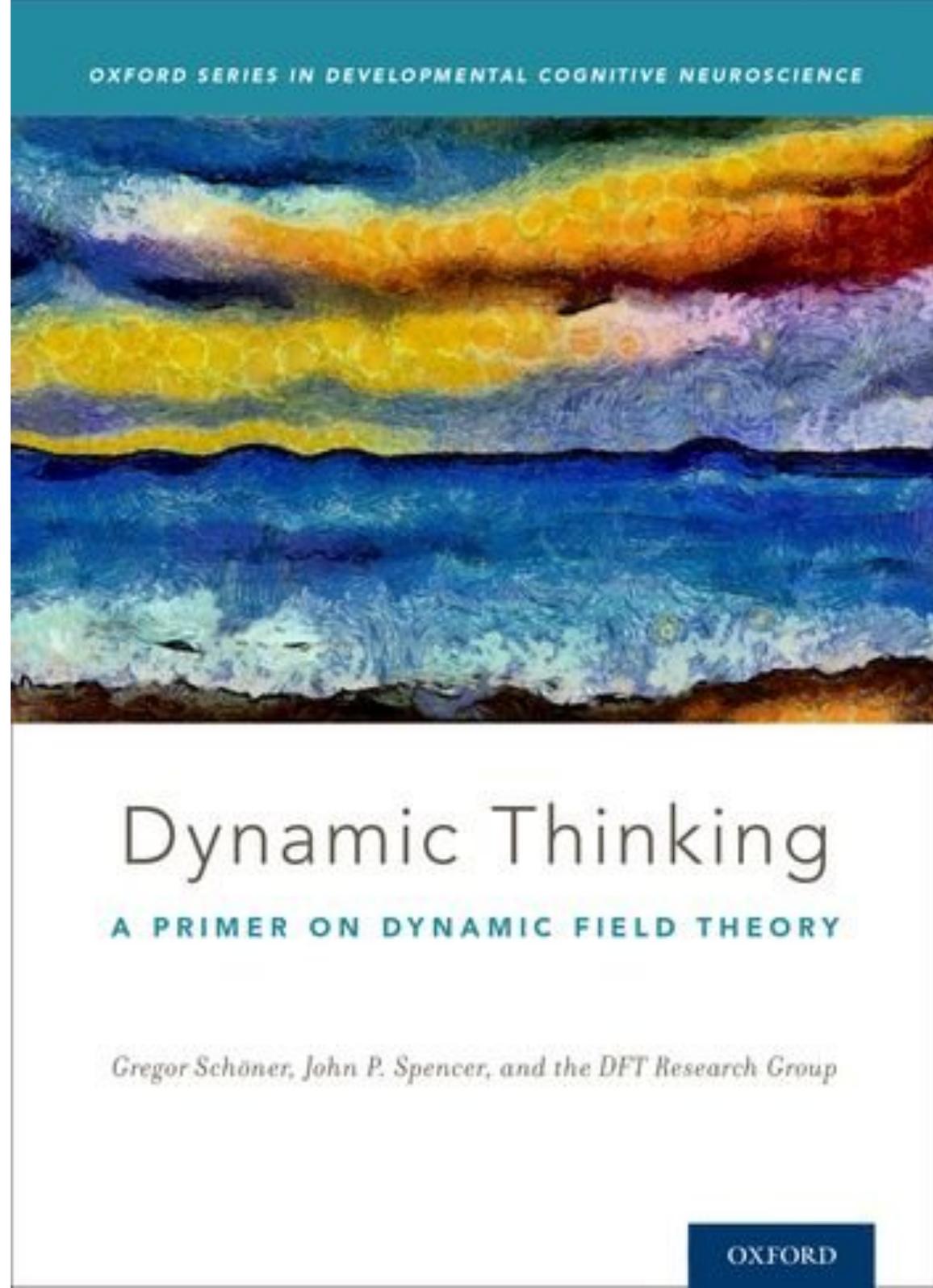
$$\tau \dot{u}(x, t) = -u(x, t) + h + s(x, t) + \int dx' w(x-x') \sigma(u(x'))$$

Relationship to the dynamics of discrete activation variables



=> simulation

■ dynamicfieldtheory.org



Attractors and their instabilities

■ input driven solution (sub-threshold)

■ self-stabilized solution (peak, supra-threshold)

■ selection / selection instability

■ working memory / memory instability

■ boost-driven detection instability



detection instability



reverse detection instability

Noise is critical near instabilities

Dynamic regimes

- which attractors and instabilities arise as input patterns are varied
- examples
 - “perceptual regime”: mono-stable sub-threshold => bistable sub-threshold/peak => mono-table peak..
 - “working memory regime” bistable sub-threshold/peak => mono-table peak.. without mono-stable sub-threshold
 - single (“selective”) vs. multi-peak regime

Case study: DFT account of sensory-motor decision making

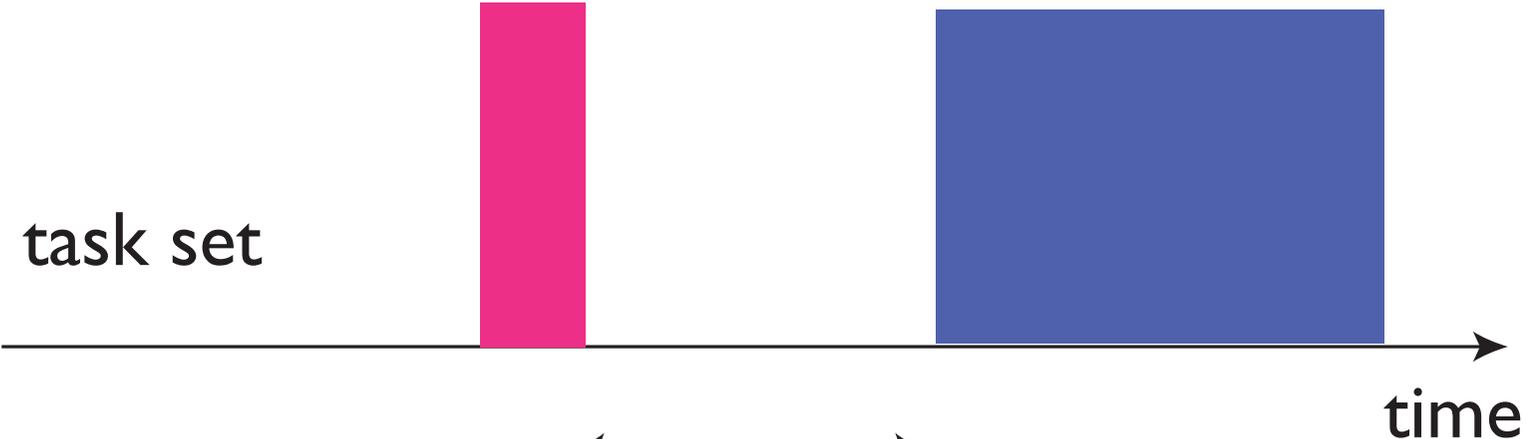
- assessed in reaction-time tasks
- information processing: how much information is processed...
- DFT: contents of task matters... embodiment
- DFT: decisions evolve continuously in time and metric space

Reaction time (RT) paradigm

imperative
signal=
go signal

response

task set



Model the task set by preshape

■ which choices are available

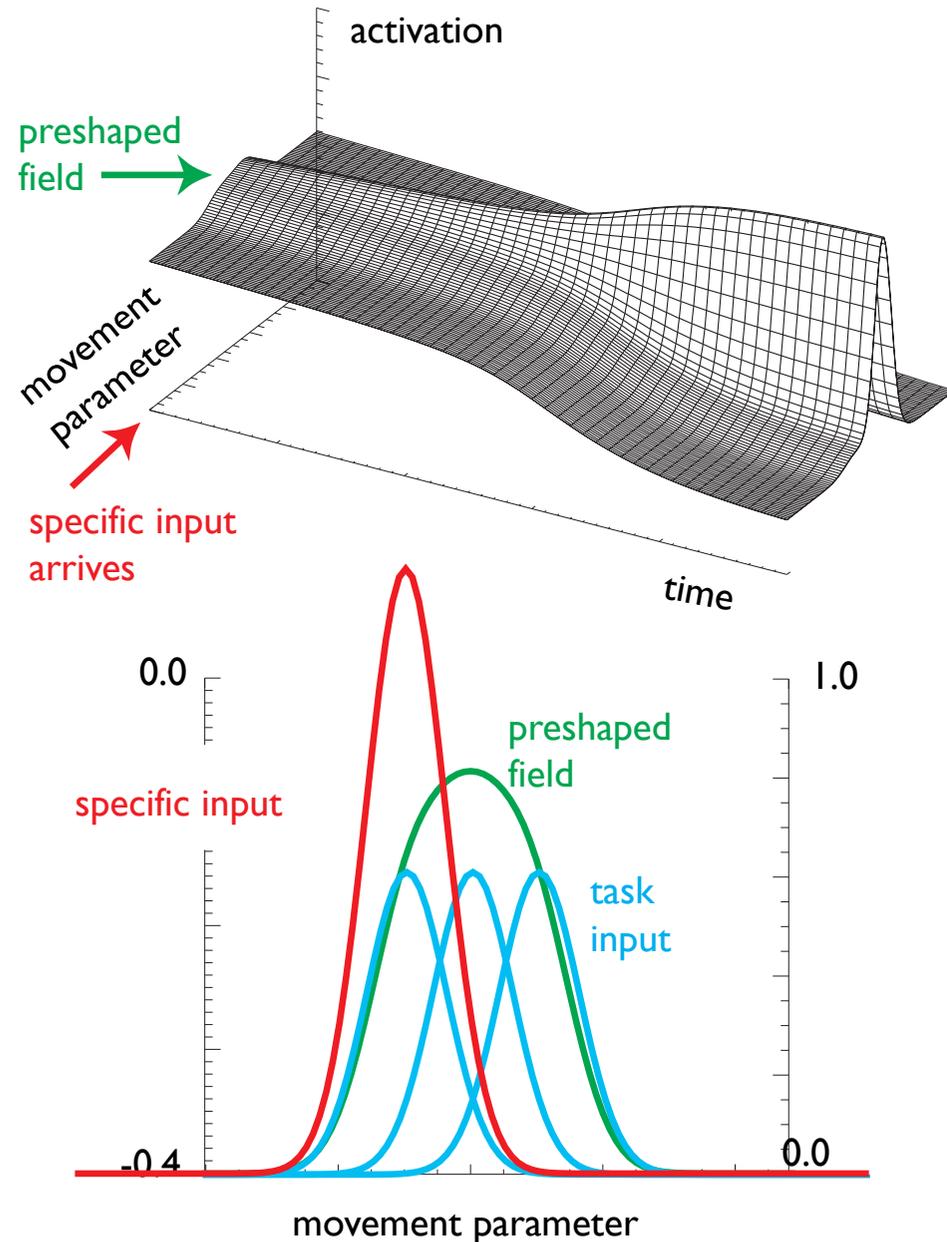
■ how many, how probable

■ how different from each other

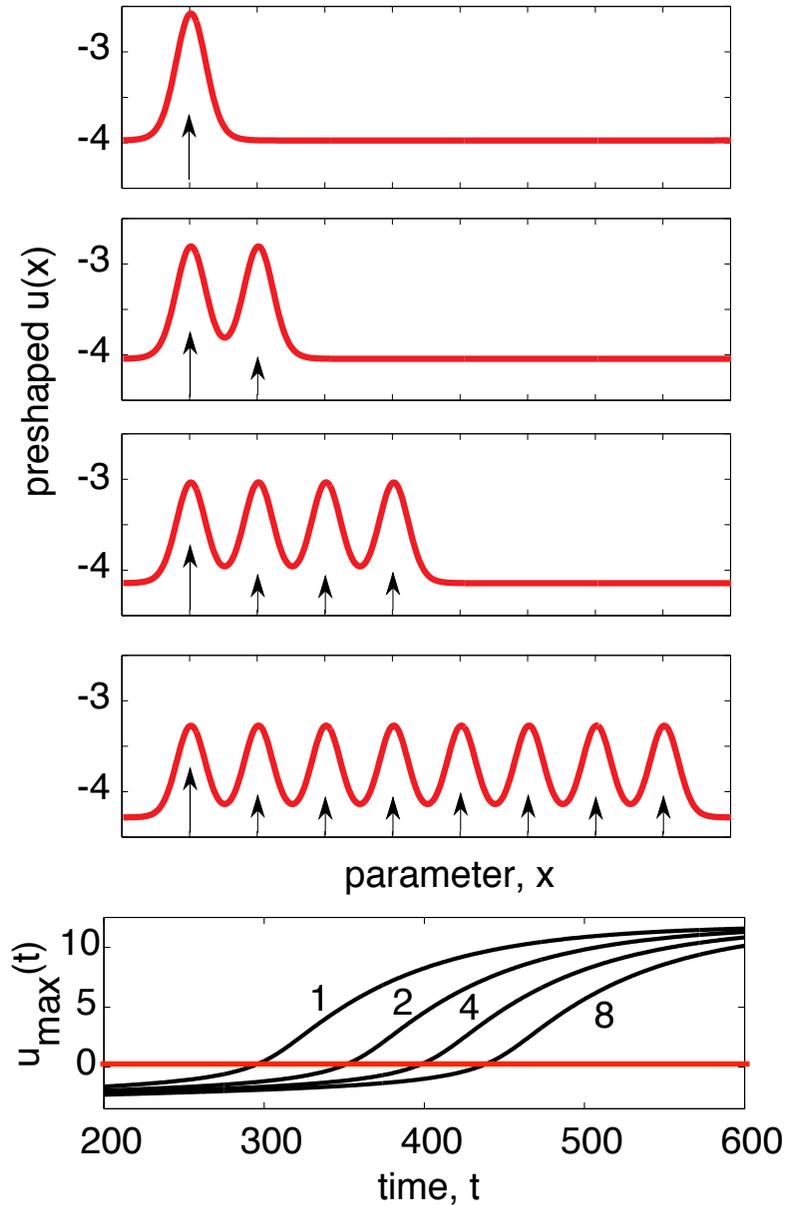
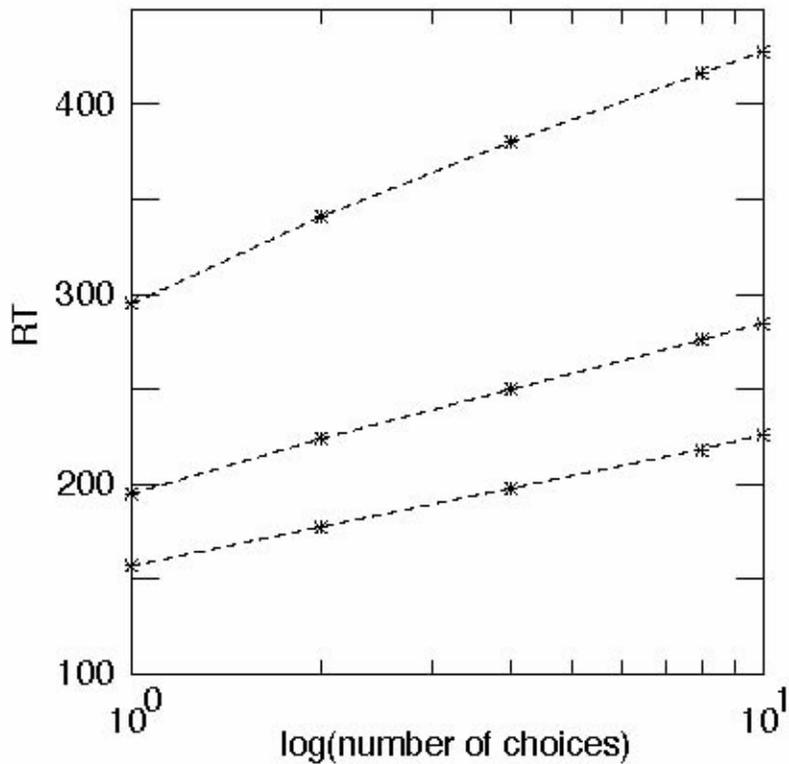
■ how easy to recognize/perform

■ choices known to the participant before the imperative signal comes

■ => preshape the field



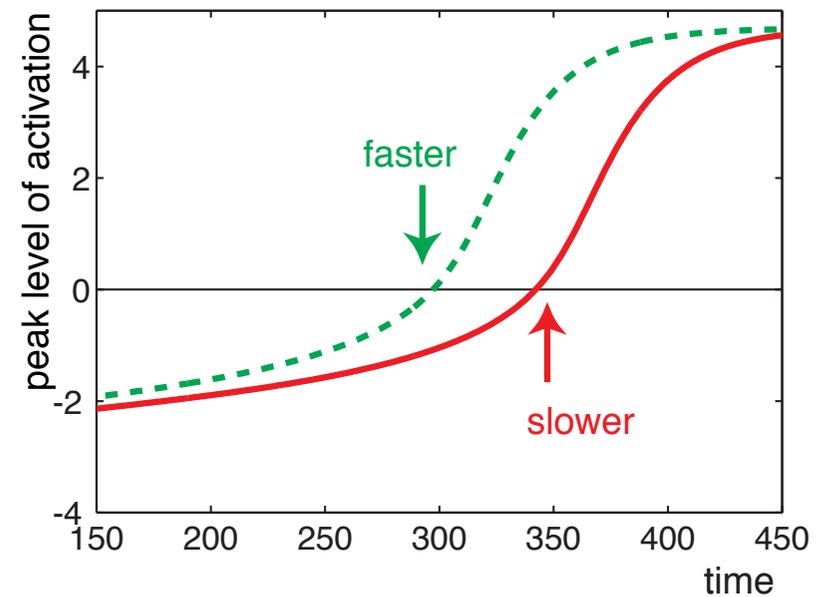
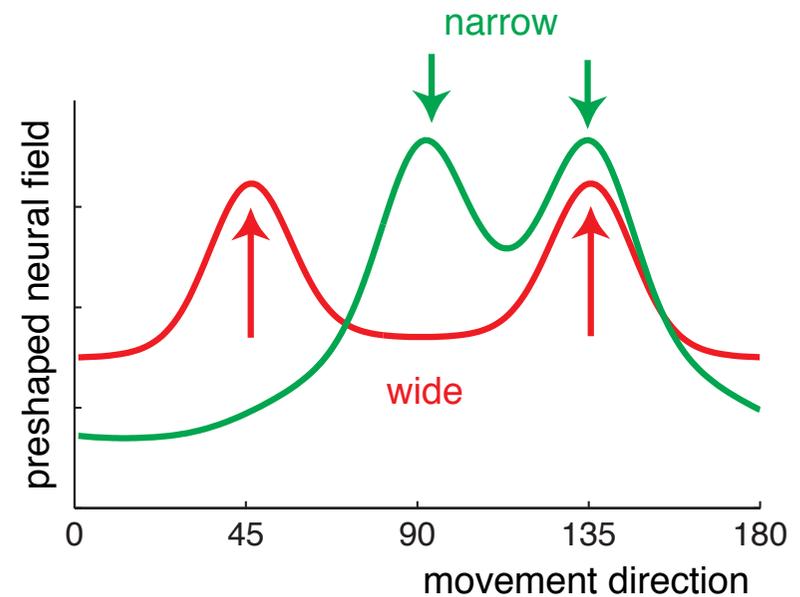
Hick's law: RT increases with # choices



[Erlhagen, Schöner, Psych Rev 2002]

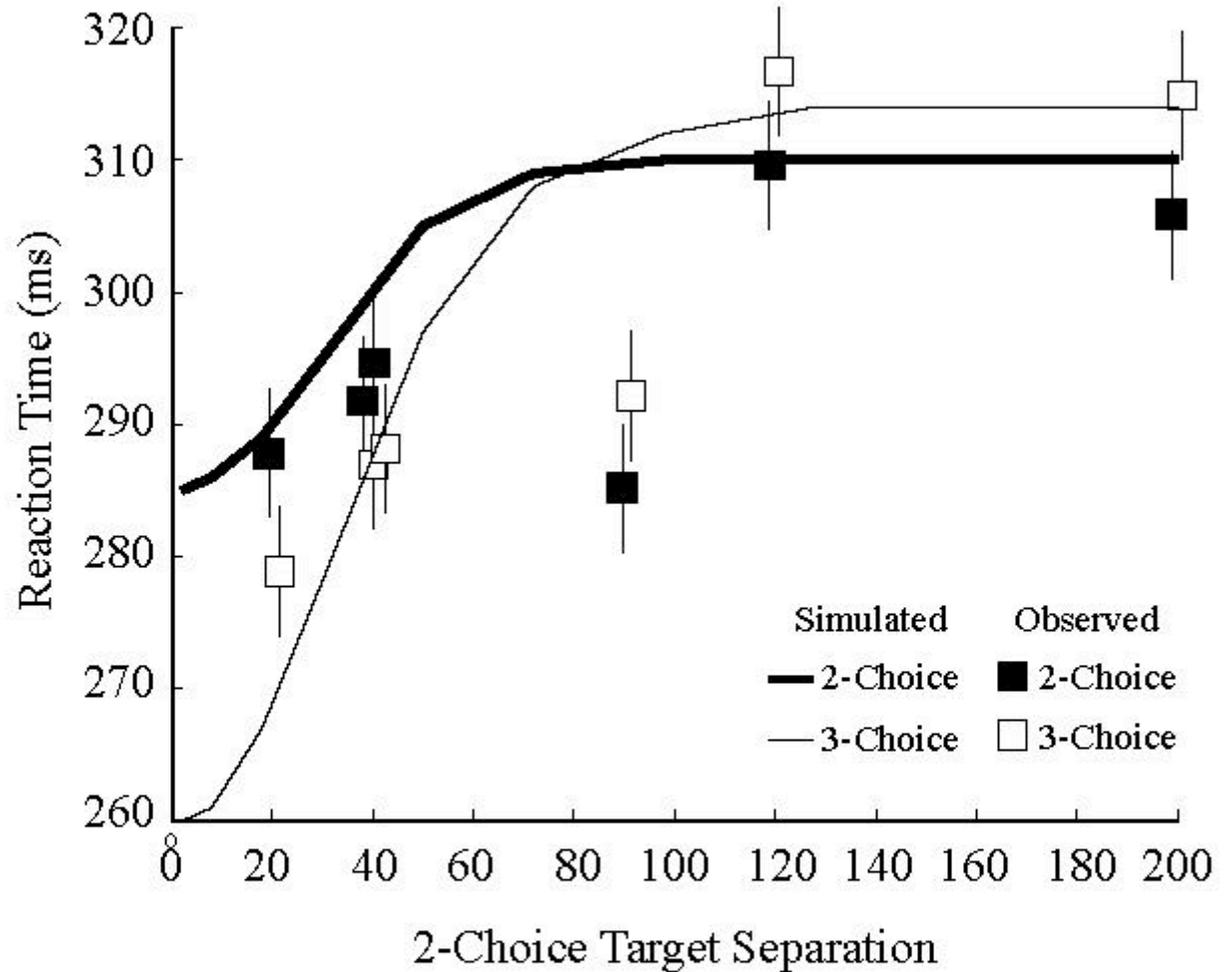
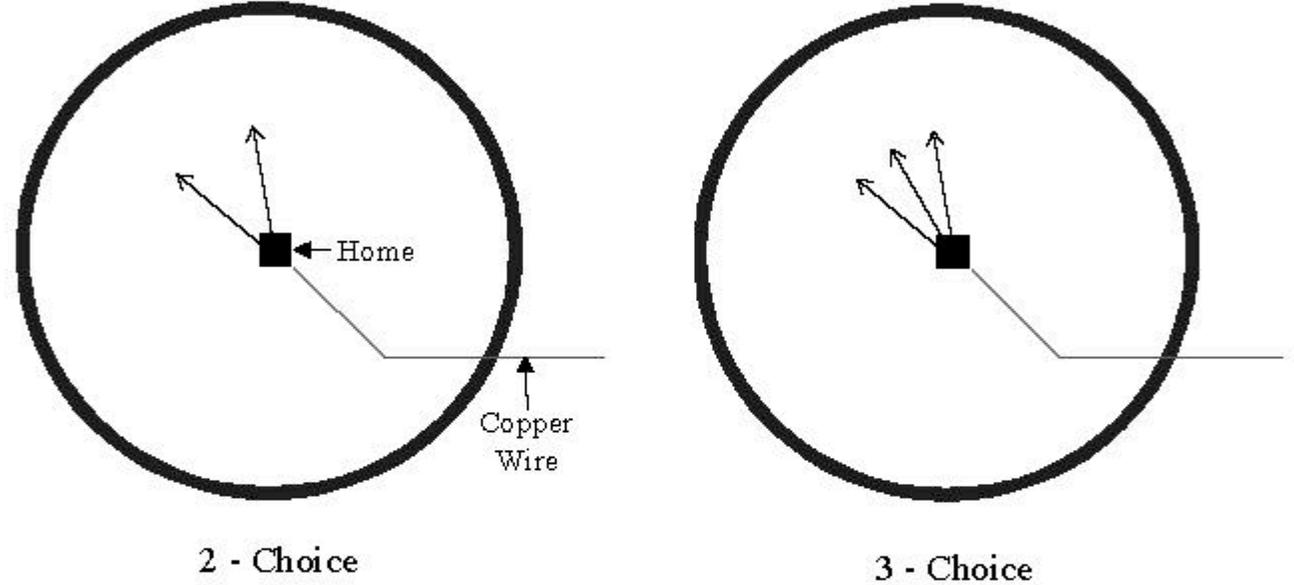
Metric effect

- predict faster response times for metrically close than for metrically far choices

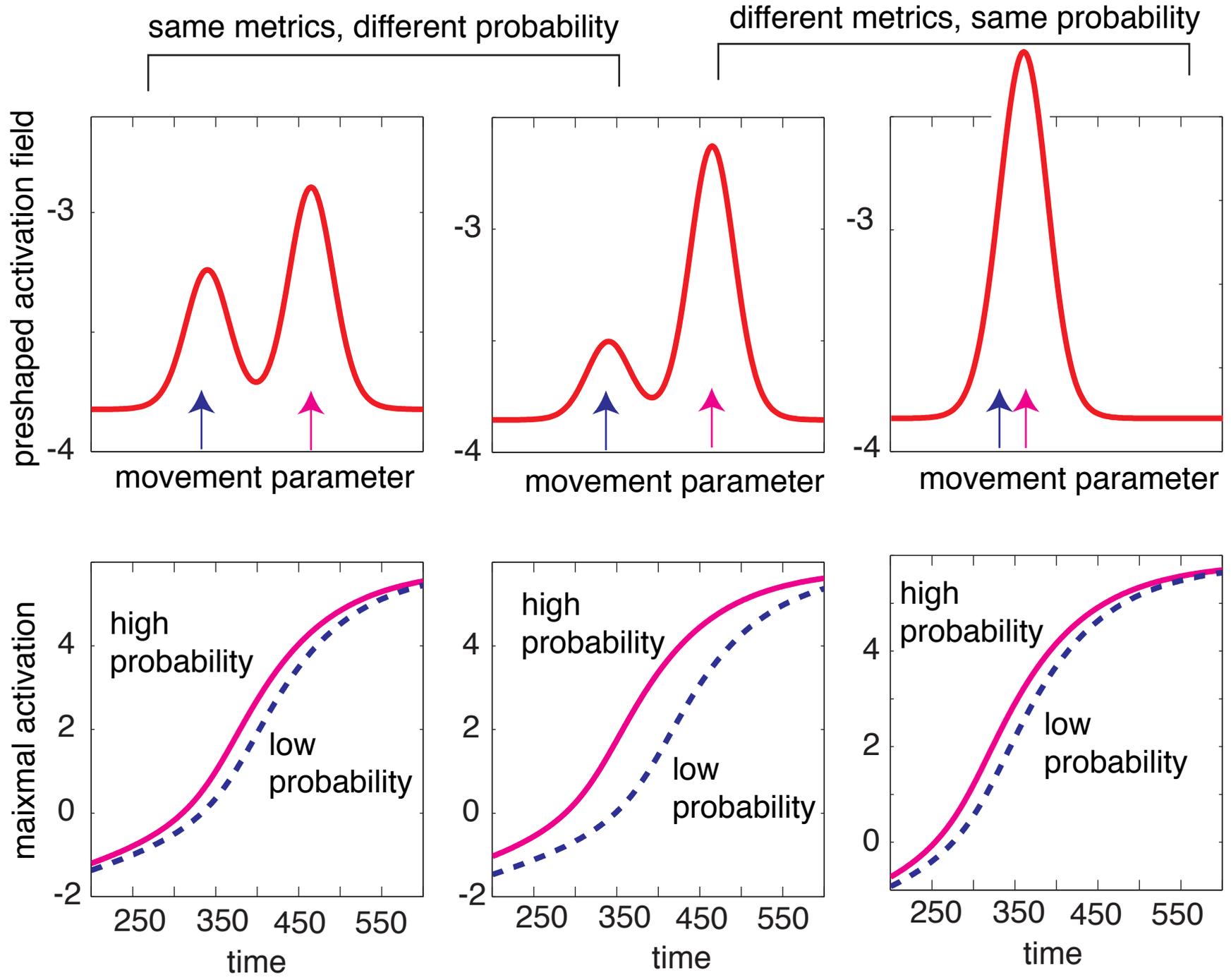


[from Schöner, Kopecz, Erlhagen, 1997]

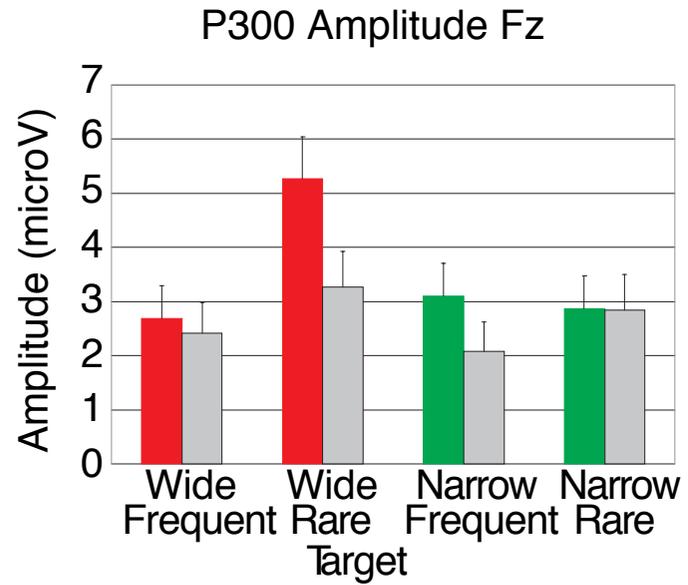
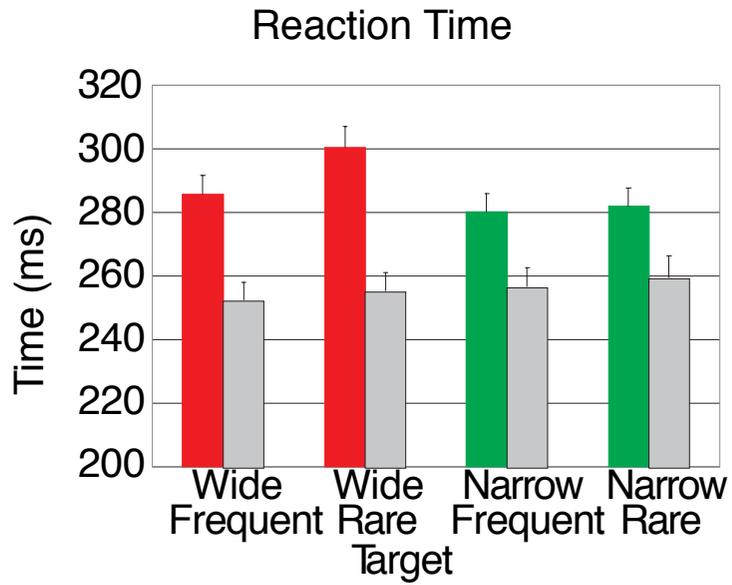
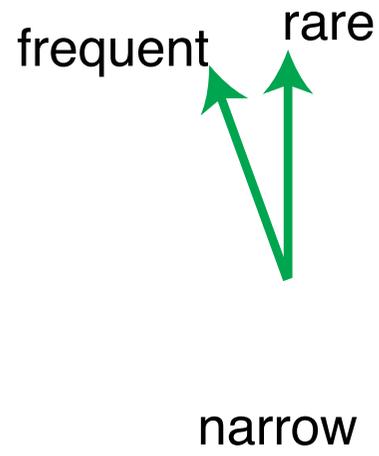
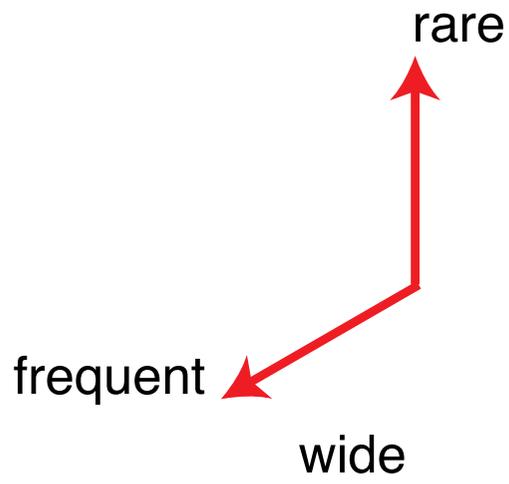
Metric effect: experiment



[McDowell, Jeka, Schöner]



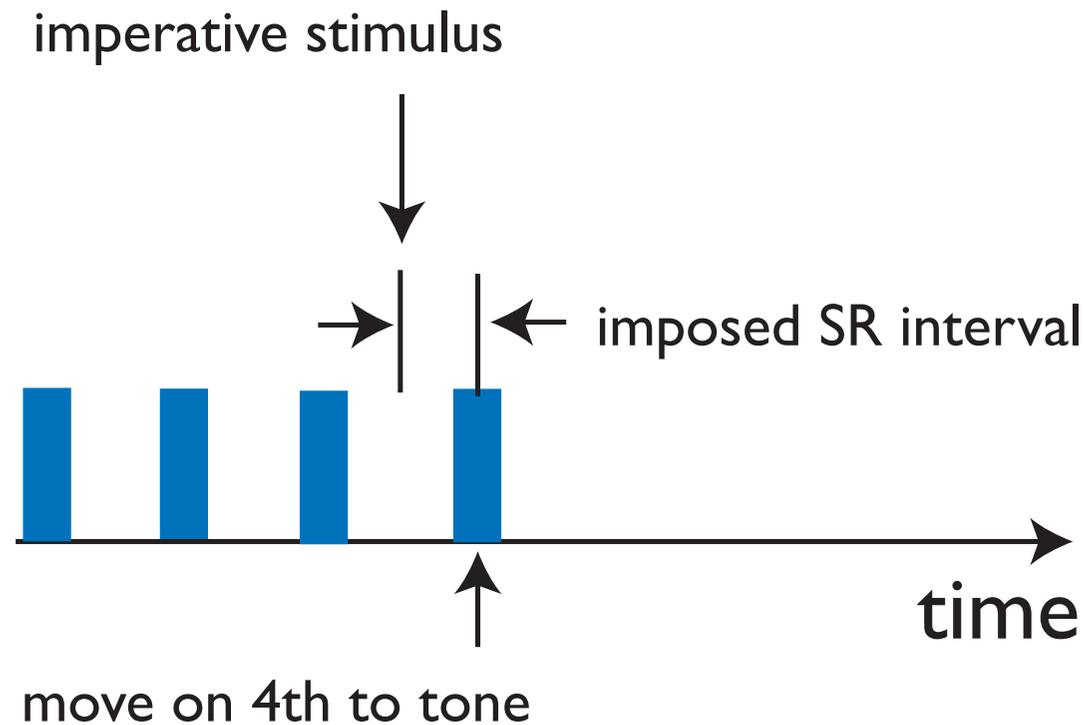
[from Erlhagen, Schöner: Psych. Rev. 2002]



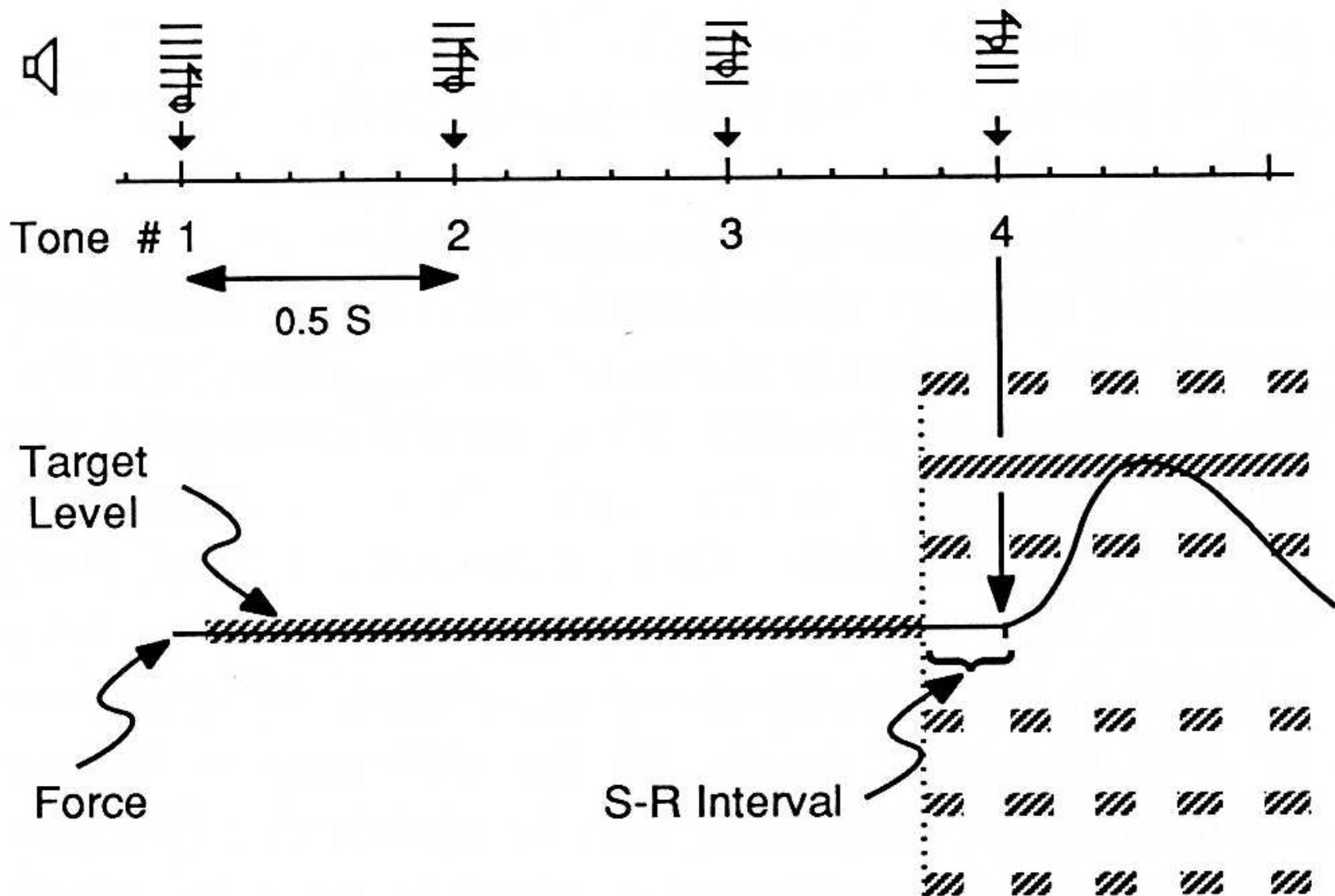
[from McDowell, Jeka, Schöner, Hatfield, 2002]

Continuous evolution of sensory-motor decisions

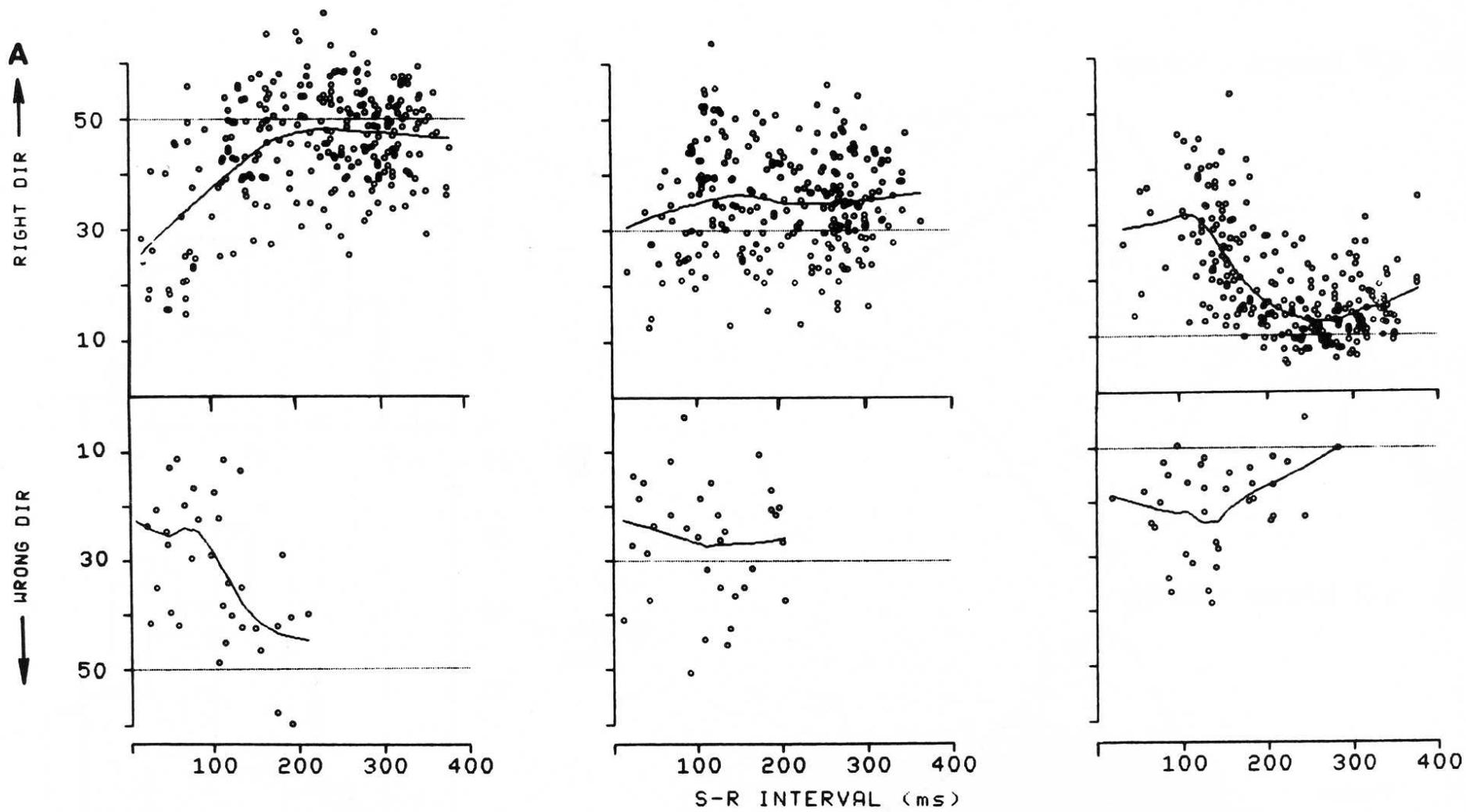
- timed movement initiation paradigm



[Ghez and colleagues, 1988 to 1990's]

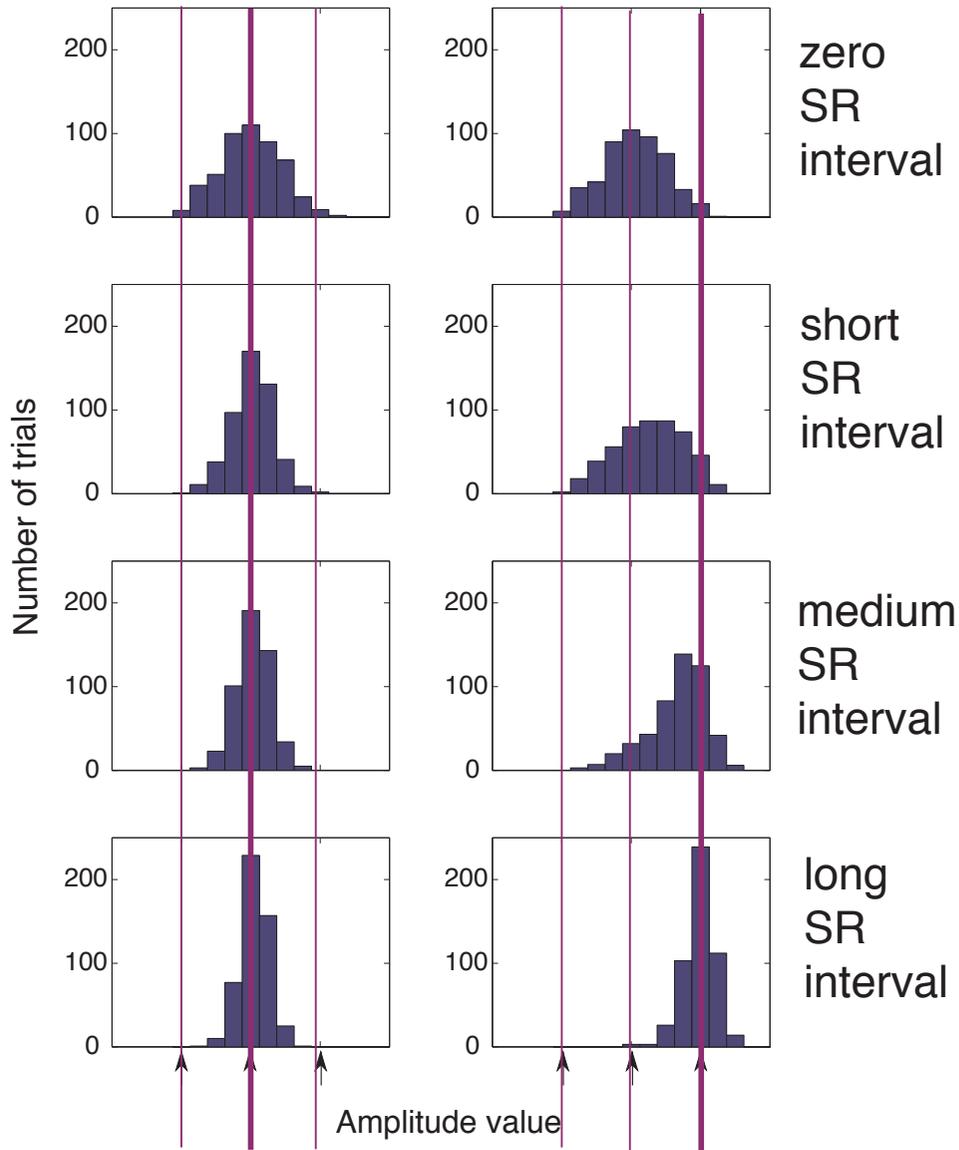


[Favilla et al. 1989]

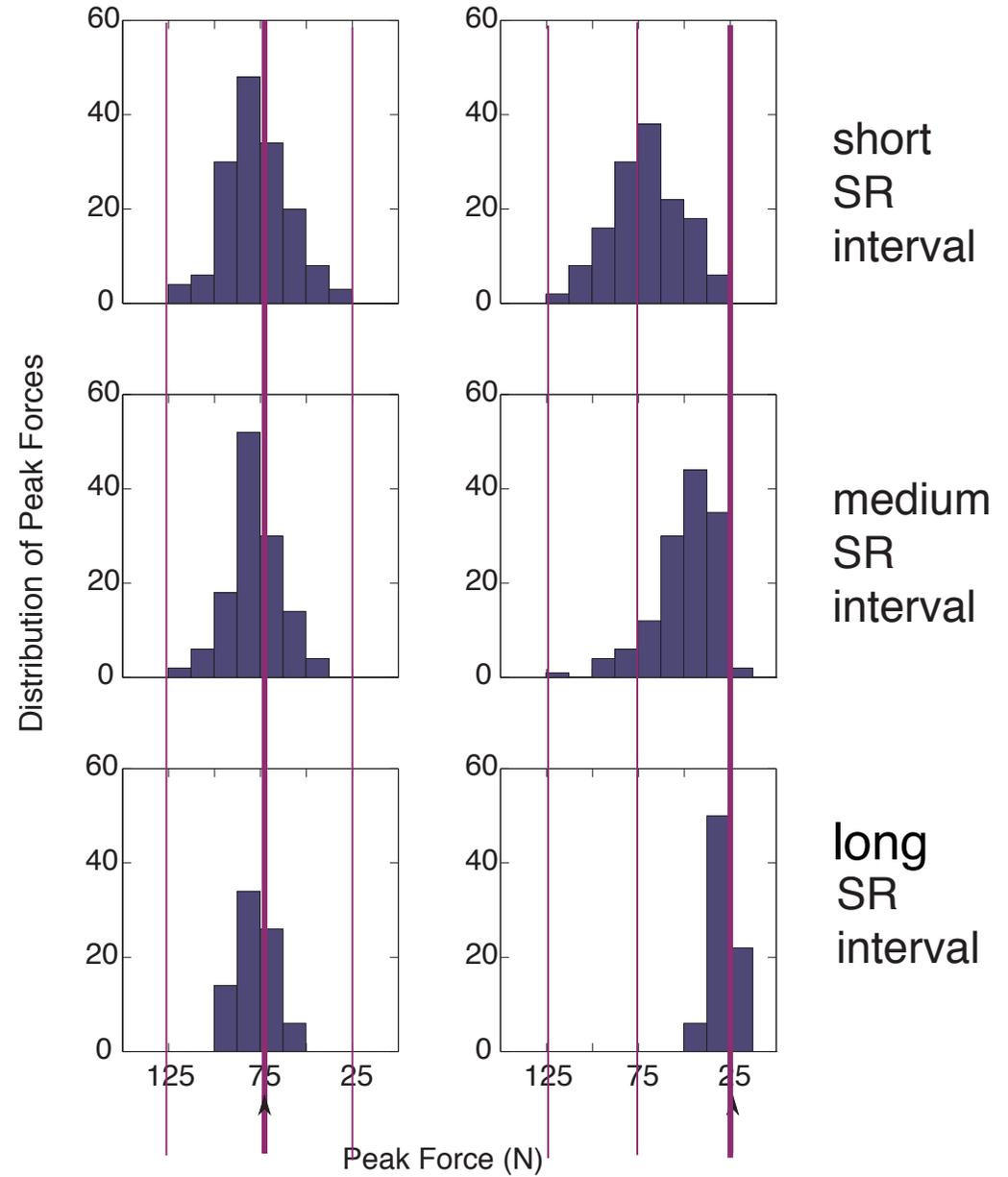


[Favilla et al. 1989]

theoretical account for Henig et al.



Experimental results of Henig et al

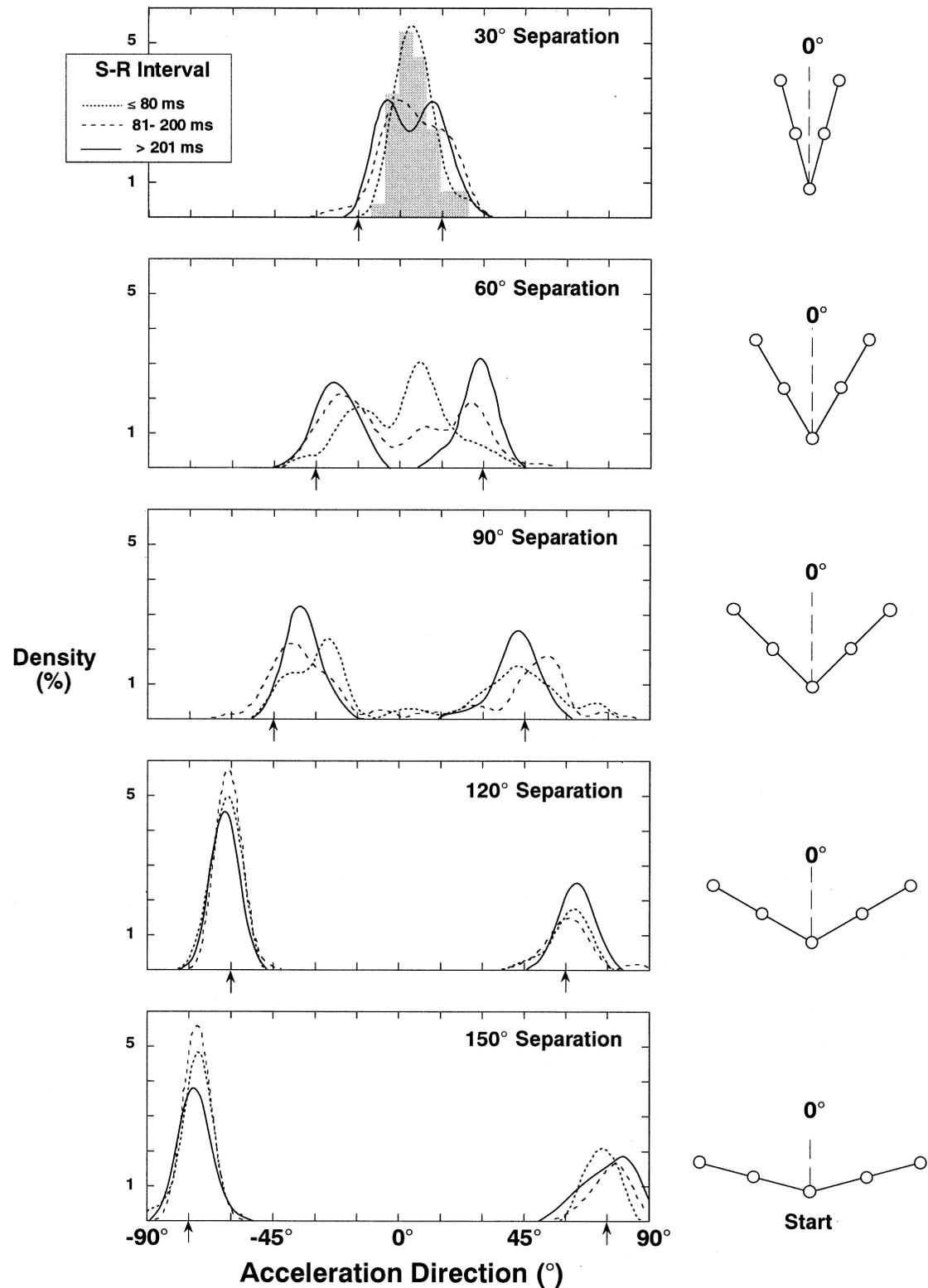


[Erlhagen, Schöner: Psychological Review 109, 545–572 (2002)]

Metric effect

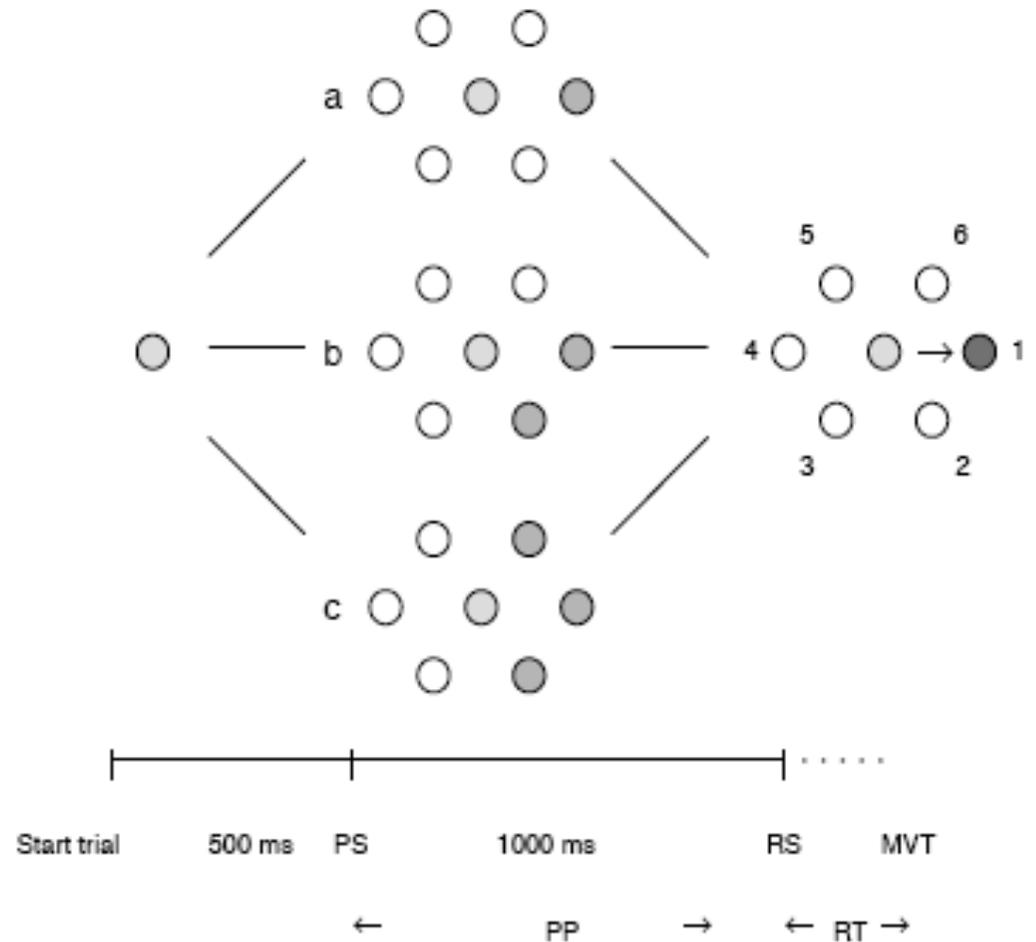
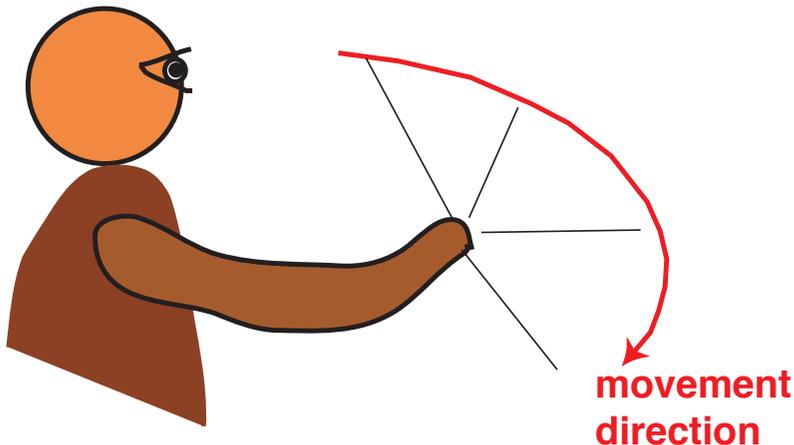
- directly observe the preshaped field ...
- and infer the width of preshape peaks

[Ghez et al 1997]

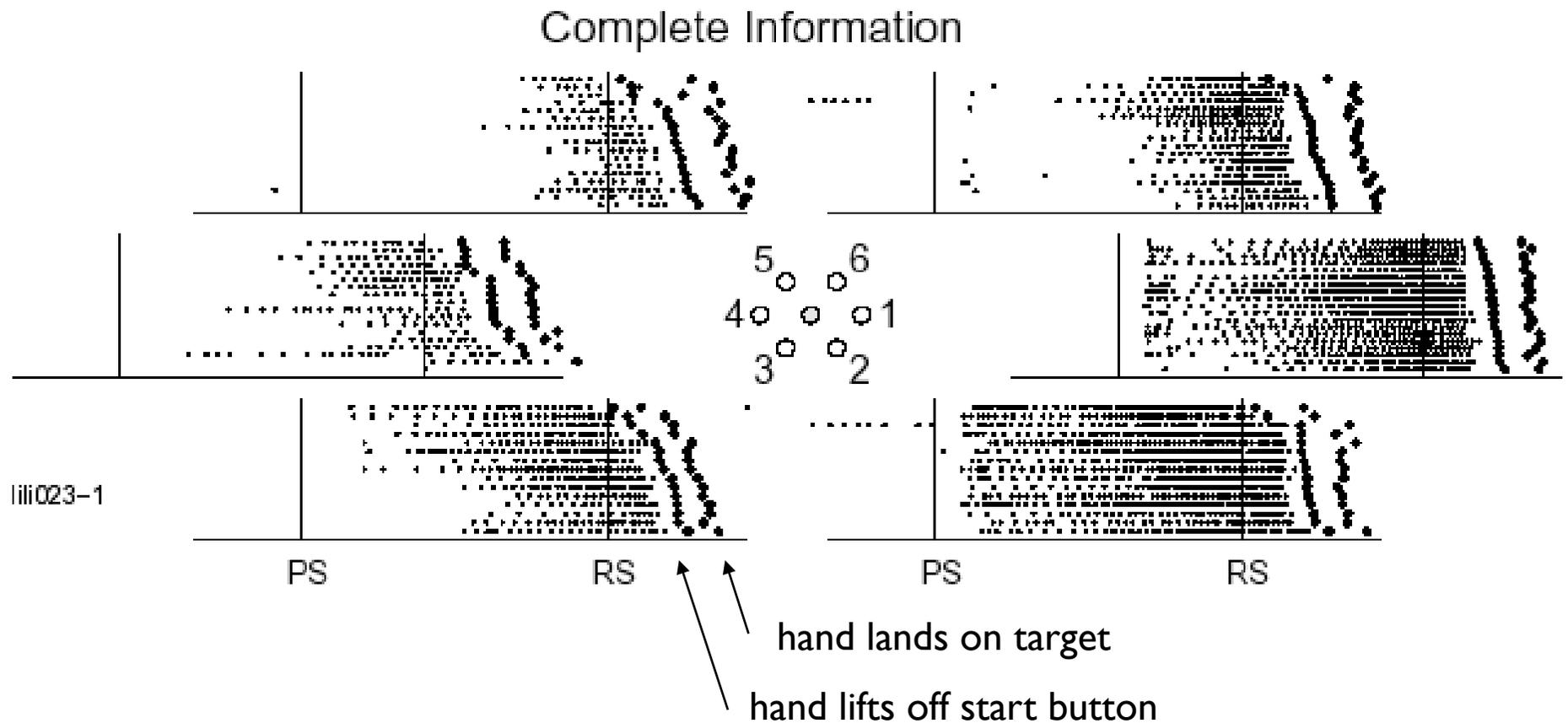


Neural observation of field

- center-out sensori-motor selection task
- varying prior information
- macaque

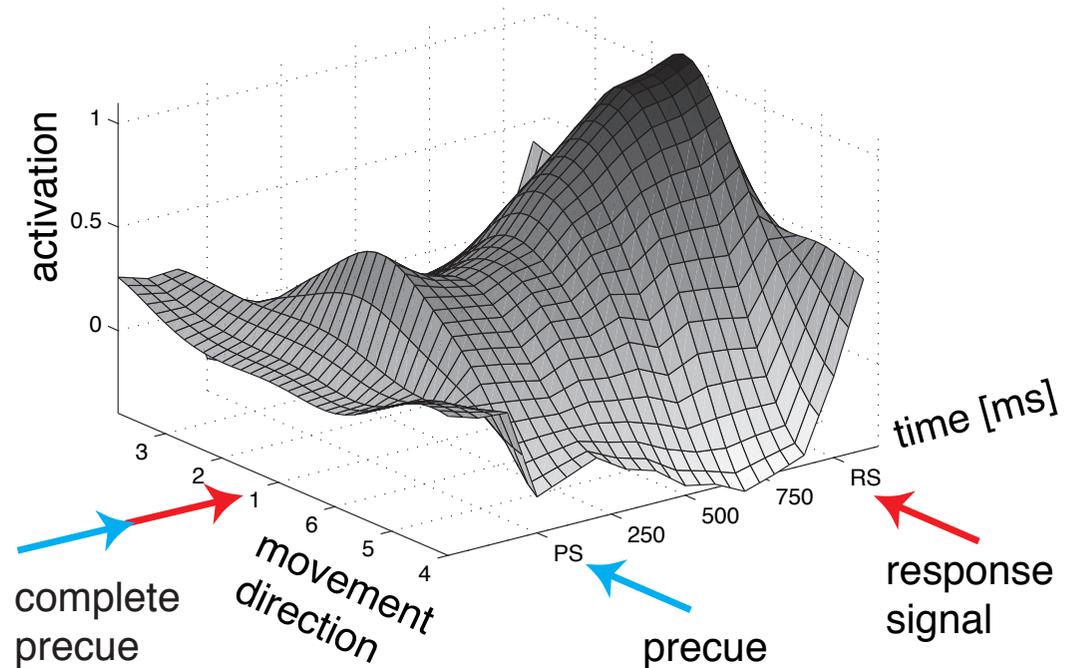
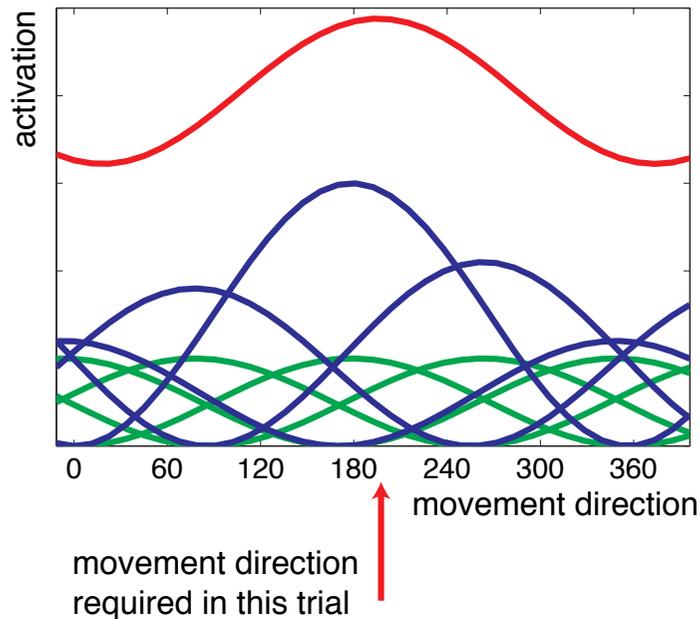


Tuning of neurons in MI to movement direction



Distribution of Population Activation (DPA) \Leftrightarrow neural field

Distribution of population activation =
 $\sum_{\text{neurons}} \text{tuning curve} * \text{current firing rate}$



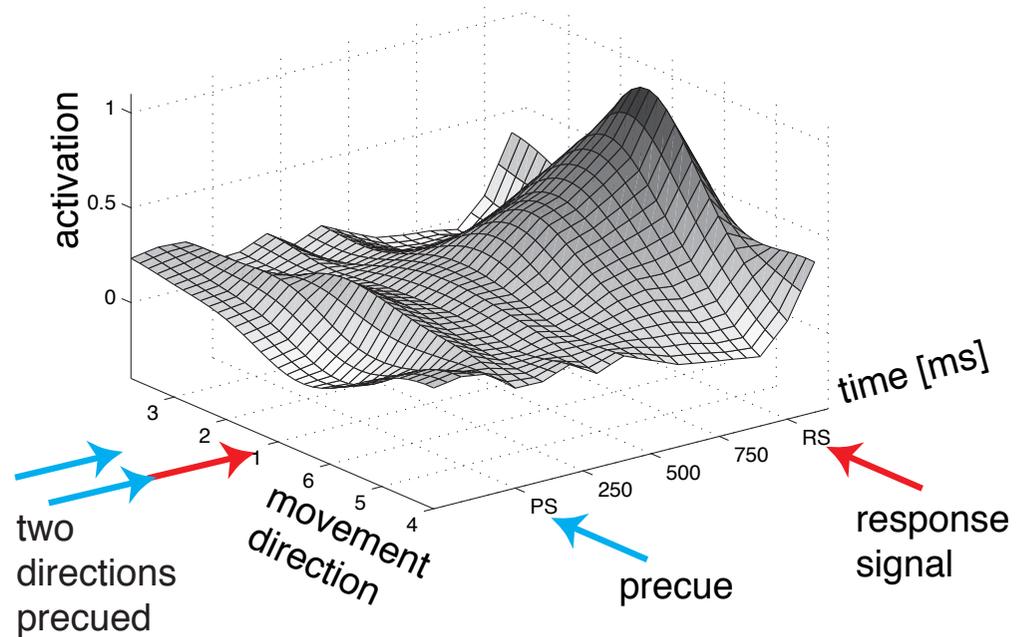
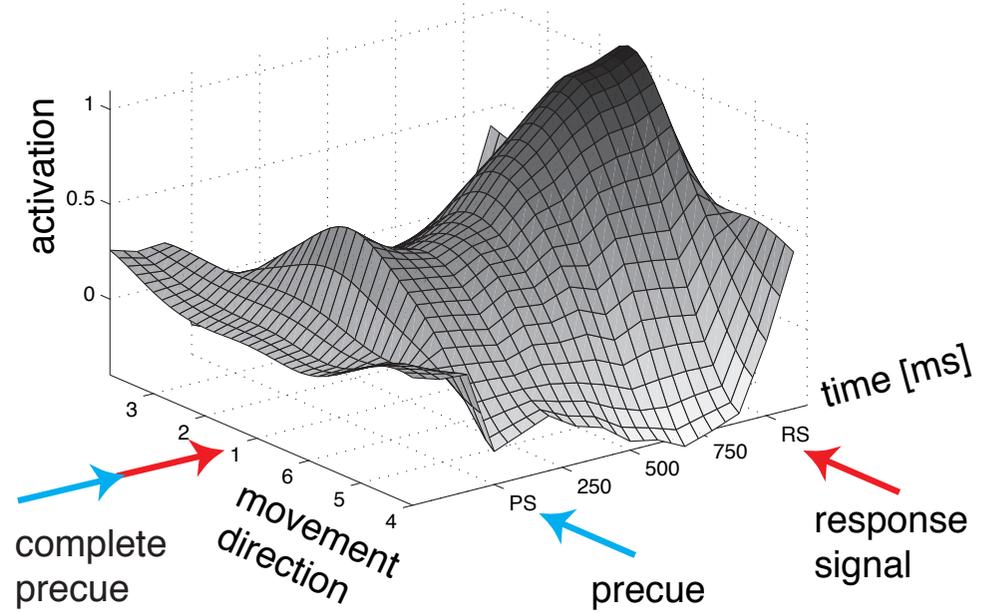
■ note: neurons are not **localized** within DPA!

[Bastian, Riehle, Schöner, 2003]

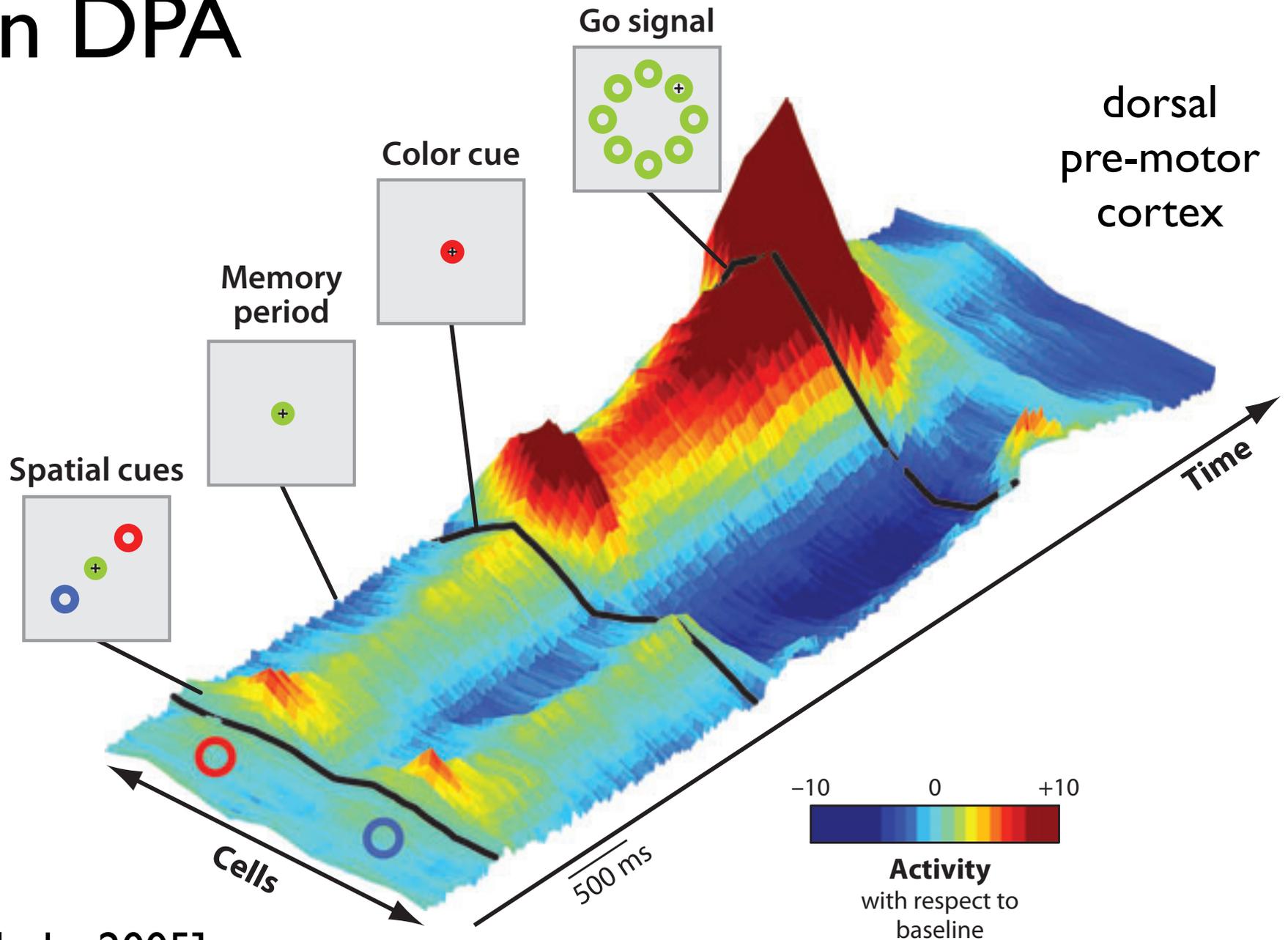
DPA

- note: neurons are not **localized** within DPA!
- [notion of projection cortical neurons really are sensitive to many dimensions
 - motor: arm configuration, force direction
 - visual: many feature dimensions such as spatial frequency, orientation, direction...
- => DPA is a **projection** from that high-dimensional space onto a single dimension]

DPA pre-shaped by pre-cue



Decision making in DPA



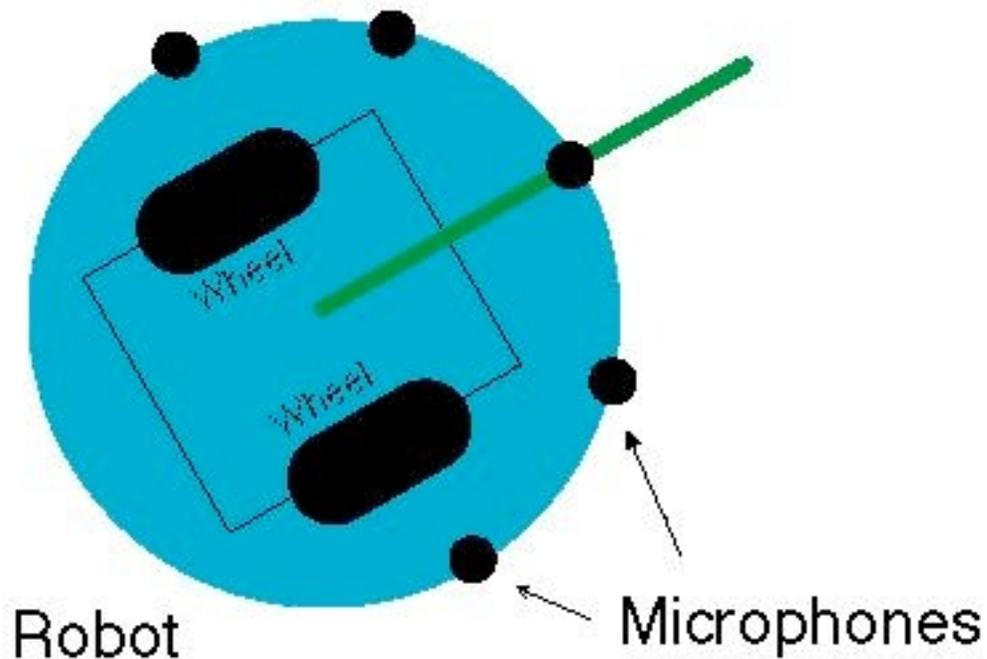
[Cisek, Kalaska 2005]

Case study: embodiment

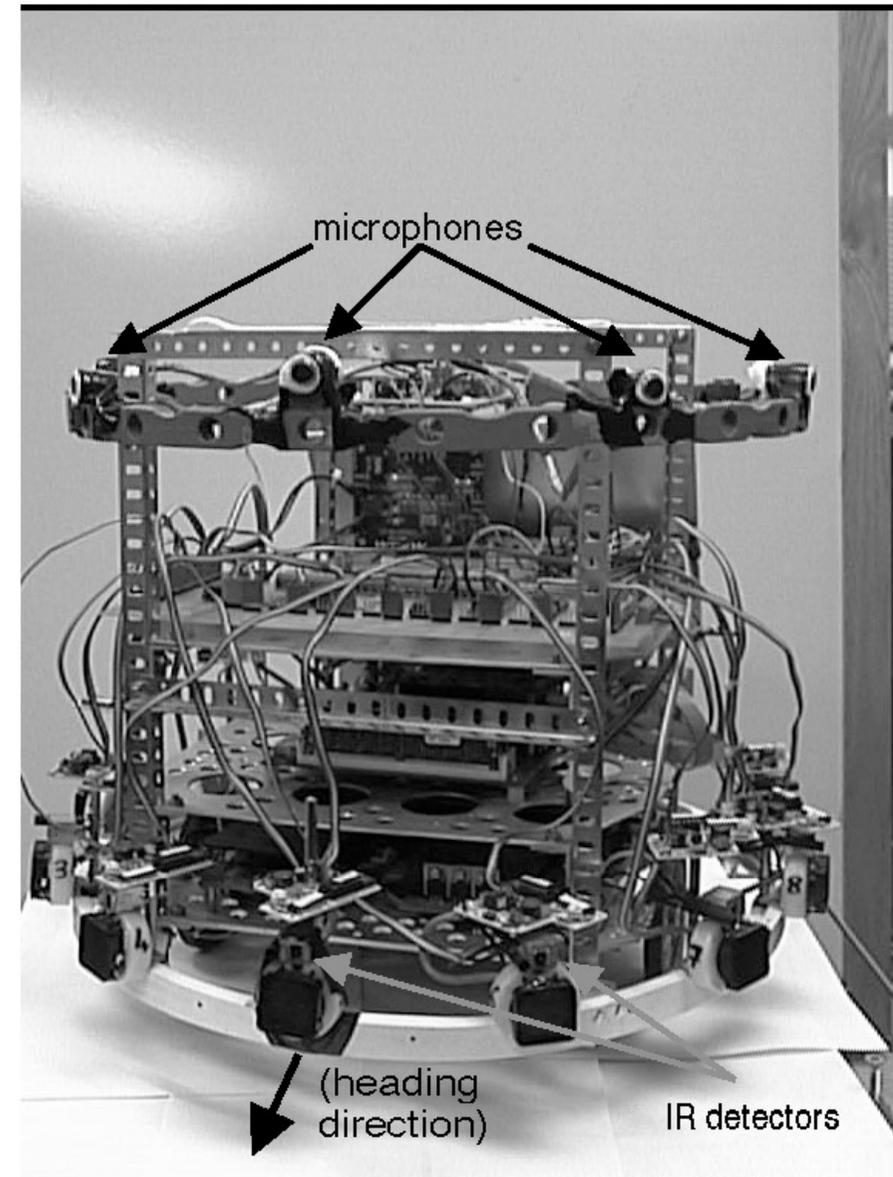
- neural dynamic fields can be linked to time-varying sensory inputs and can control motor systems in closed loop

Driving fields from sensory signals

- robot that orients toward sound sources

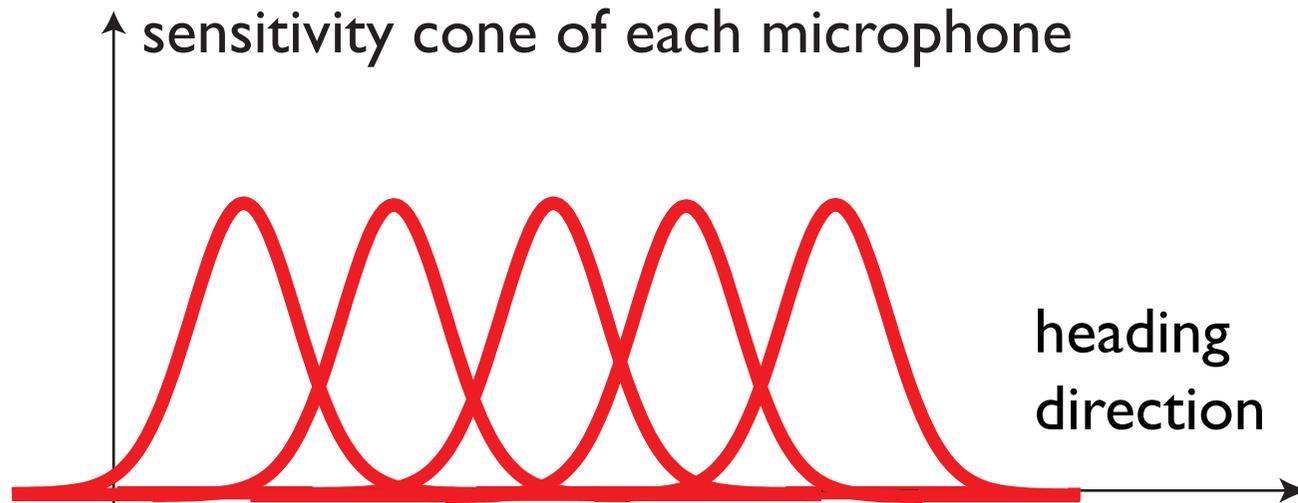


[from Bicho, Mallet, Schöner, Int J Rob Res, 2000]



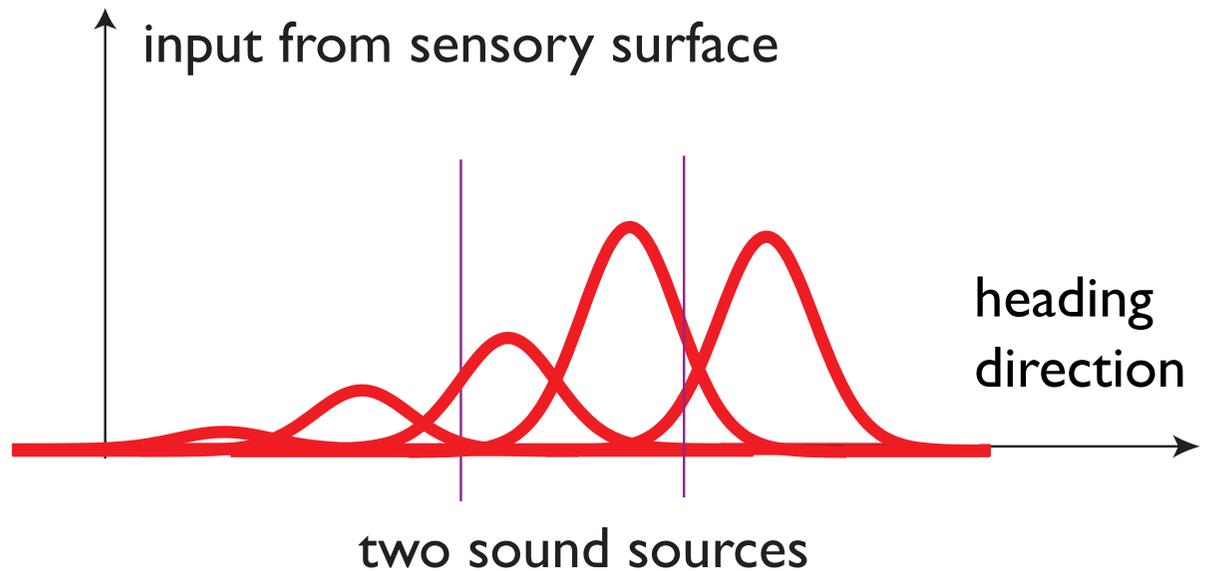
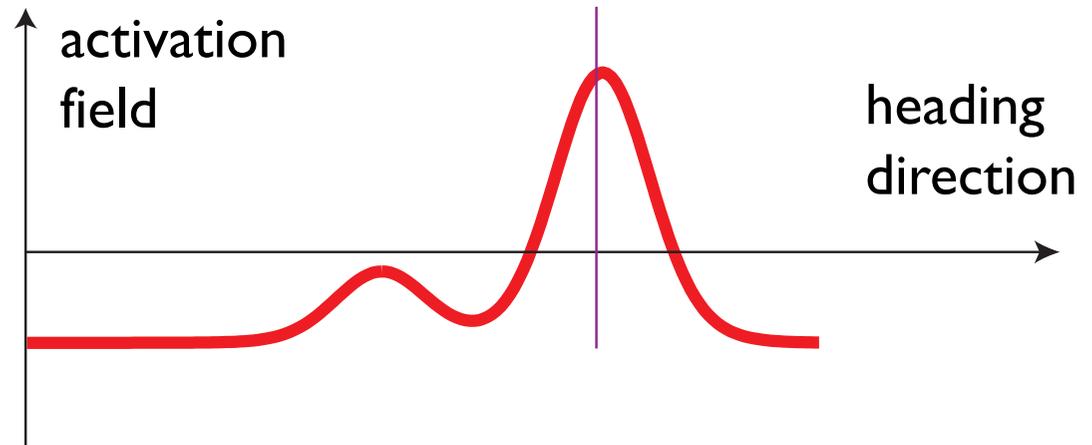
Sensory surface

- each microphone samples heading direction

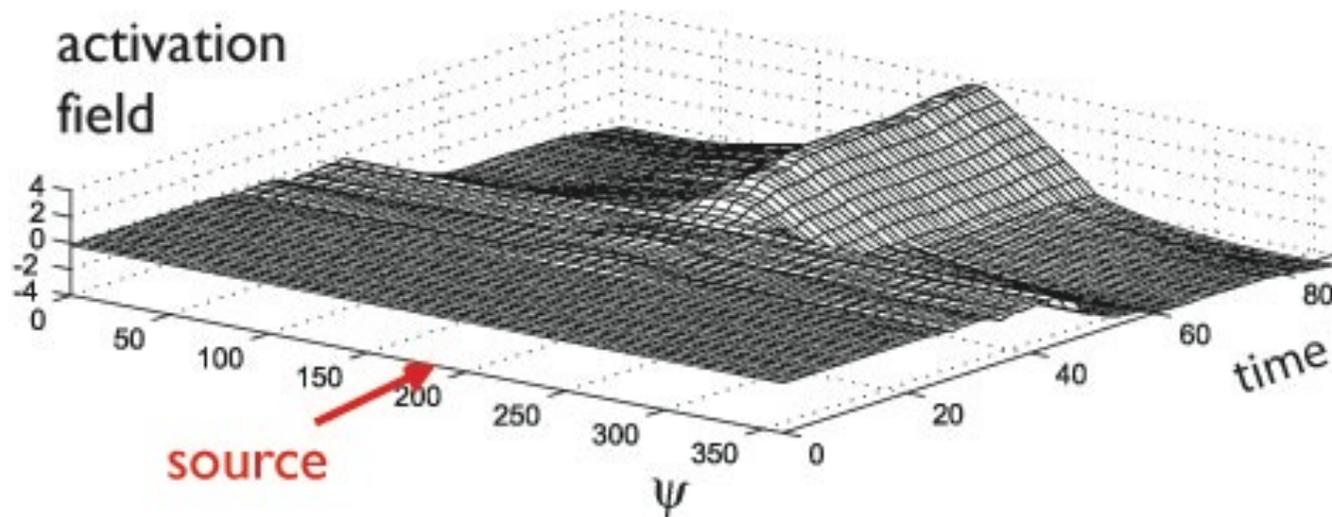
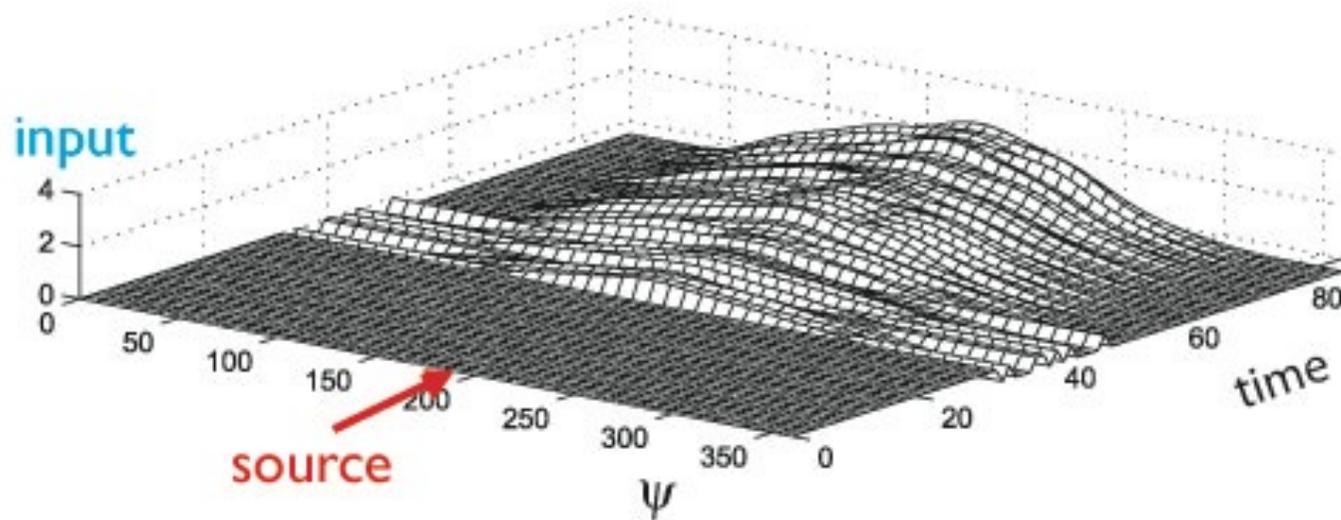


Sensory input

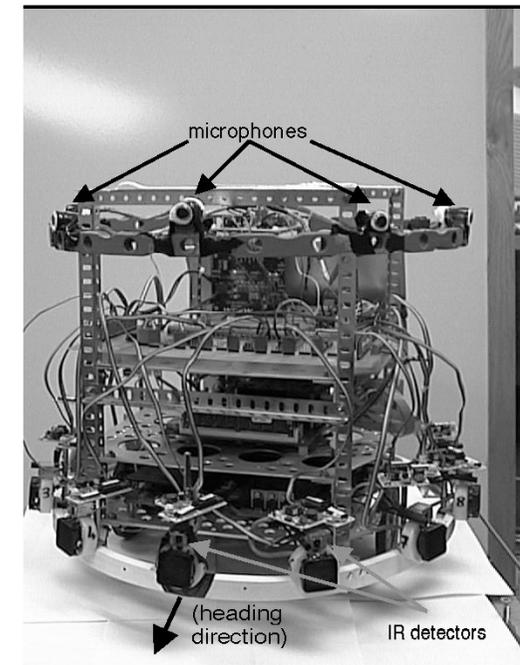
- each microphone provides input to the field =
loudness *
sensitivity cone



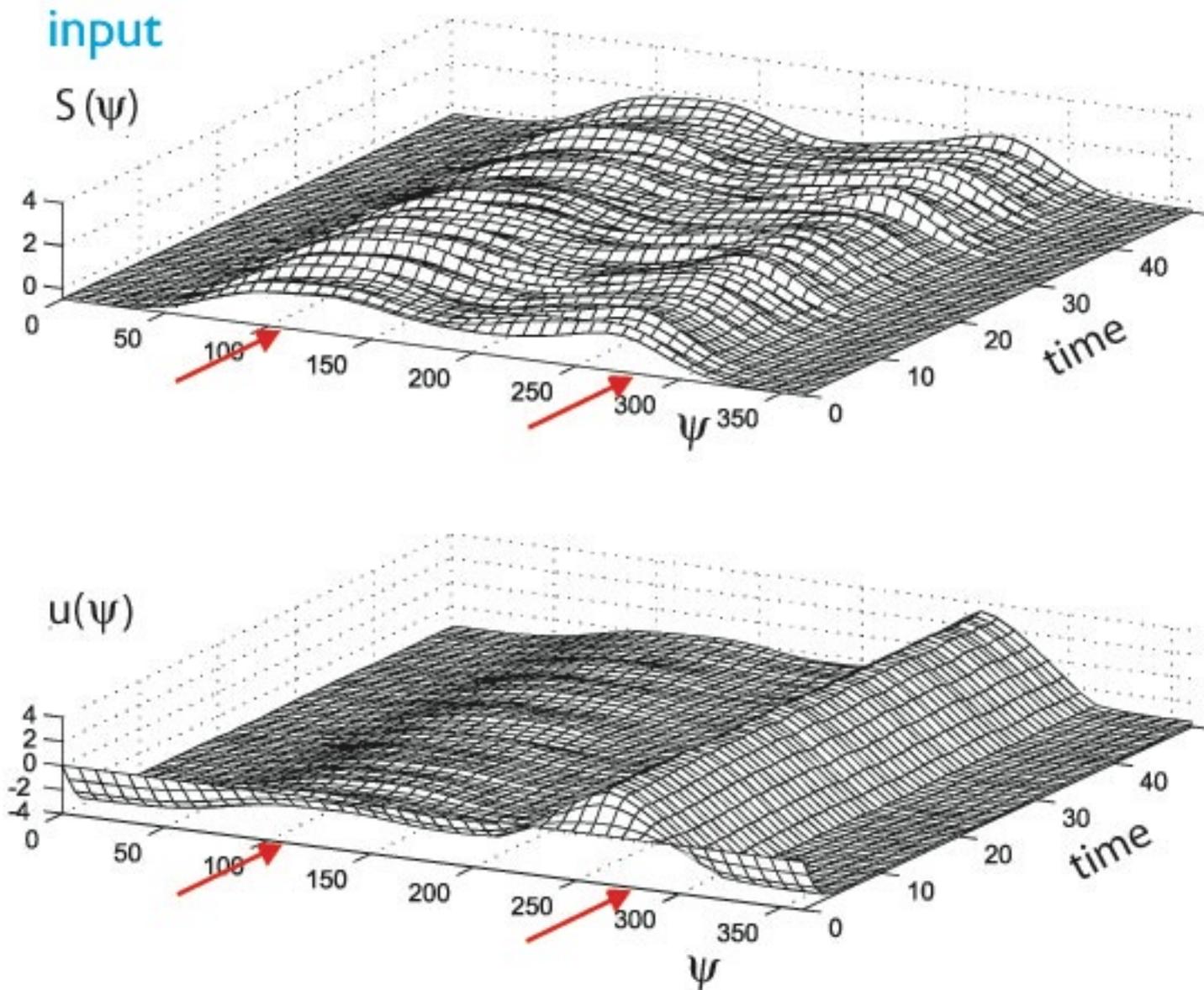
Detection instability as intensity of sound source increases



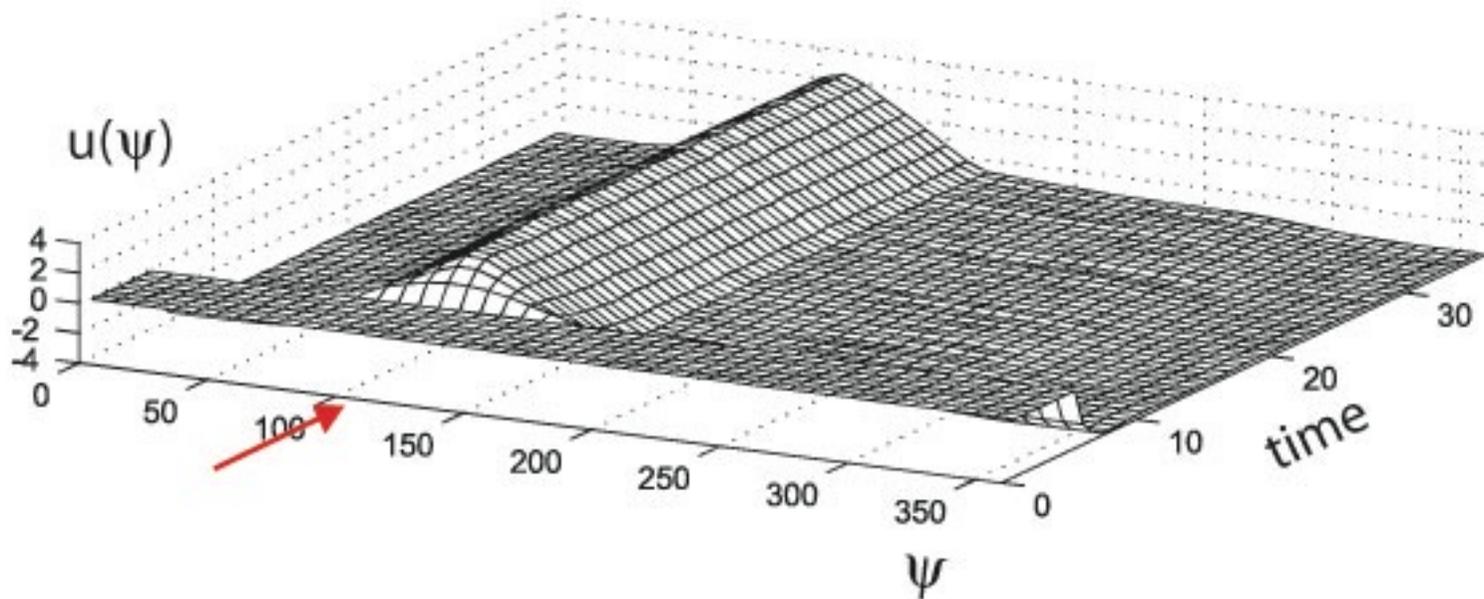
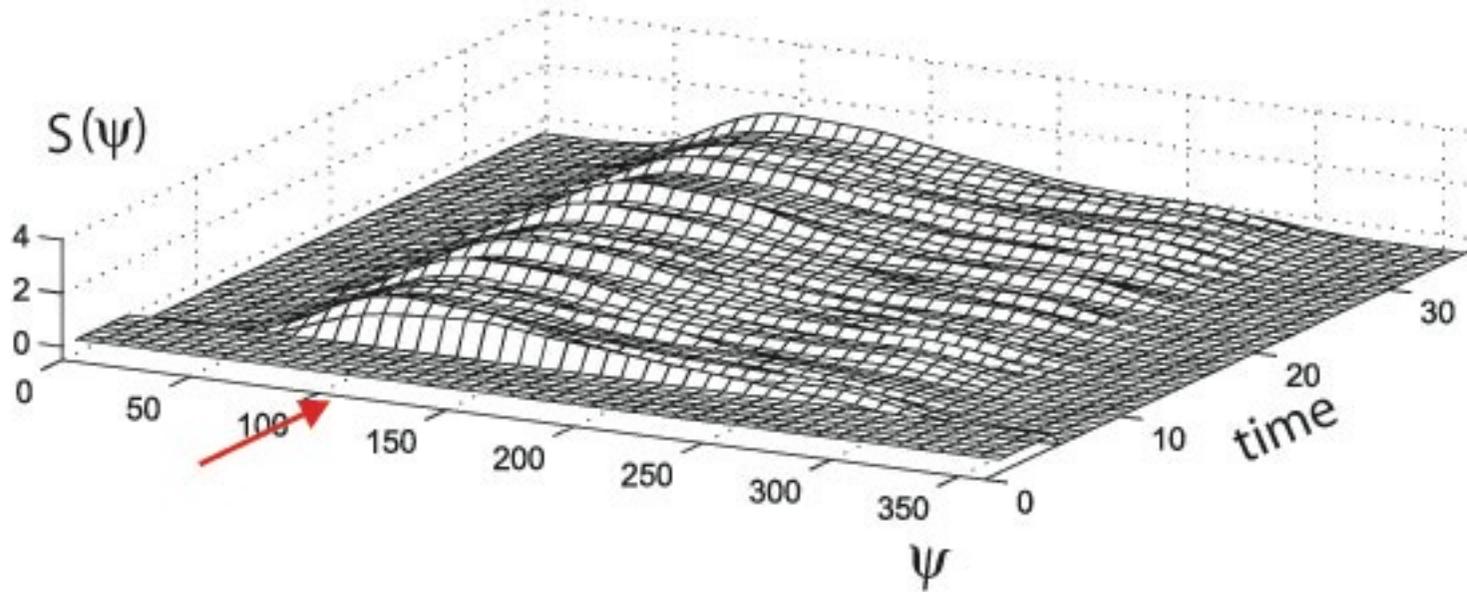
[from Bicho, Mallet, Schöner: Int. J. Rob. Res., 2000]



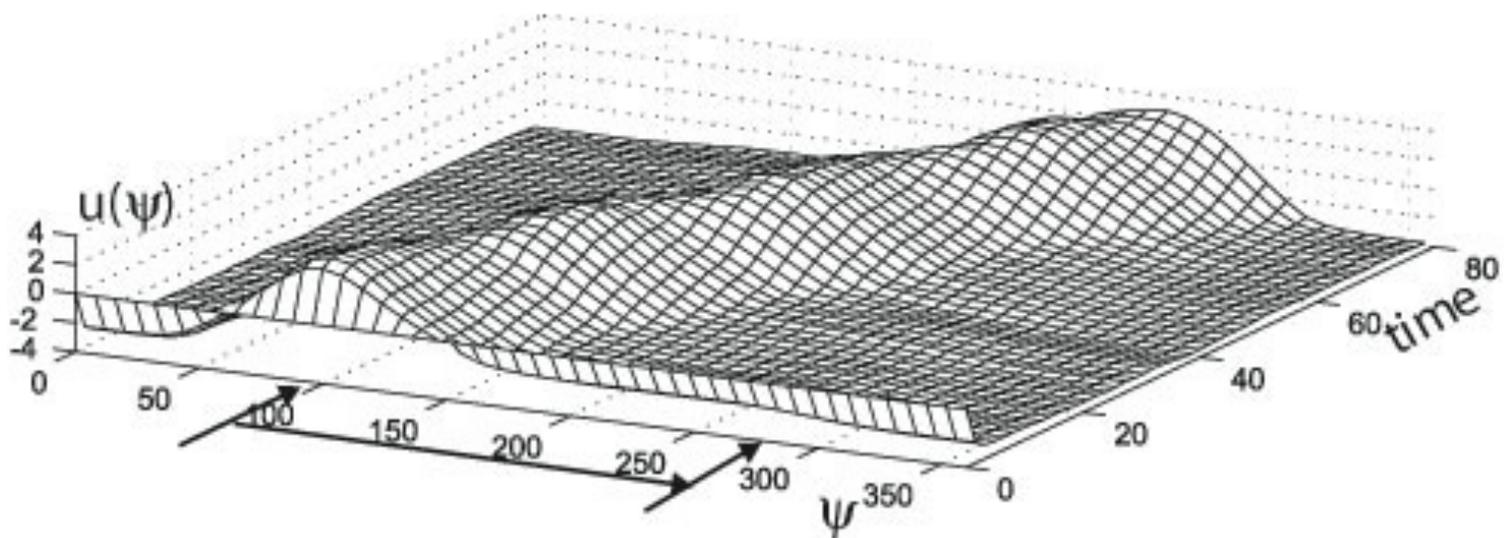
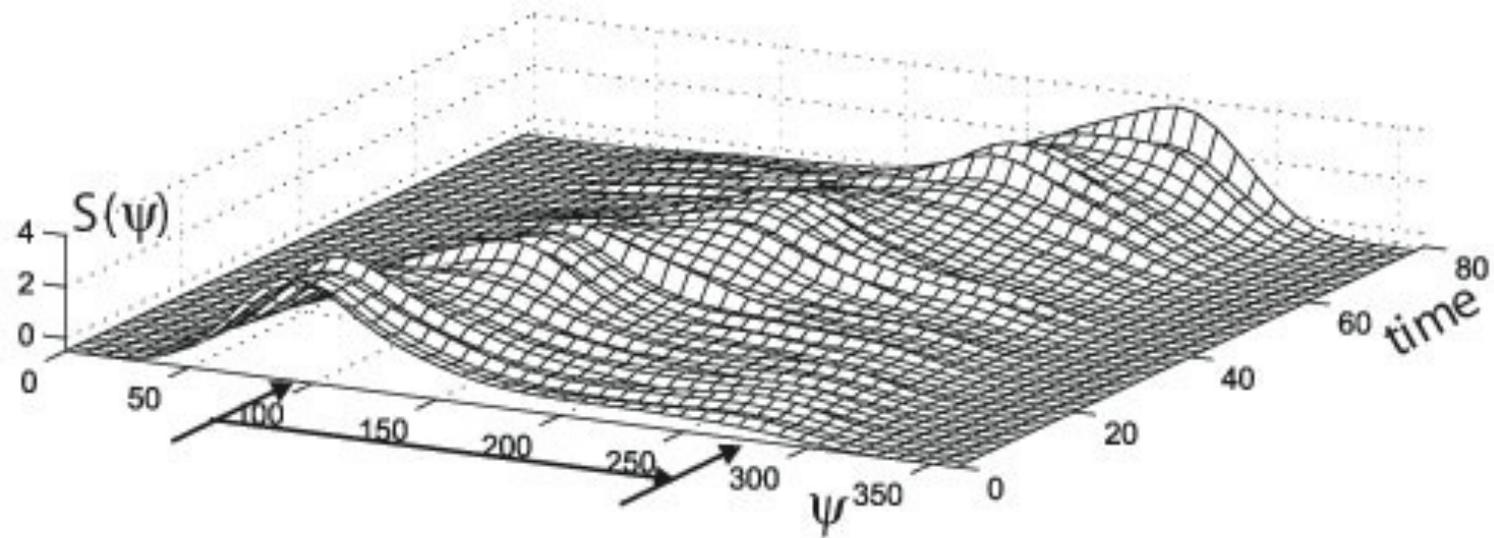
Target selection in the presence of two sources



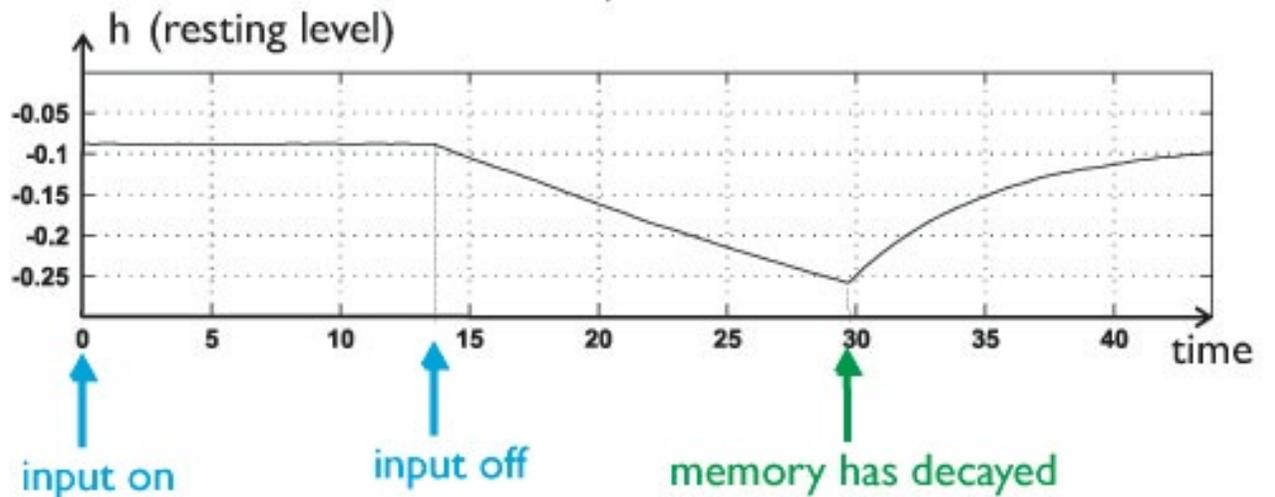
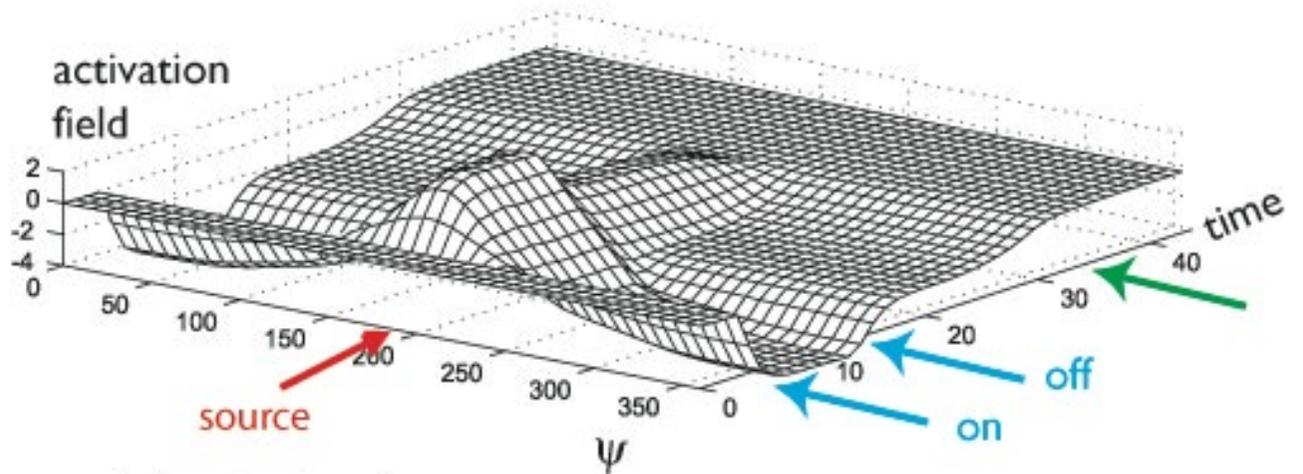
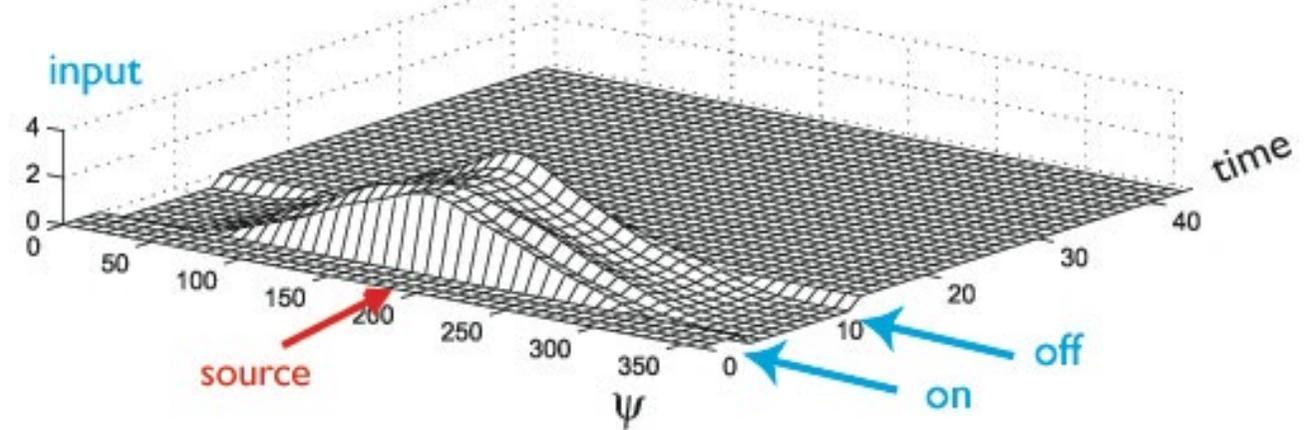
Robust estimation in the presence of outliers



Tracking moving sound source



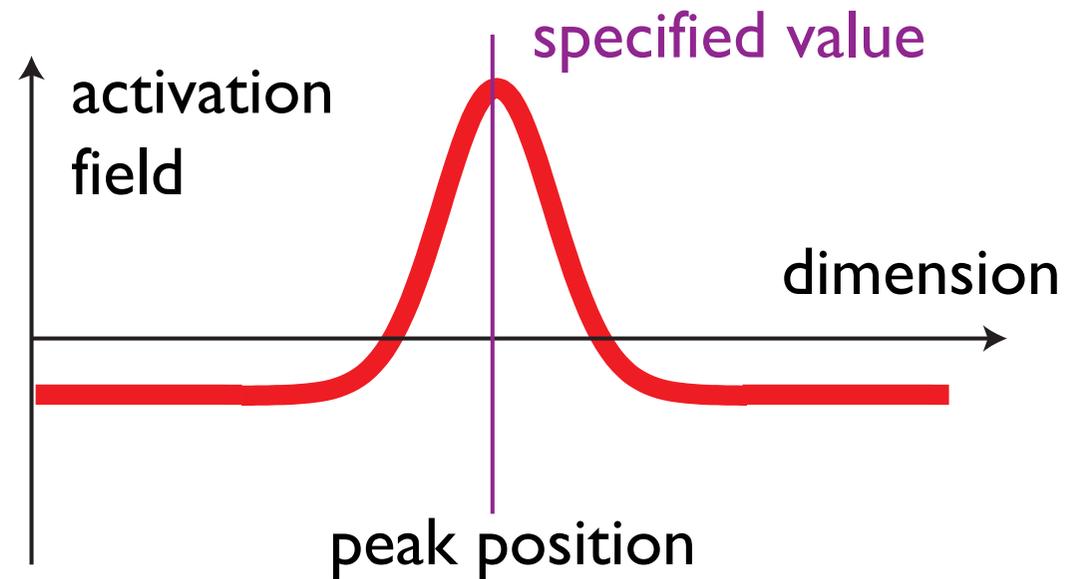
Working memory



[from Bicho, Mallet, Schöner: Int J Rob Res 19:424(2000)]

How to generate the behavior?

- “reading out” the peak location to specify heading?



Challenges

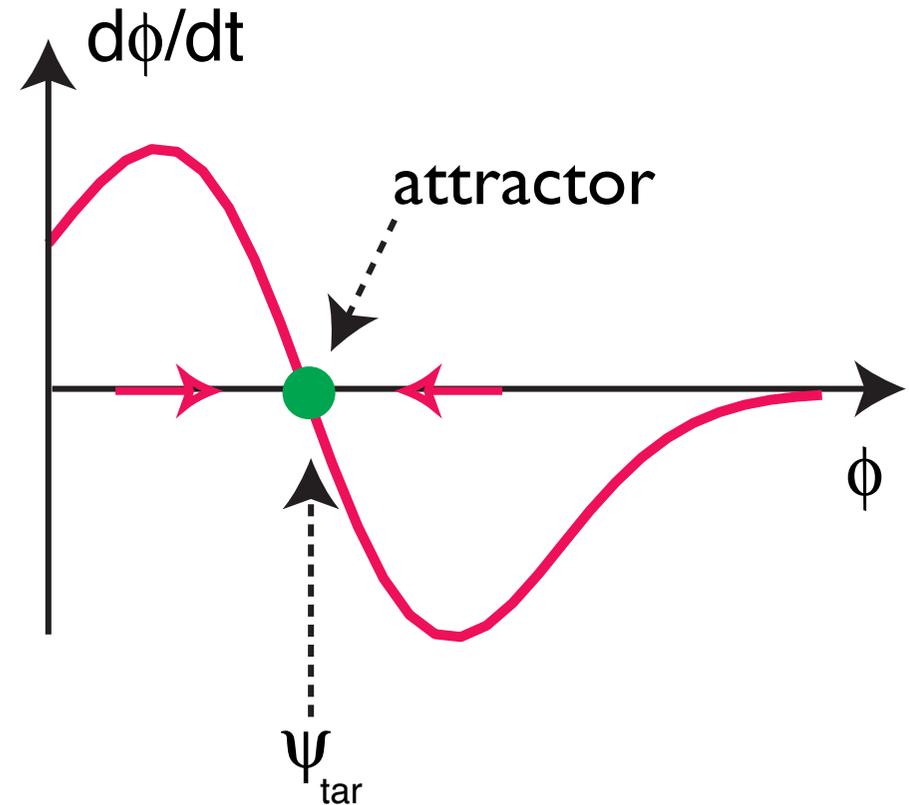
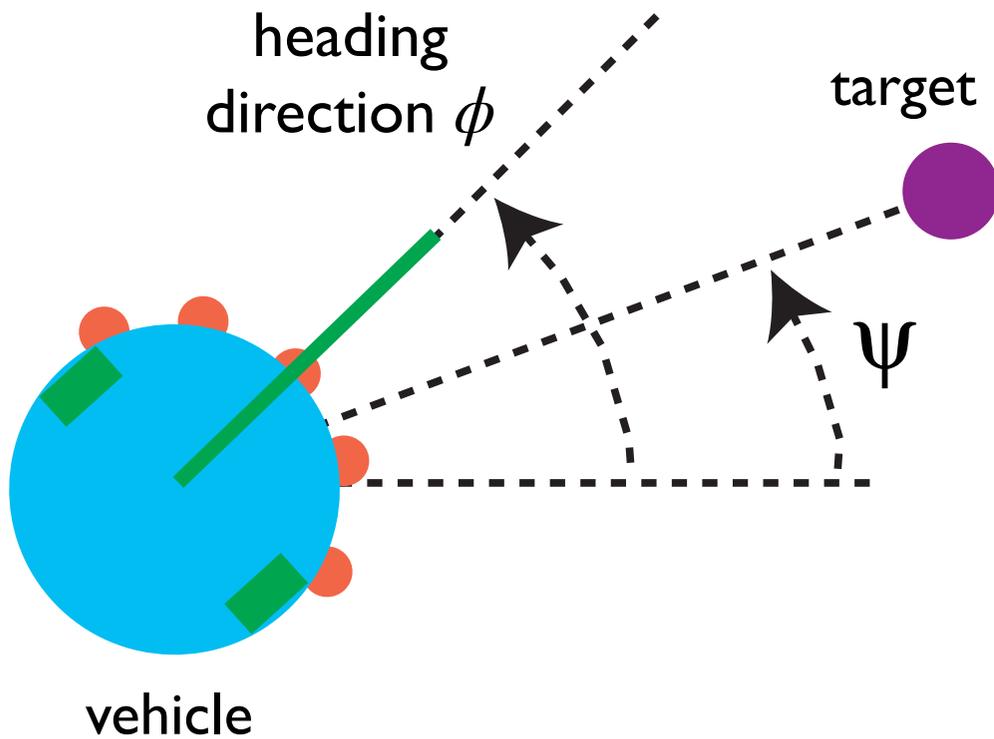
- 1) any actual motor behavior involves dynamics.. stability!
- 2) in organism, motor behavior ultimately involves muscles, which receive descending activation that is graded (rate code)... and temporally structured (timing)

Challenges

- 1) any actual motor behavior involves dynamics.. stability!
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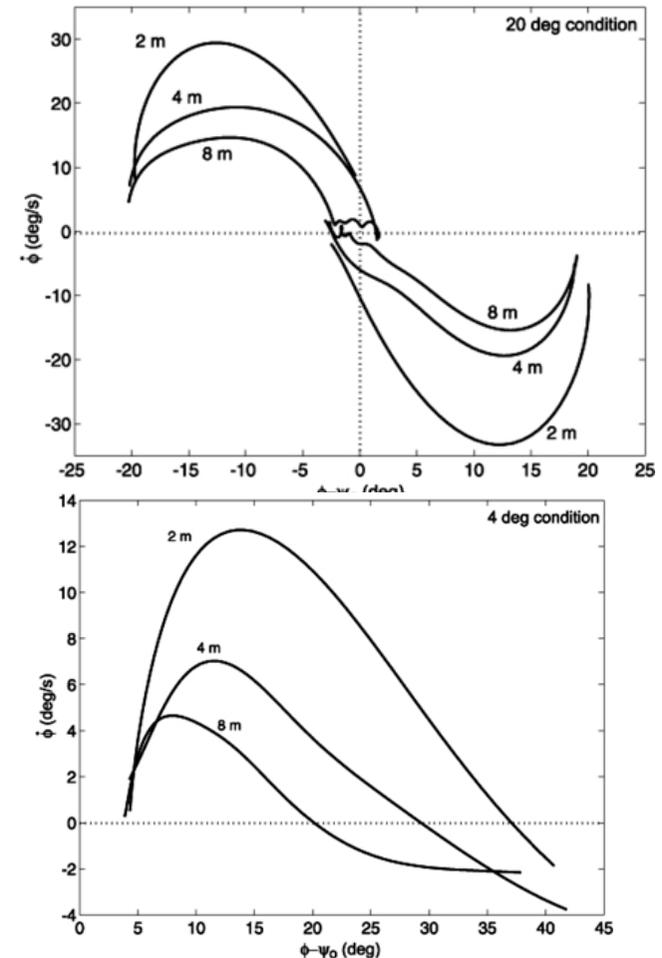
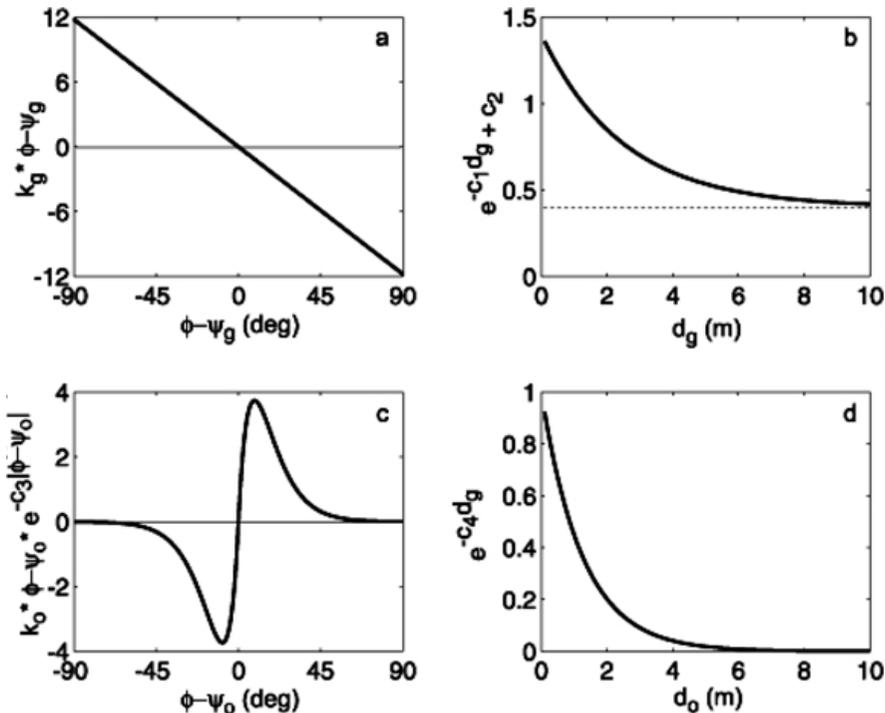
Generating behavior entails dynamics

- behavioral dynamics of a vehicle
- with an attractor at desired heading



Human locomotion described by dynamics of heading direction

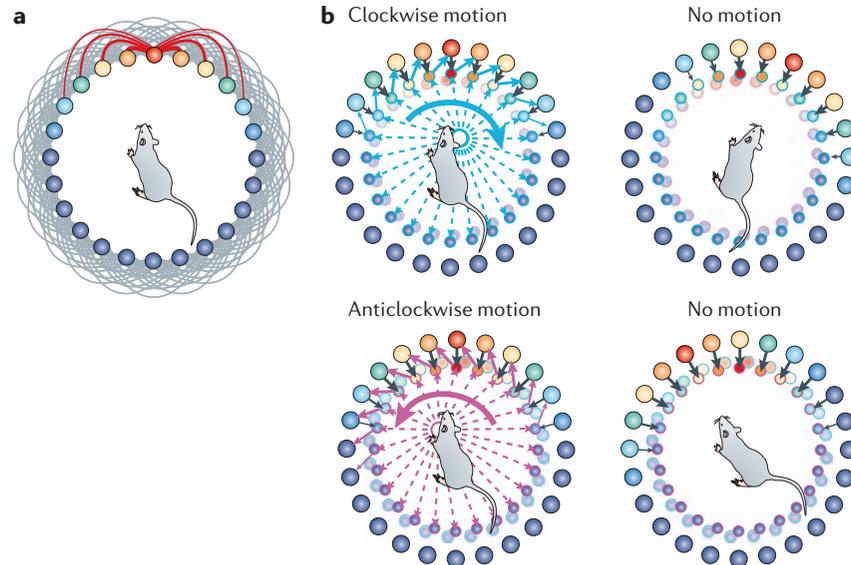
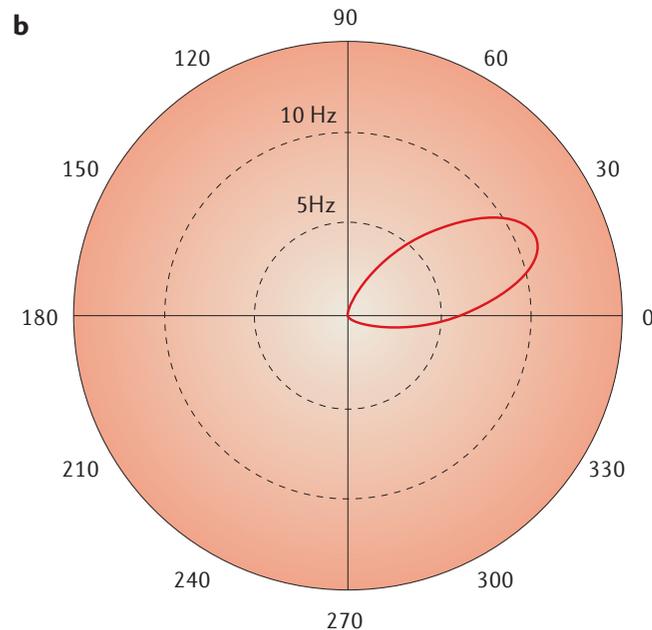
- humans walking in virtual reality under the influence of targets and obstacles



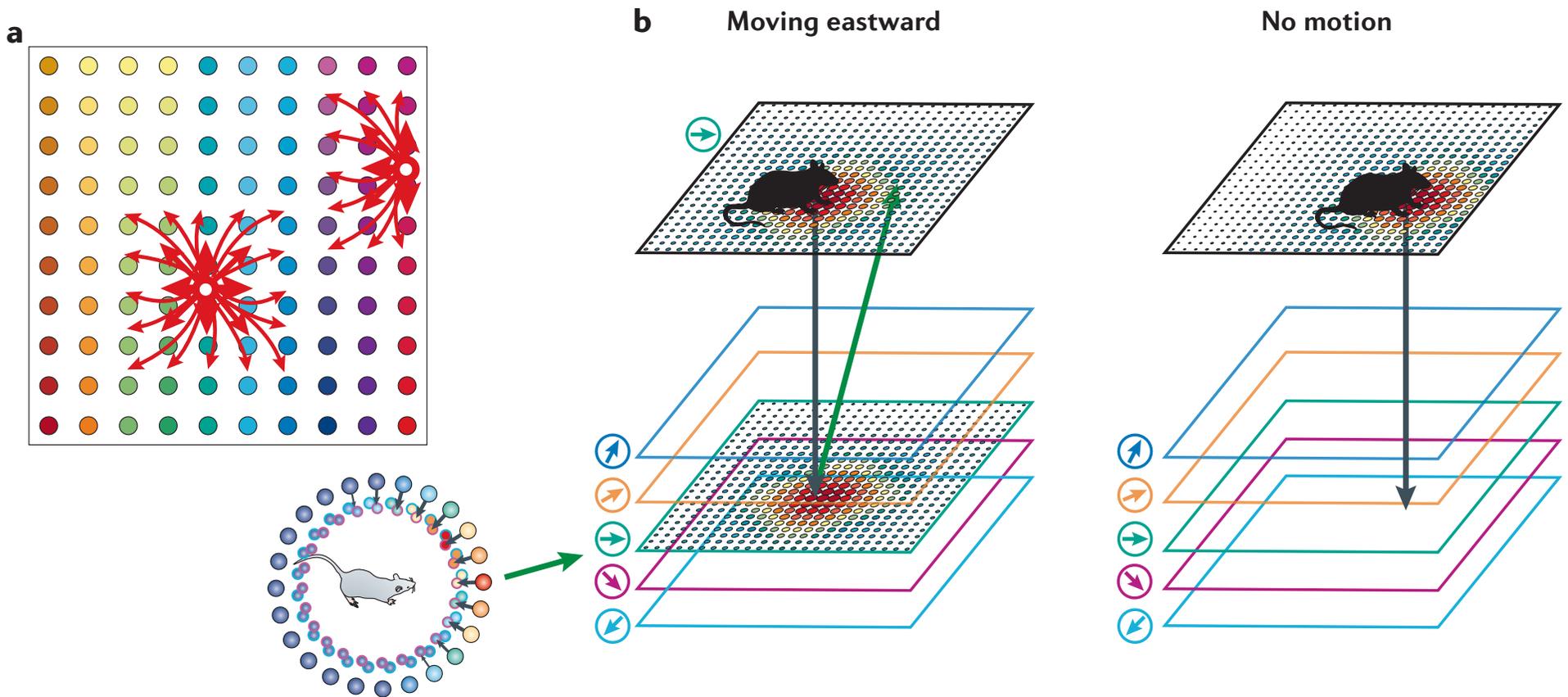
[Warren, Fajen et al, 2003]

Heading direction

- Neural evidence for head-orientation cells... that function as heading direction representation
- Neural attractor dynamics (neural field) for heading direction



Neural dynamics of path integration

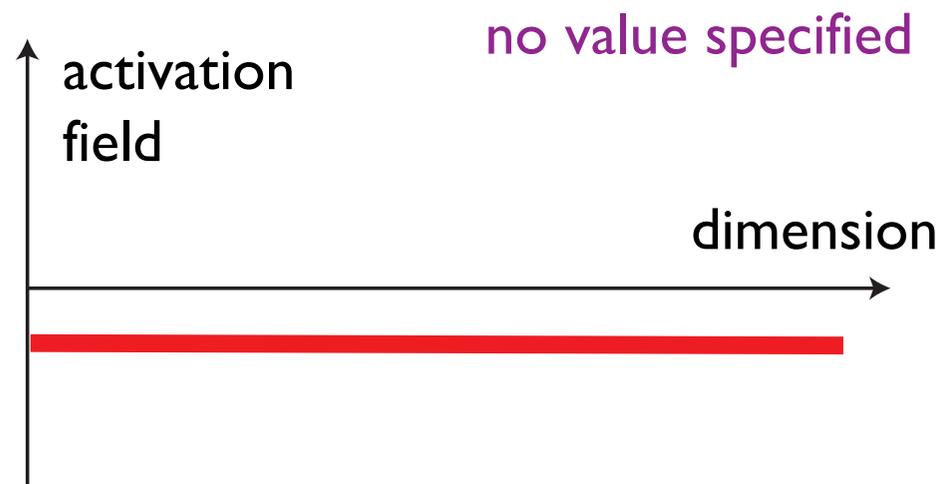
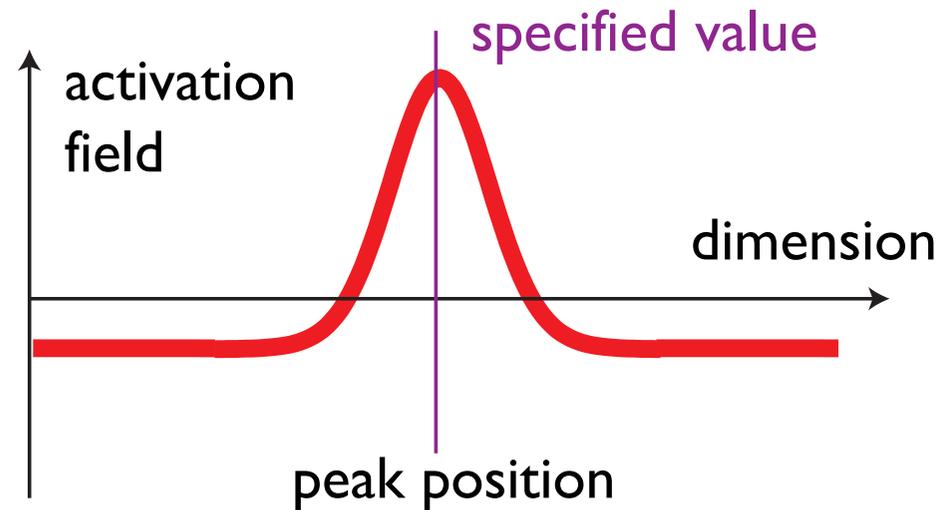


[McNaughton et al., *Nature reviews neuroscience* 2006]

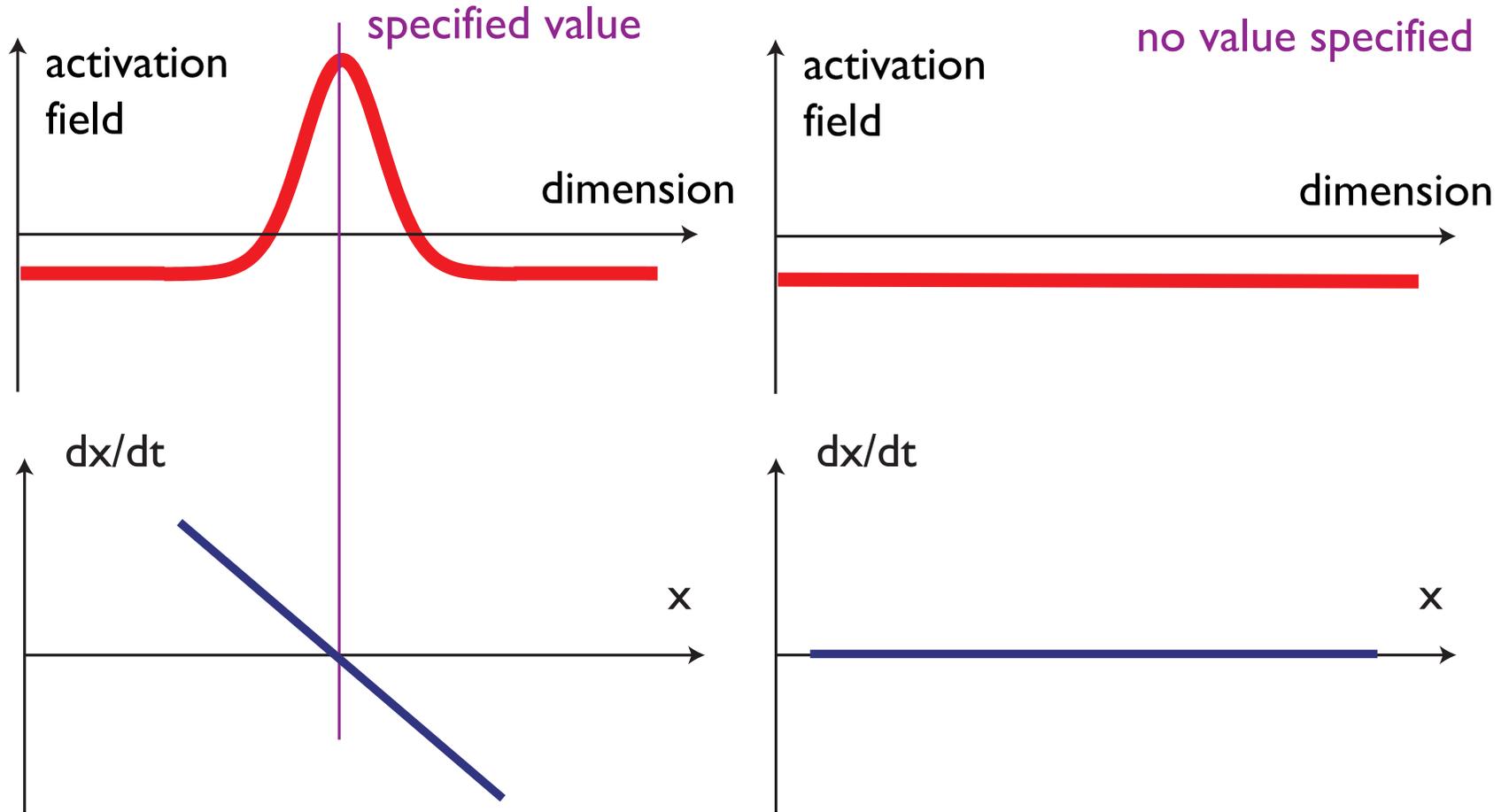
From field to behavioral dynamics

- standard idea: $\sigma(u) \sim$ probability density
- but: normalization!
- \Rightarrow problem when there is no peak: divide by zero!

$$\phi_{\text{peak}} = \frac{\int d\phi \phi \sigma(u(\phi, t))}{\int d\phi' \sigma(u(\phi', t))}$$

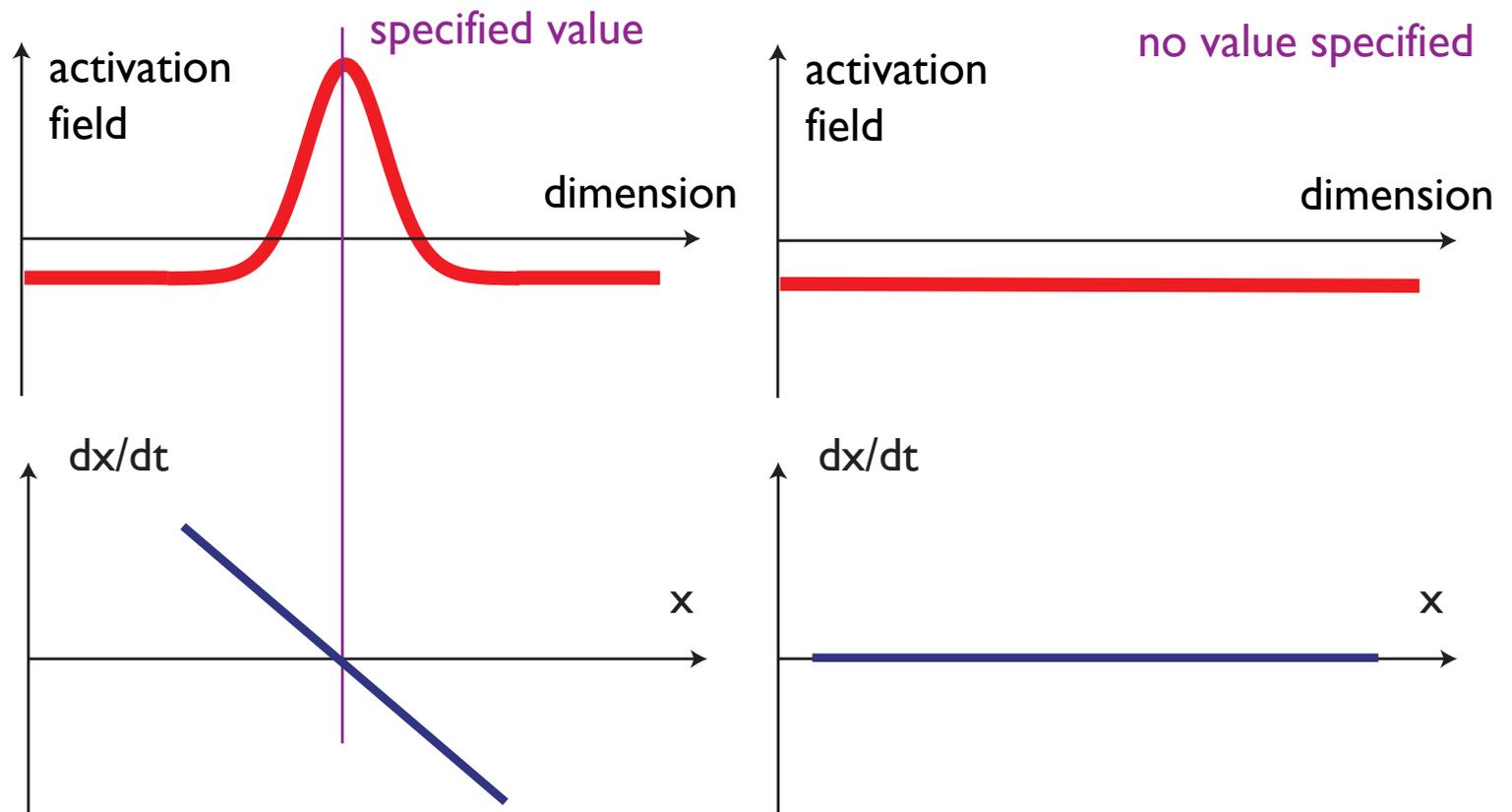


Erect an attractor rather than “read out”



$$\dot{\phi} = - \left[\int d\phi' \sigma(u(\phi', t)) \right] (\phi - \phi_{\text{peak}})$$

$$= - \int d\phi' (\phi - \phi') \sigma(u(\phi', t))$$

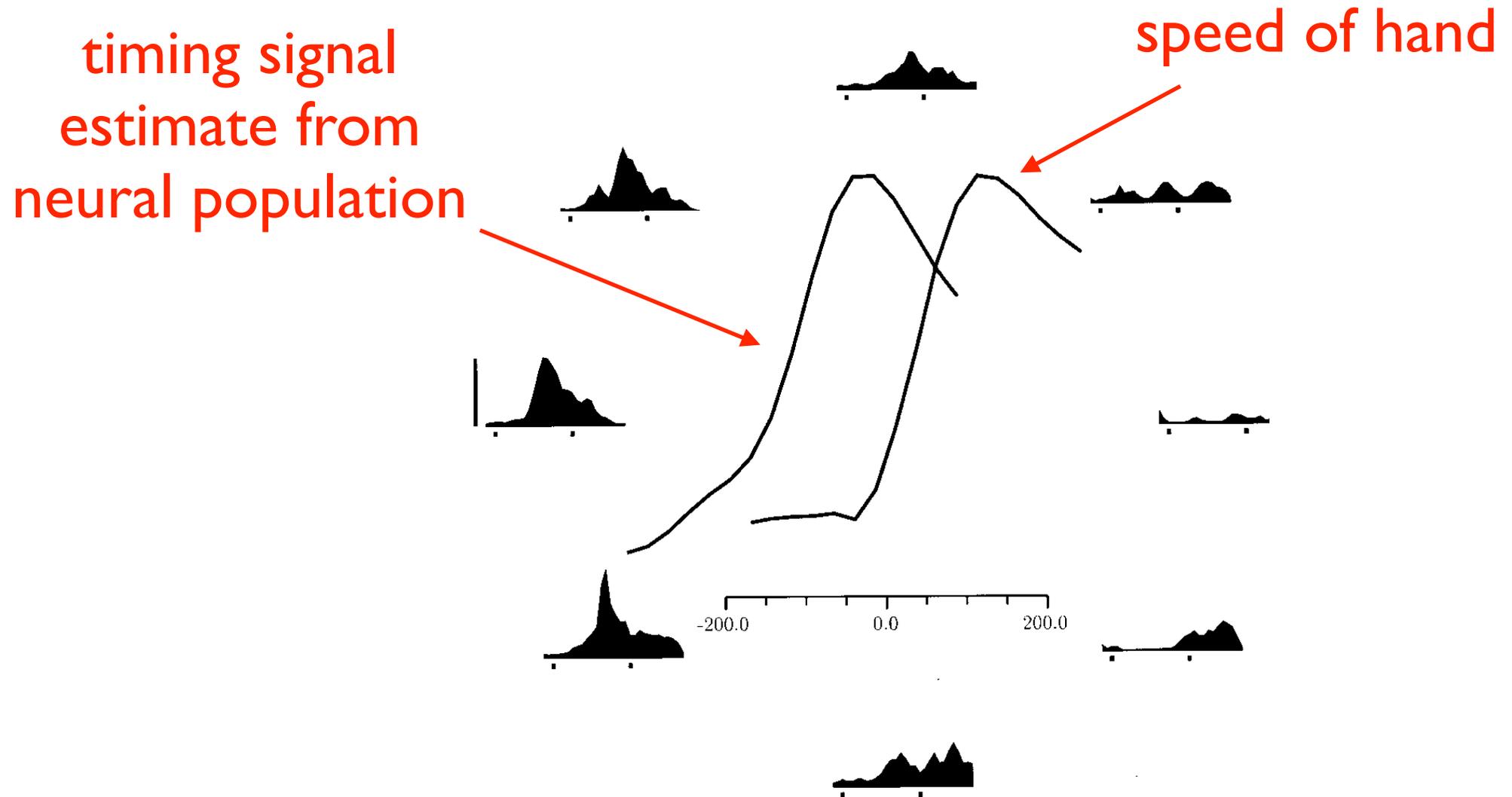




Challenges

- 1) any actual motor behavior involves dynamics.. stability!
- 2) in organism, motor behavior ultimately involves muscles, which receive descending activation that is graded (rate code)... and temporally structured (timing)

Neural timers in MC



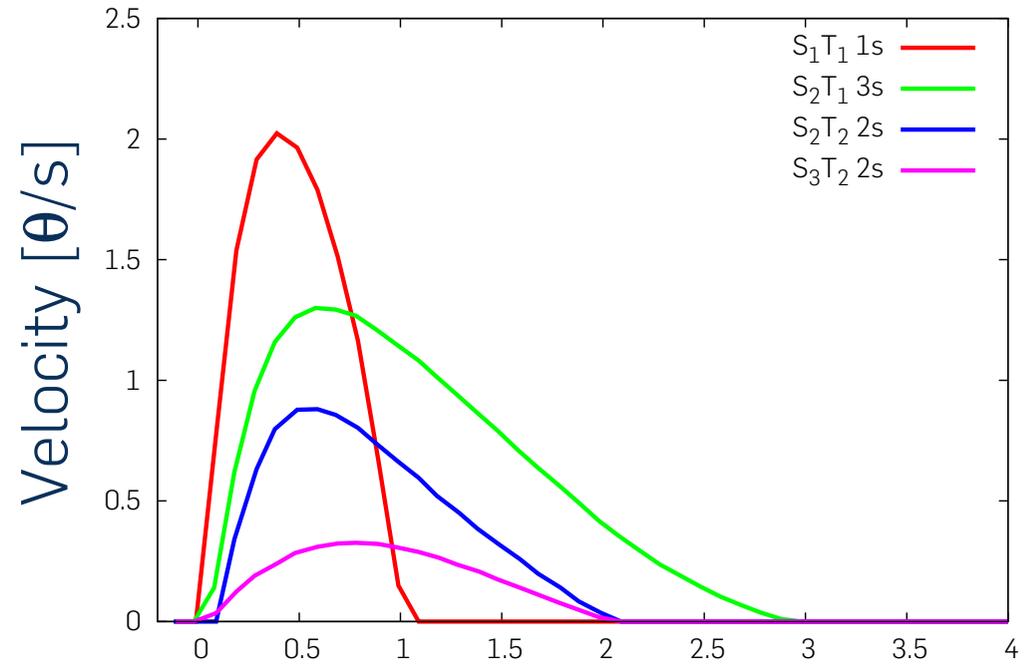
[Moran, Schwartz, J Neurophys 1999]

Neural oscillator model of timing

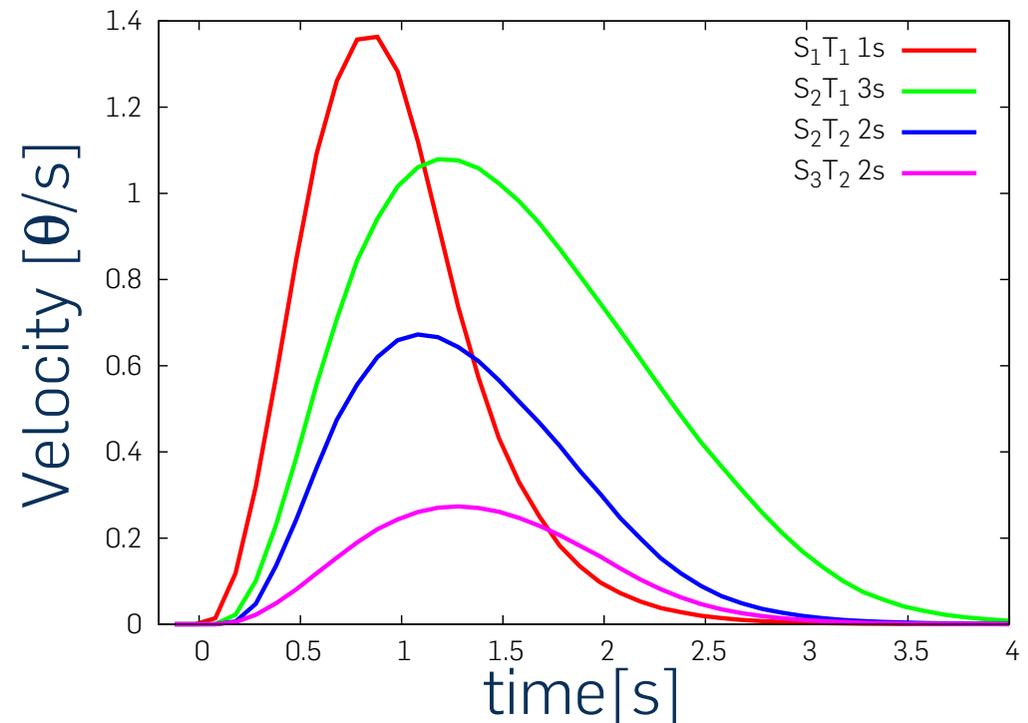
- standard excitatory-inhibitory neural population dynamics => oscillations/active transients
- field of such oscillators for different peak velocities/ amplitudes

[Zibner, Tekülve, Schöner, ICDL 2015; Schöner, Tekülve, Zibner, 2019]

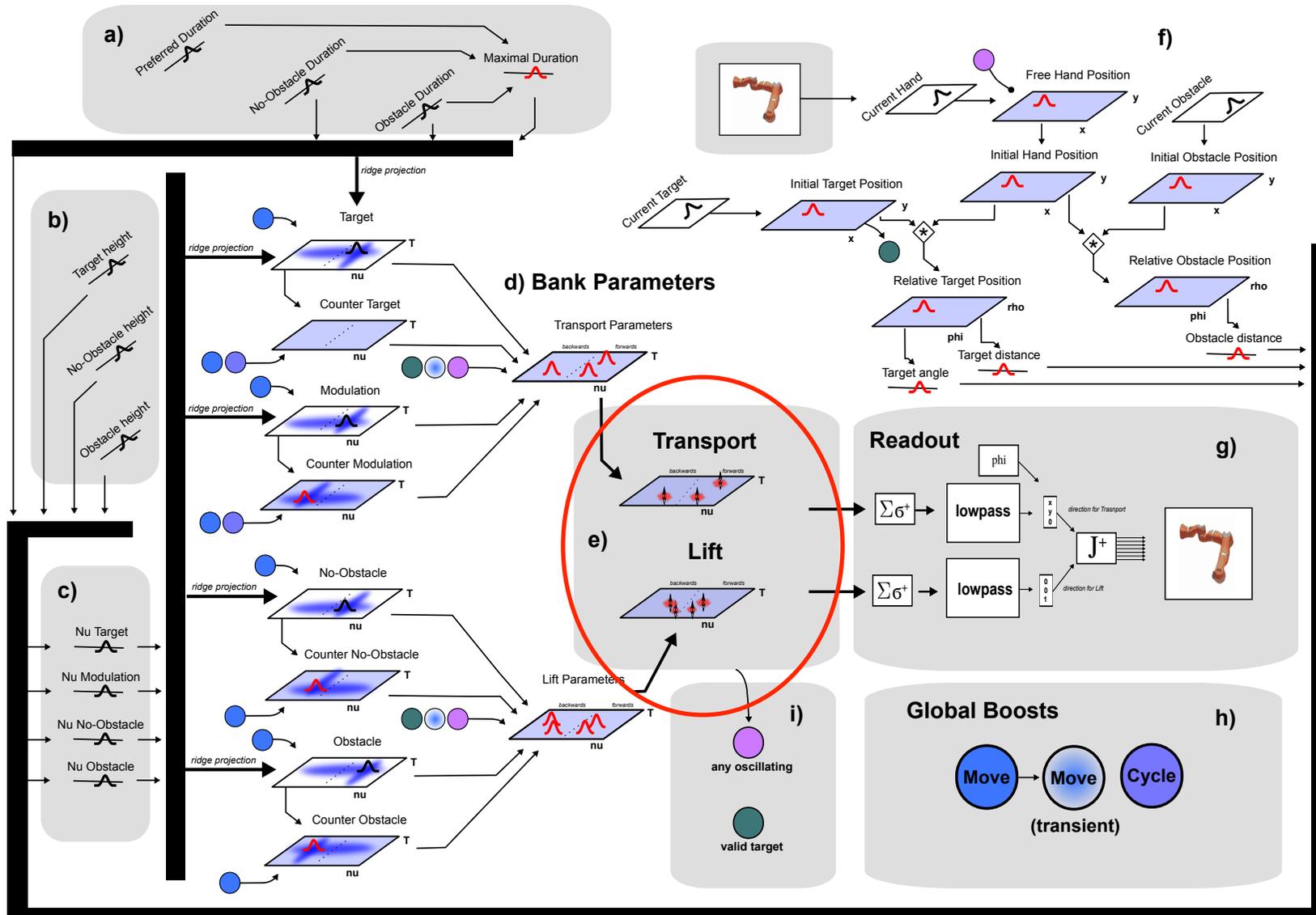
virtual velocity from oscillator



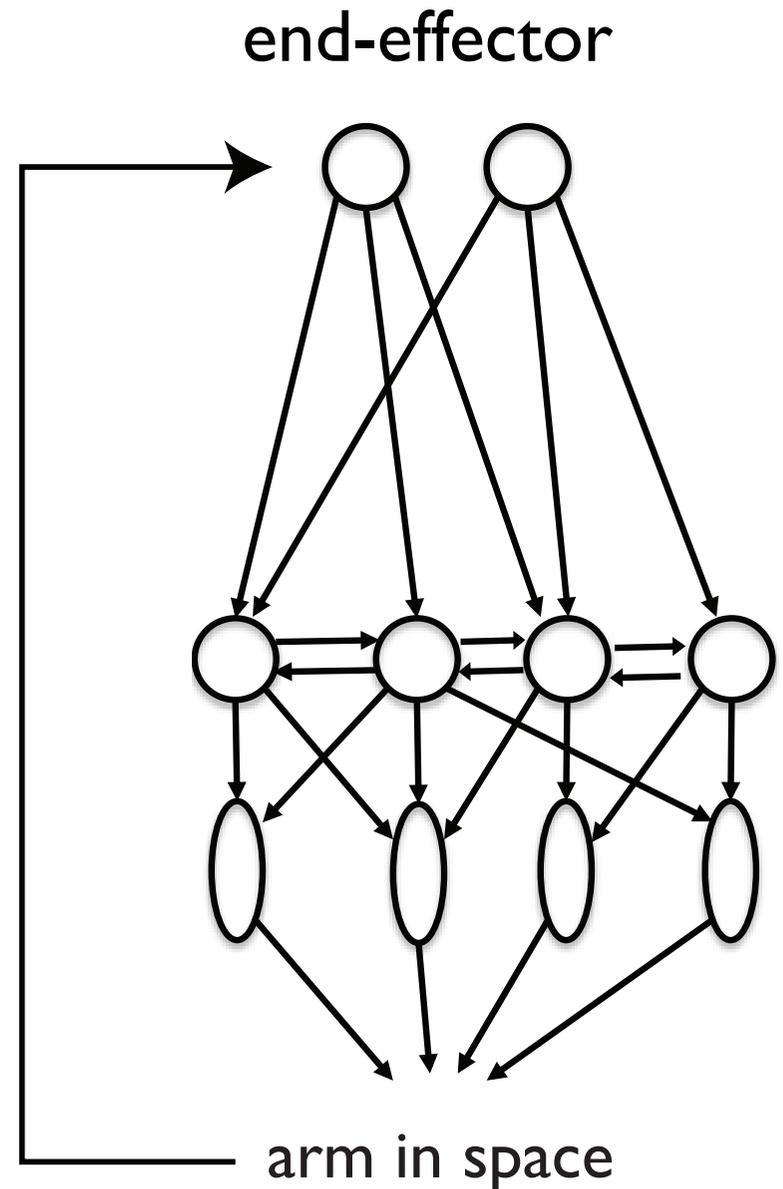
hand velocity



Localized input triggers transients/oscillations in such fields



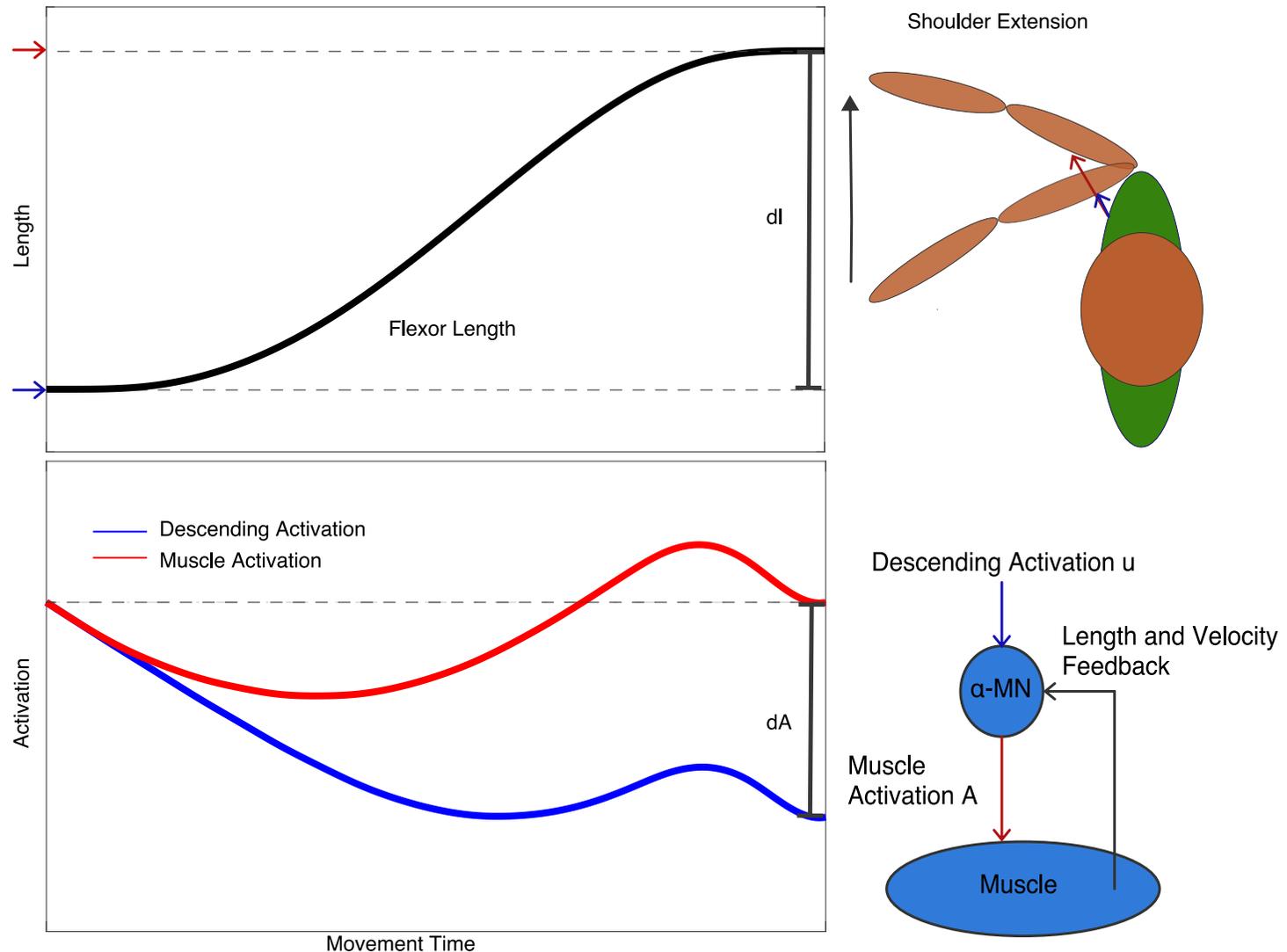
descending activation: DoF problem



[Martin, Scholz, Schöner. *Neural Computation* (2009)]

[Martin, Reimann, Schöner *Biological Cybernetics* 2019]

Muscles: dynamical system with an attractor at a postural state



Conclusion

- sensory-motor cognition from neural dynamic fields that are coupled to sensory surfaces and act on the motor surfaces (through behavioral dynamics)
- instabilities make decisions
 - detection
 - selection
 - working memory

Outlook

- how do we go from sensory-motor cognition to “real” cognition?