DFT Foundations 1: Space and Time

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Foundations 1: Space and time

- Space
- Time:
  - Neural dynamics
  - Interaction
- Instabilities
- Tutorial: discrete activation variables
- Supplement: mathematical formalization
- Tutorial: simulating fields

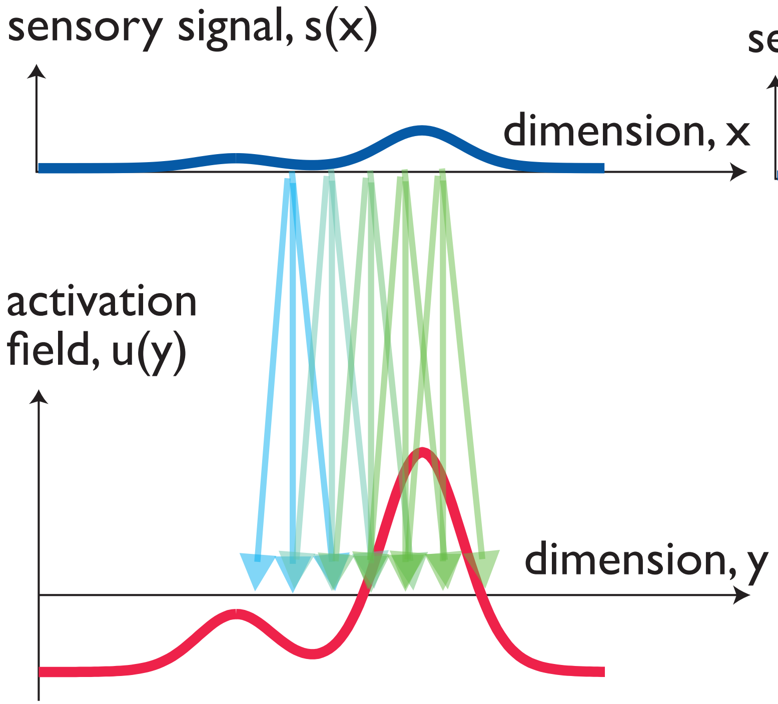
Space

- activation in neural populations carries functional meaning
- activation: $u(x, t)$ where x spans low-dimensional spaces

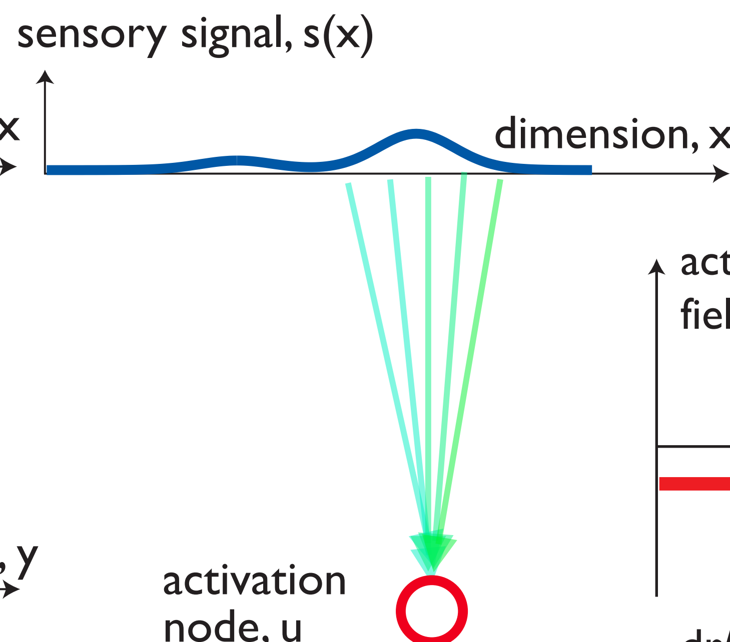
[Schöner TopiCS 2019]

Where do the spaces come from?

- connectivity from sensory surfaces / to motor surfaces

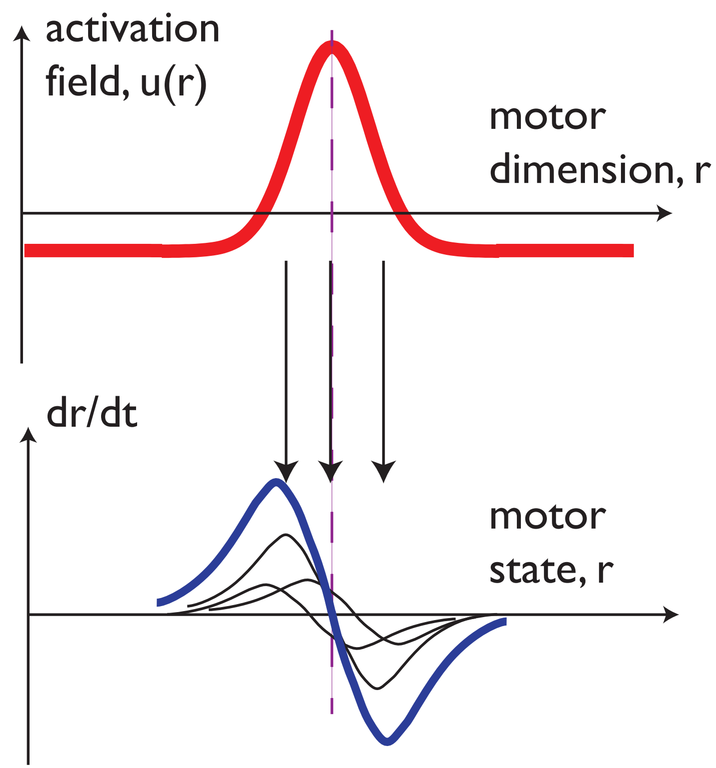


feature space



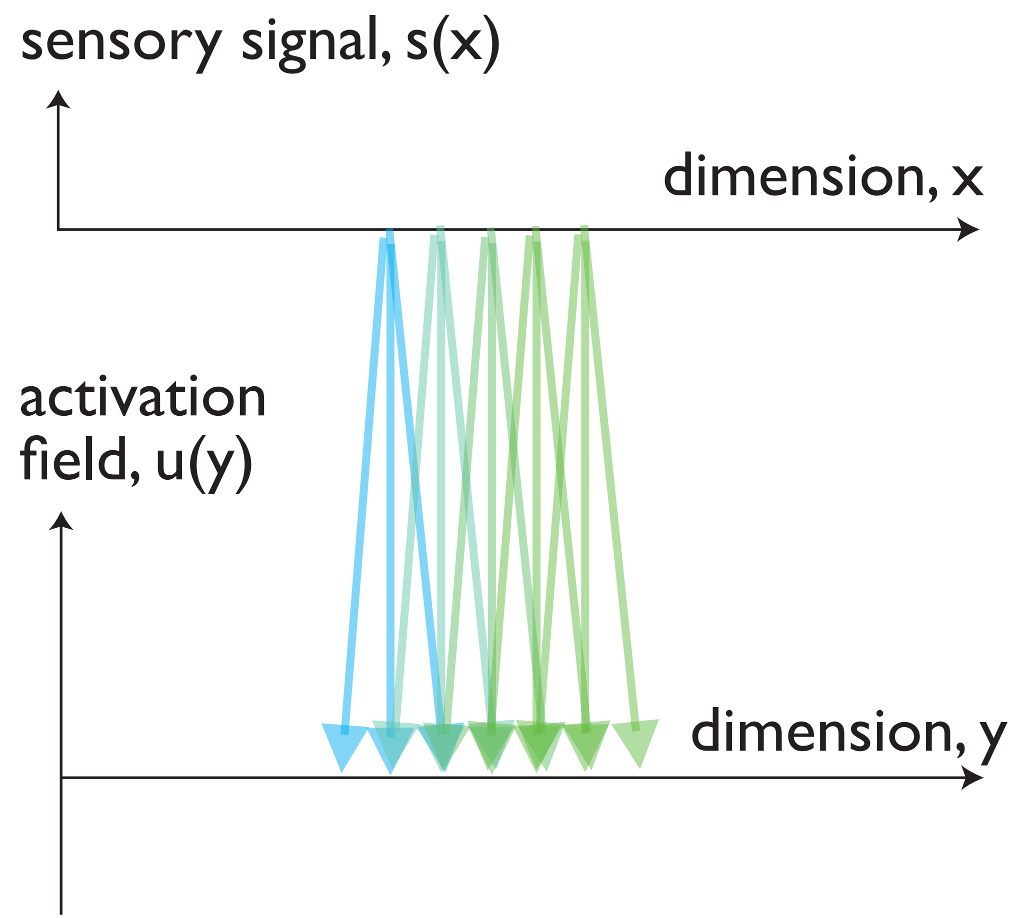
concept

motor space



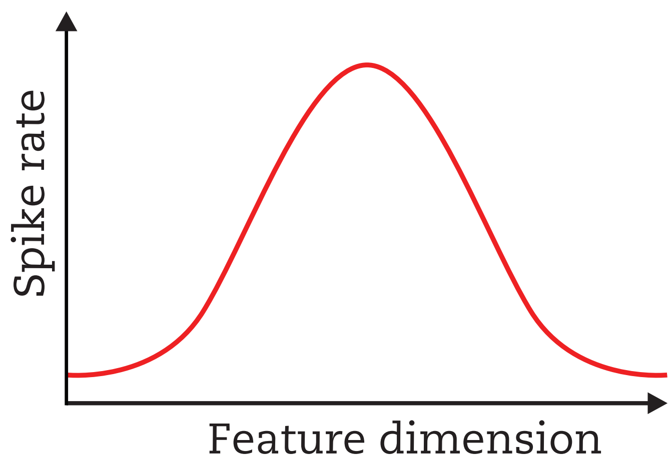
Neural fields

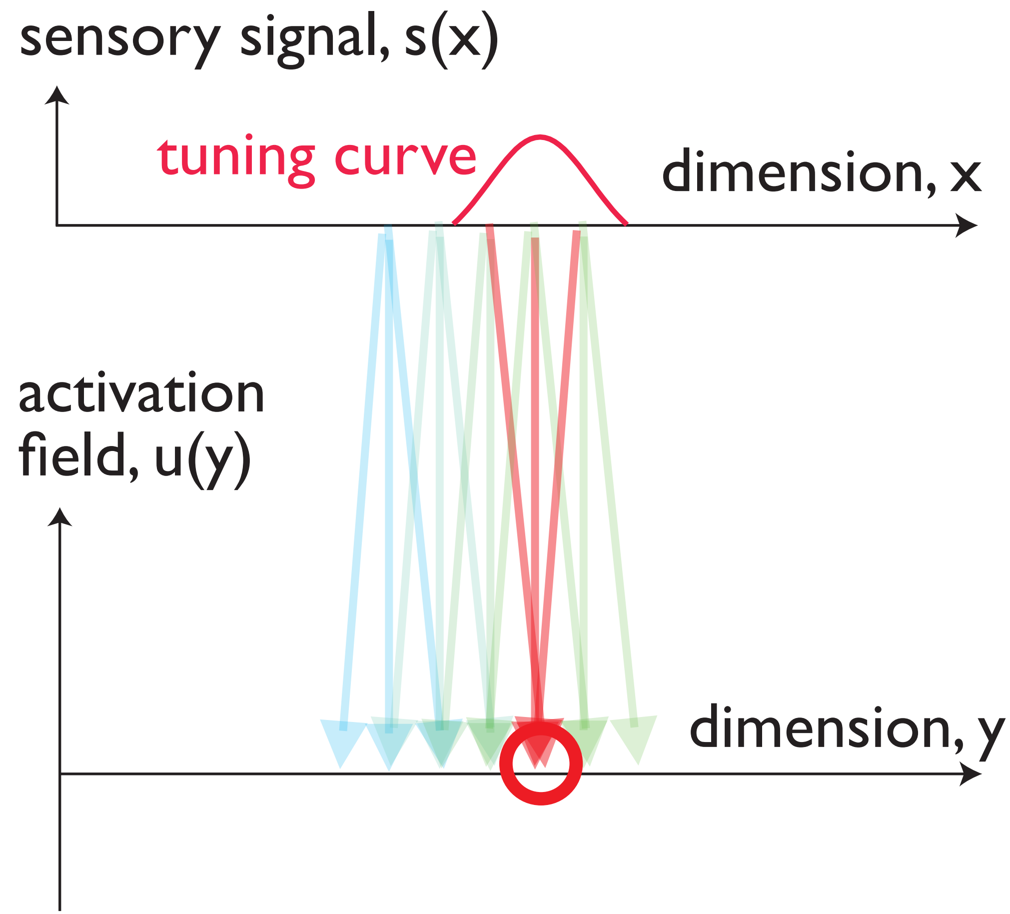
- forward connectivity from the sensory surface extracts perceptual feature dimensions



Neural fields

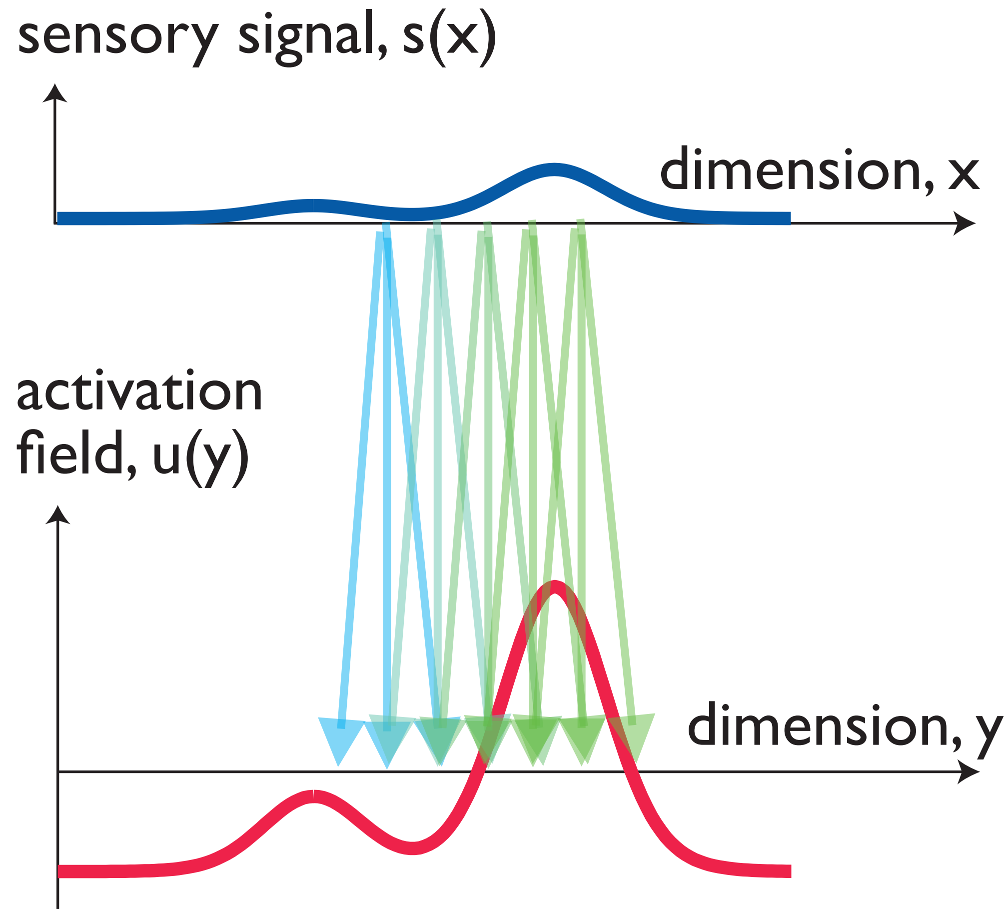
- as described by tuning curves or receptive fields





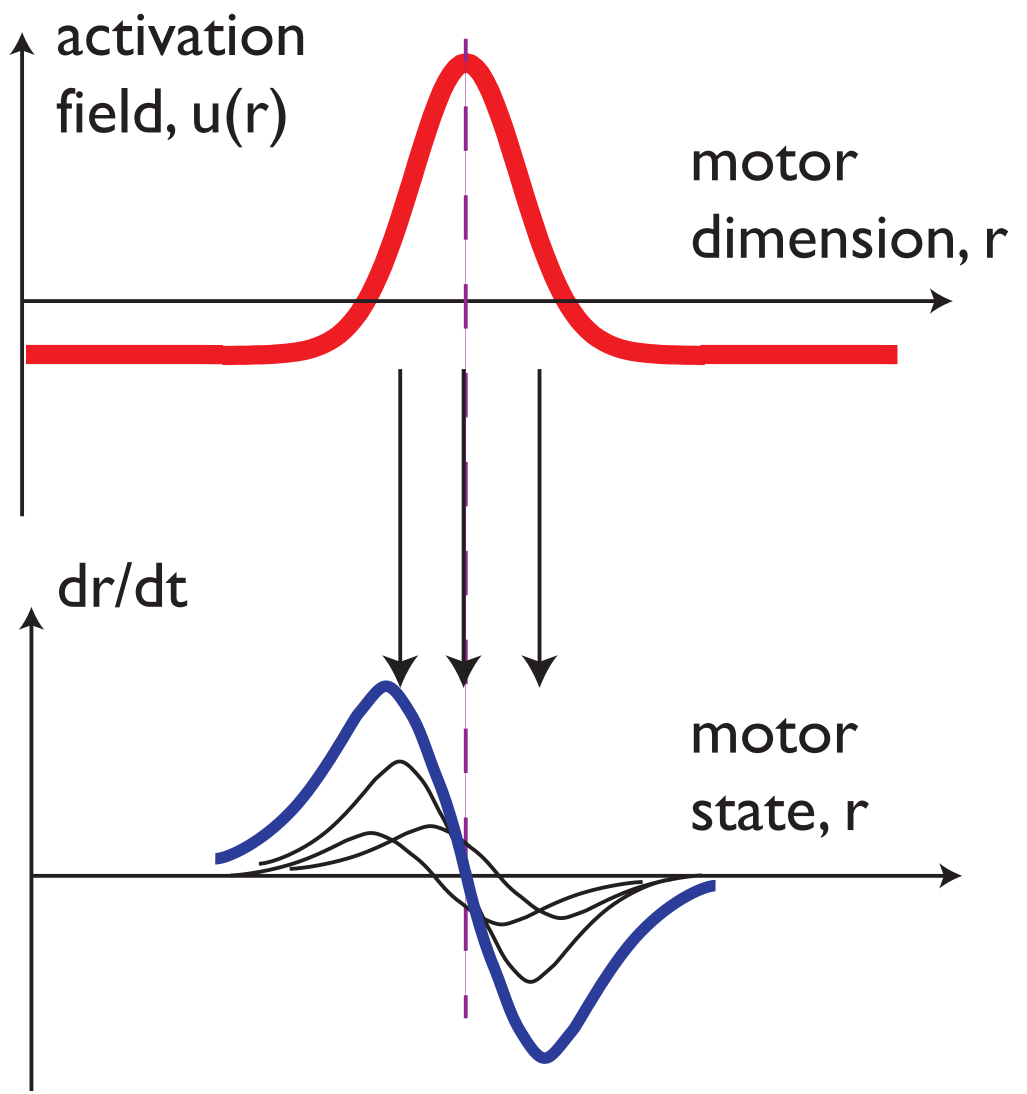
Neural fields

- => neural map from sensory surface to feature dimension
- neglect the sampling by individual neurons => activation field



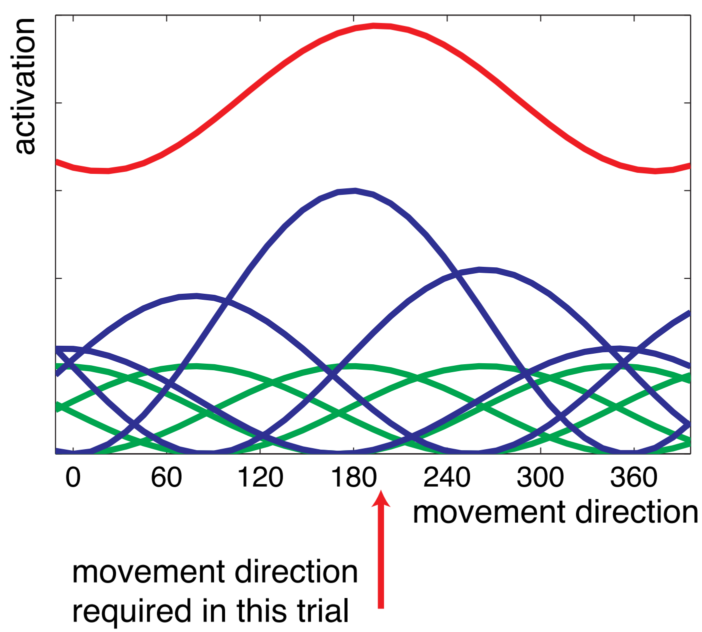
Neural fields

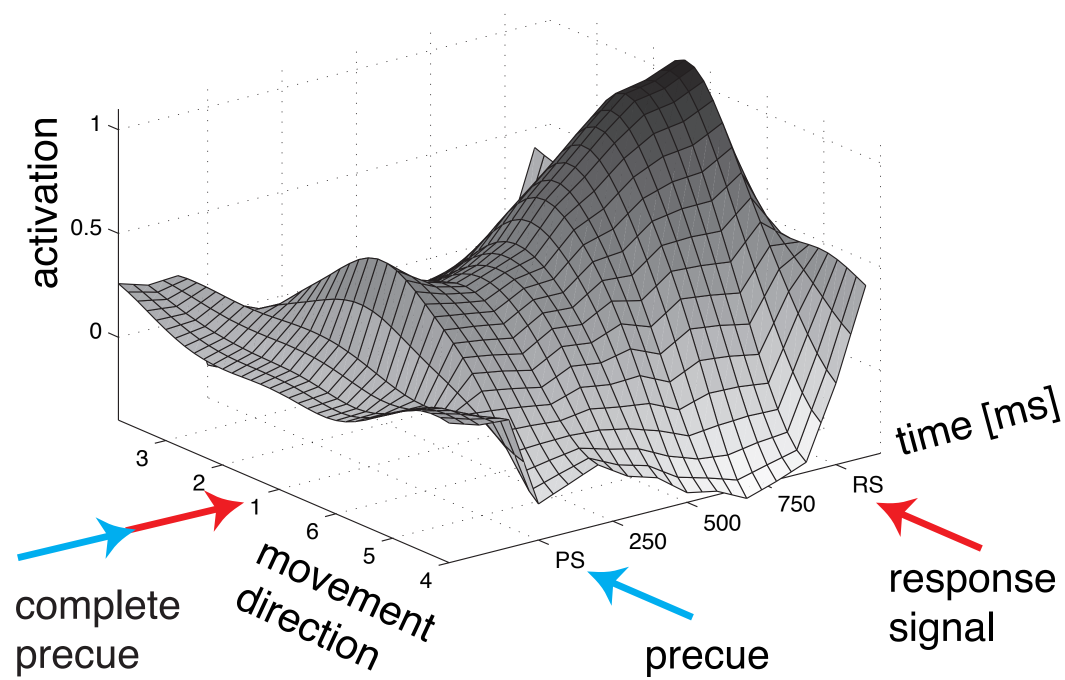
- analogous for projection onto to motor surfaces…
- which actually involves behavioral dynamics (e.g., through neural oscillators and peripheral reflex loops)



Distribution of Population Activation (DPA) <=> neural field

Distribution of population activation = $\sum_{\text{neurons}}$ tuning curve * current firing rate

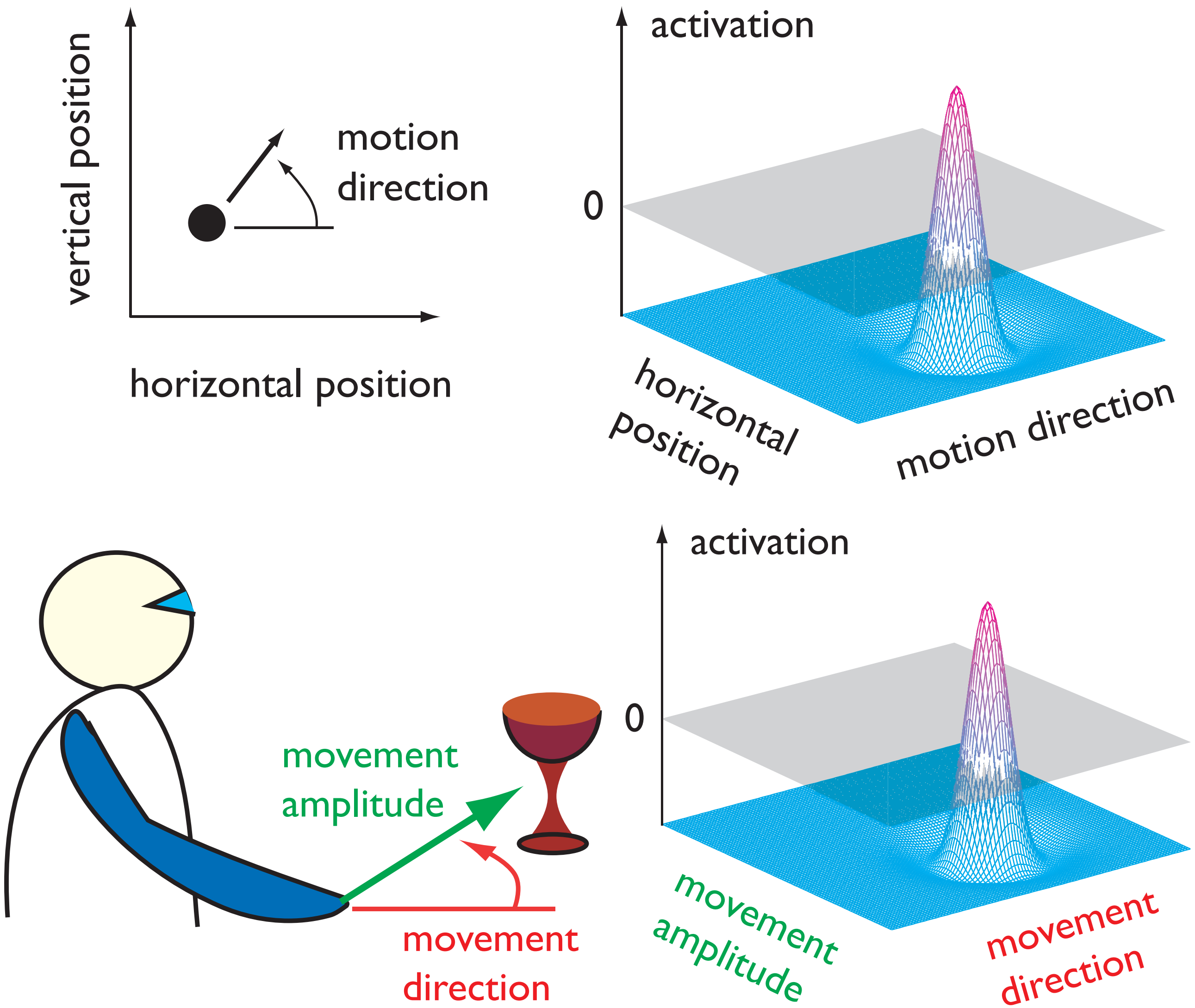




- note: neurons are not localized within DPA!

[Bastian, Riehle, Schöner, 2003]

Hypothesis: mental states are localized in these low-dimensional spaces



~ Gärdenfors

Roadmap

Foundations 1: Space and time

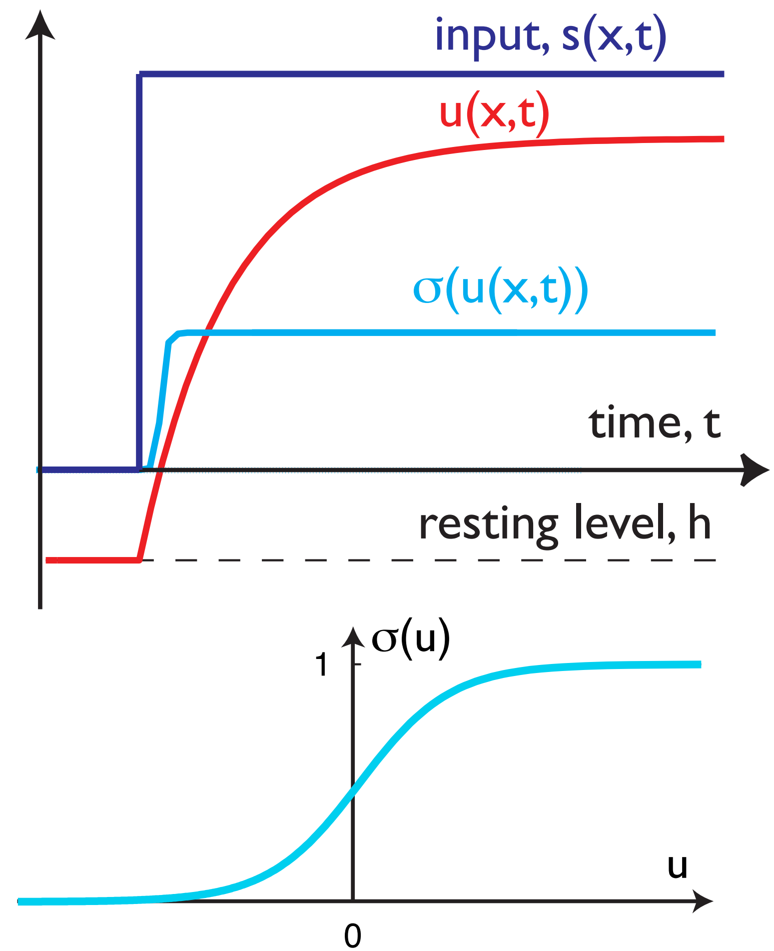
- Space
- Time:
  - Neural dynamics
  - Interaction
- Instabilities
- Tutorial: discrete activation variables
- Supplement: mathematical formalization
- Tutorial: simulating fields

Time

Activation

- ~ population level membrane potential
- defined relative to sigmoid
  - above threshold: transmitted
  - below threshold: not transmitted

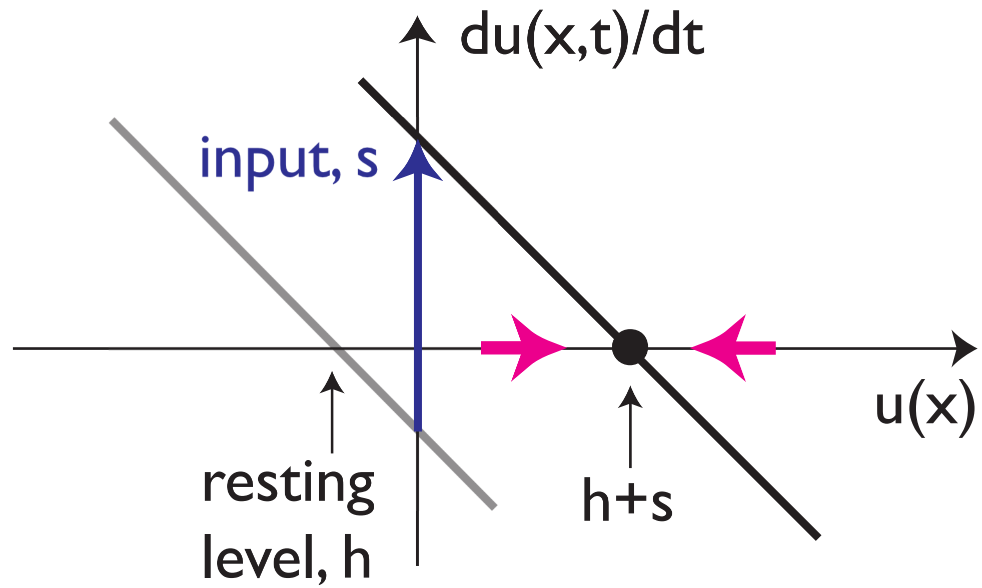
$$\tau \dot{u}(x,t) = -u(x,t) + h + s(x,t)$$



Neural dynamics

- originates from membrane dynamics
- inputs add to the rate of change of activation
  - positive: excitatory
  - negative: inhibitory

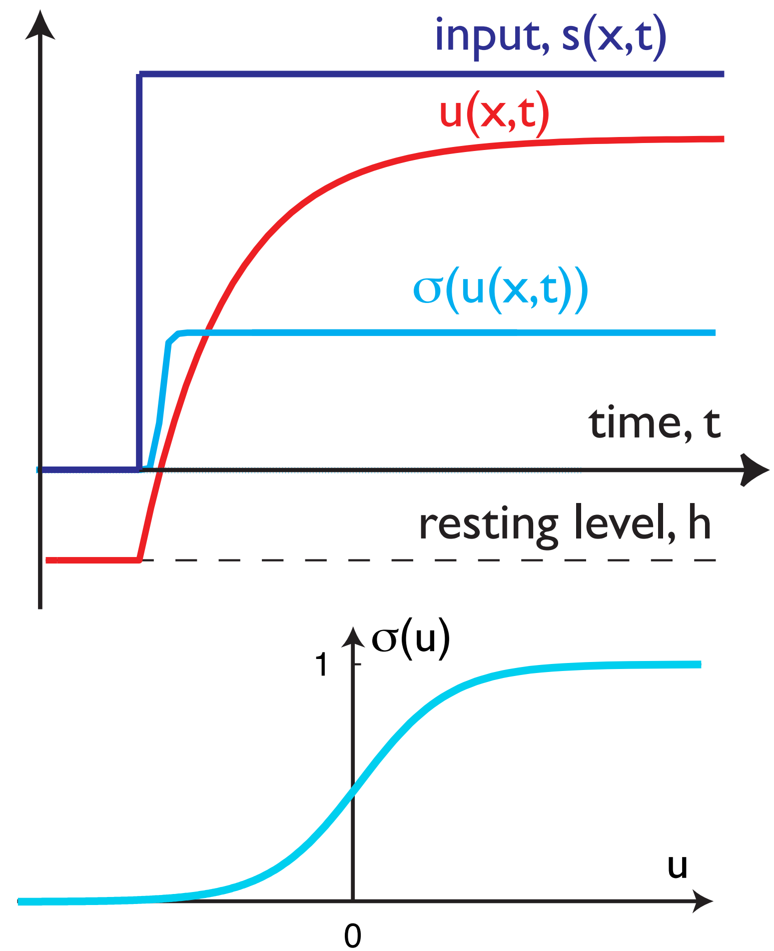
$$\tau \dot{u}(x,t) = -u(x,t) + h + s(x,t)$$



Time courses

$$\tau \dot{u}(x,t) = -u(x,t) + h + s(x,t)$$

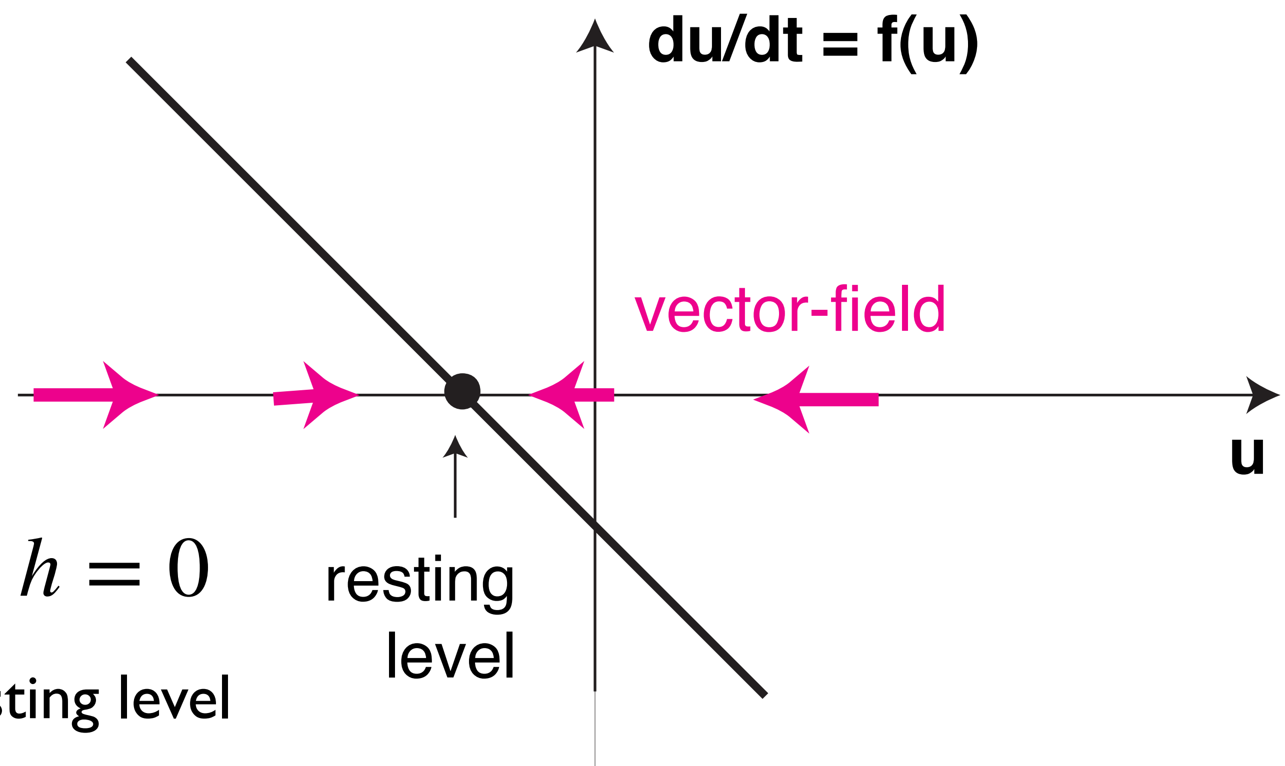
- so far: only transmits and smooths time courses of input



Tutorial

Qualitative dynamics

- dynamical system: the present determines the future
- fixed point = constant solution = stationary state
- stable fixed point = attractor: nearby solutions converge to the fixed point



$$\tau\dot{u}(t) = -u(t) + h$$

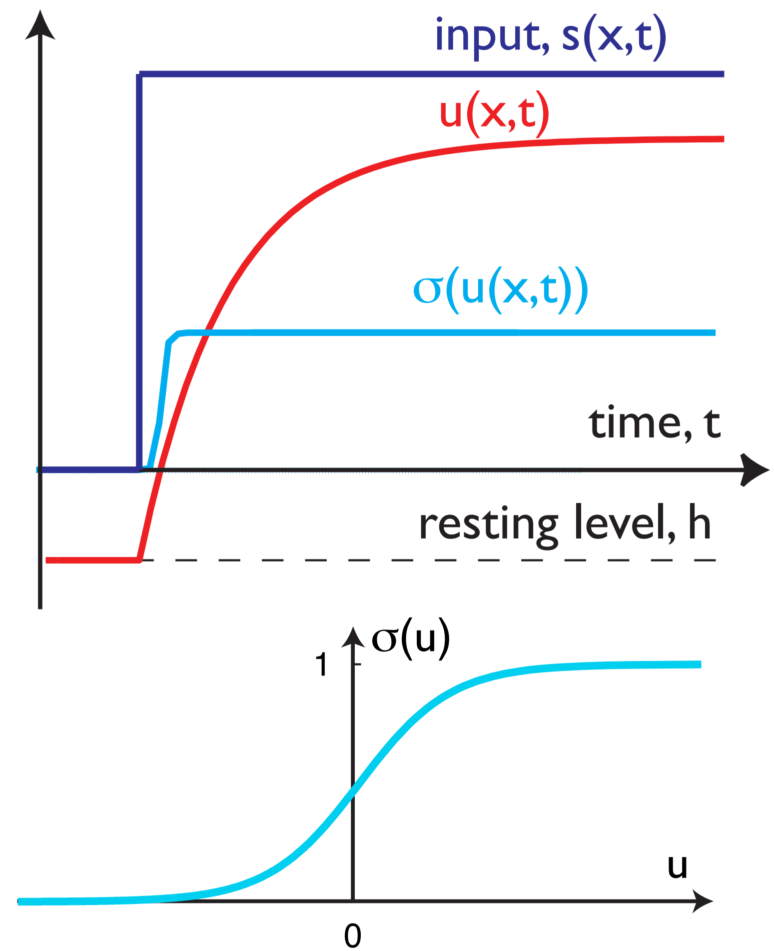
$$\tau\dot{u}_{\mathrm{fp}} = -u_{\mathrm{fp}} + h = 0$$

$$\Rightarrow u_{\mathrm{fp}} = h$$ resting level

Time courses

$$\tau\dot{u}(x,t) = -u(x,t) + h + s(x,t)$$

- input shifts the attractor
- => activation tracks this shift
- => $\sigma(u(t))$ transmitted to down-stream neurons



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Interaction

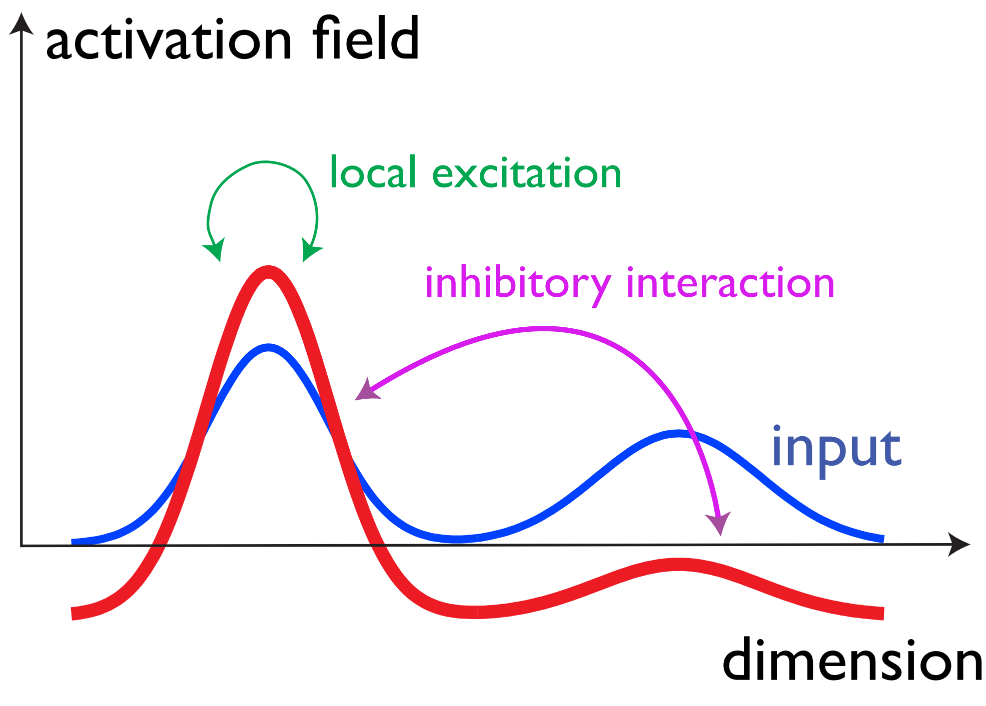
…beyond input driven activation

$$\tau\dot{u}(x,t) = -u(x,t) + h + s(x,t)$$

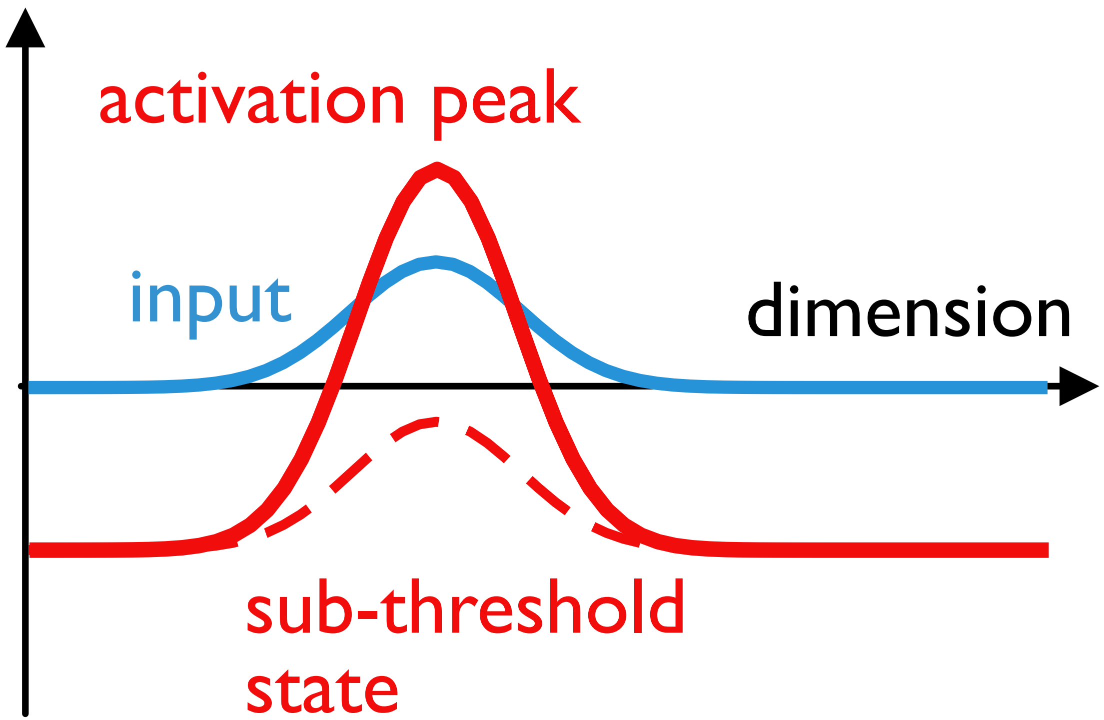
$$+\int w(x-x')\sigma(u(x',t))dx'$$

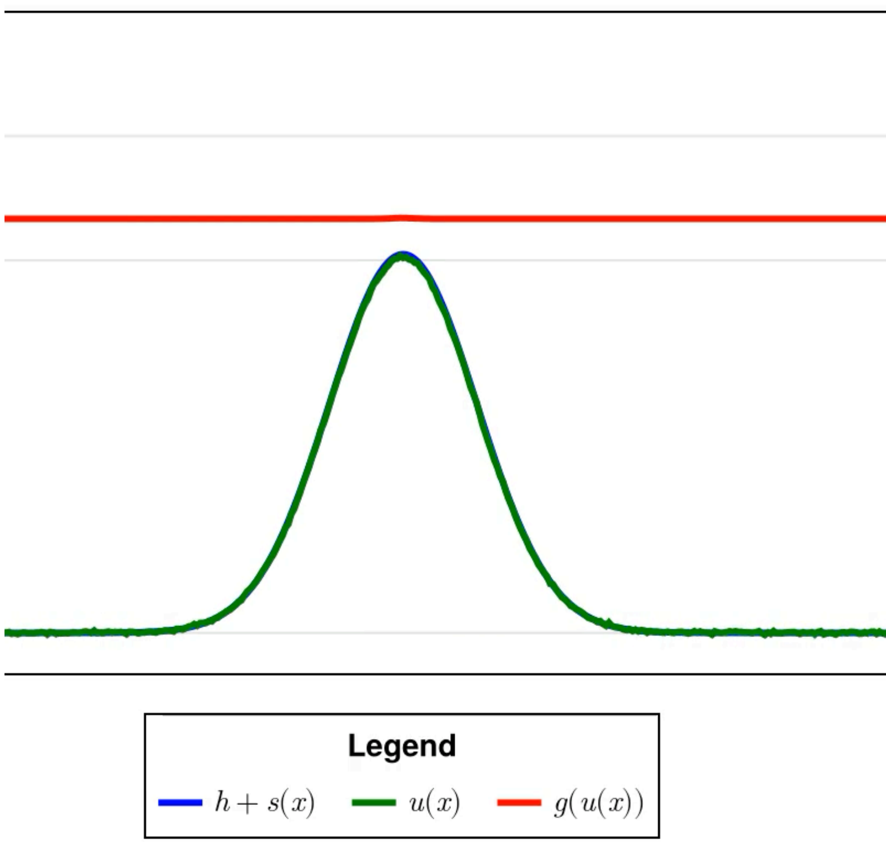
interaction

- strong recurrent connectivity within populations
- excitatory for neighbors in space
- inhibitory for activation at a spatial distance

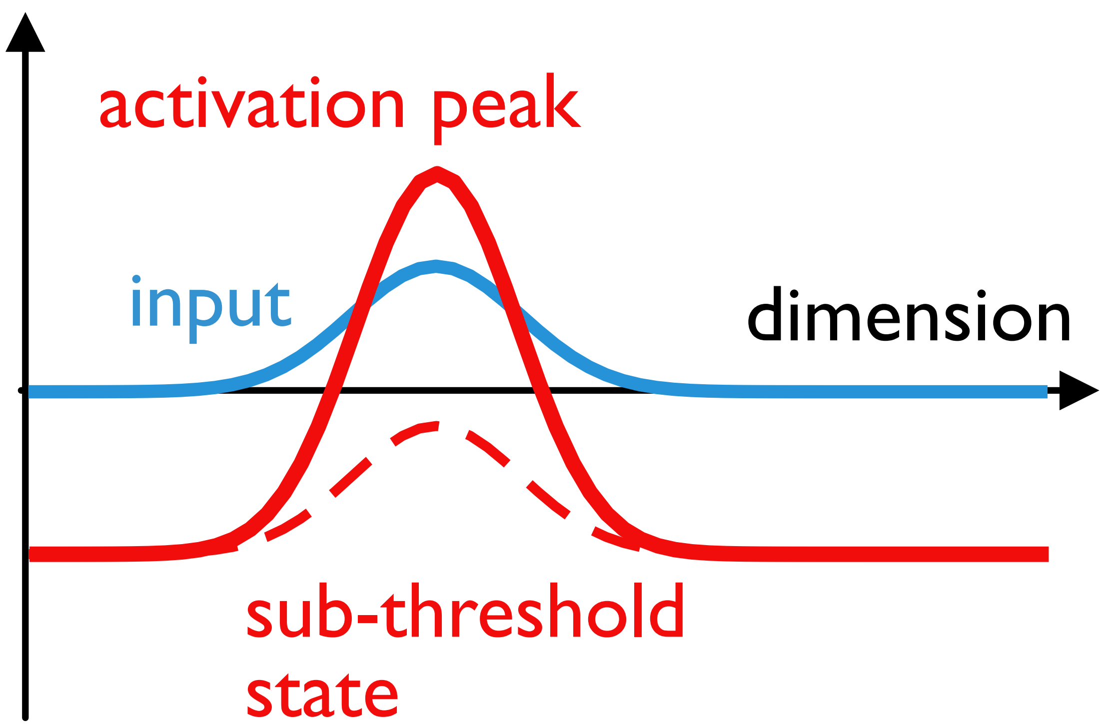


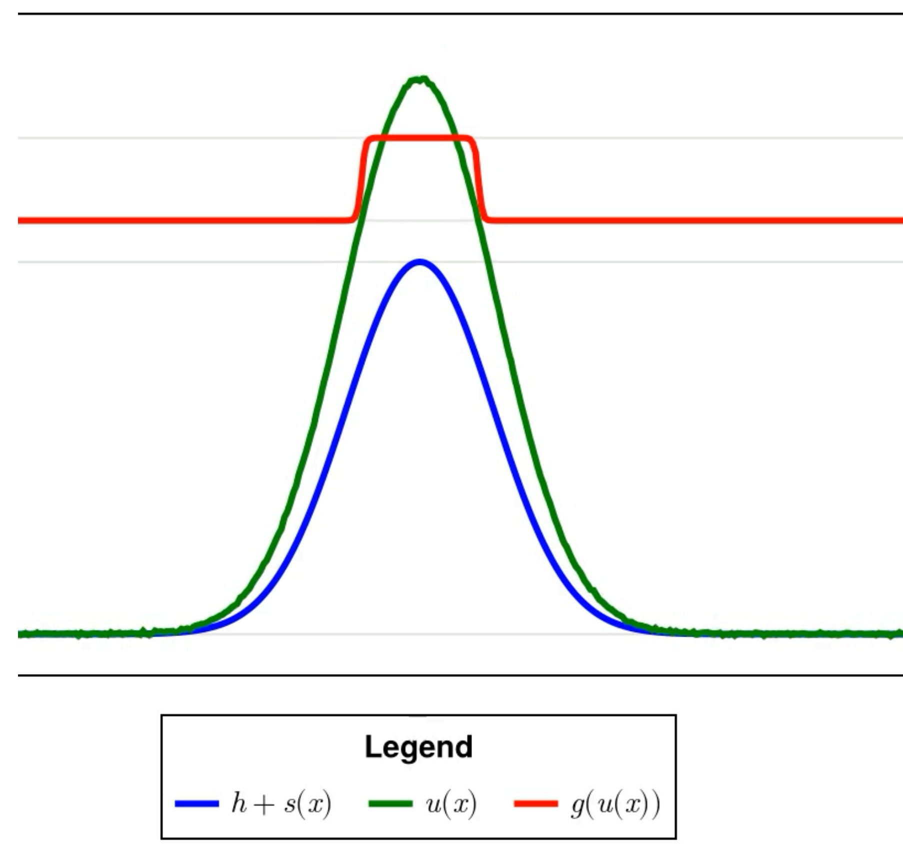
- detection instability of sub-threshold state=> switch to peak
- peak persists below detection instability => bistable





■ reverse detection instability of peak





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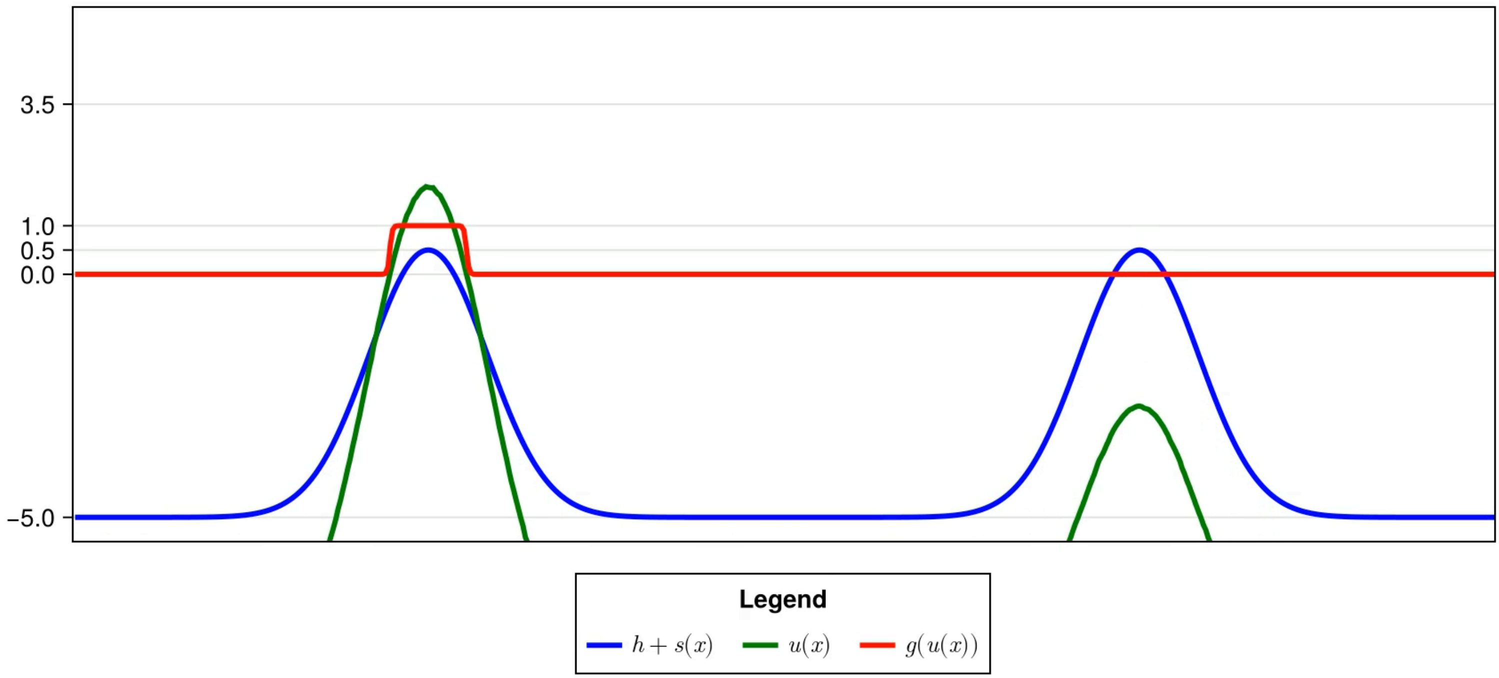
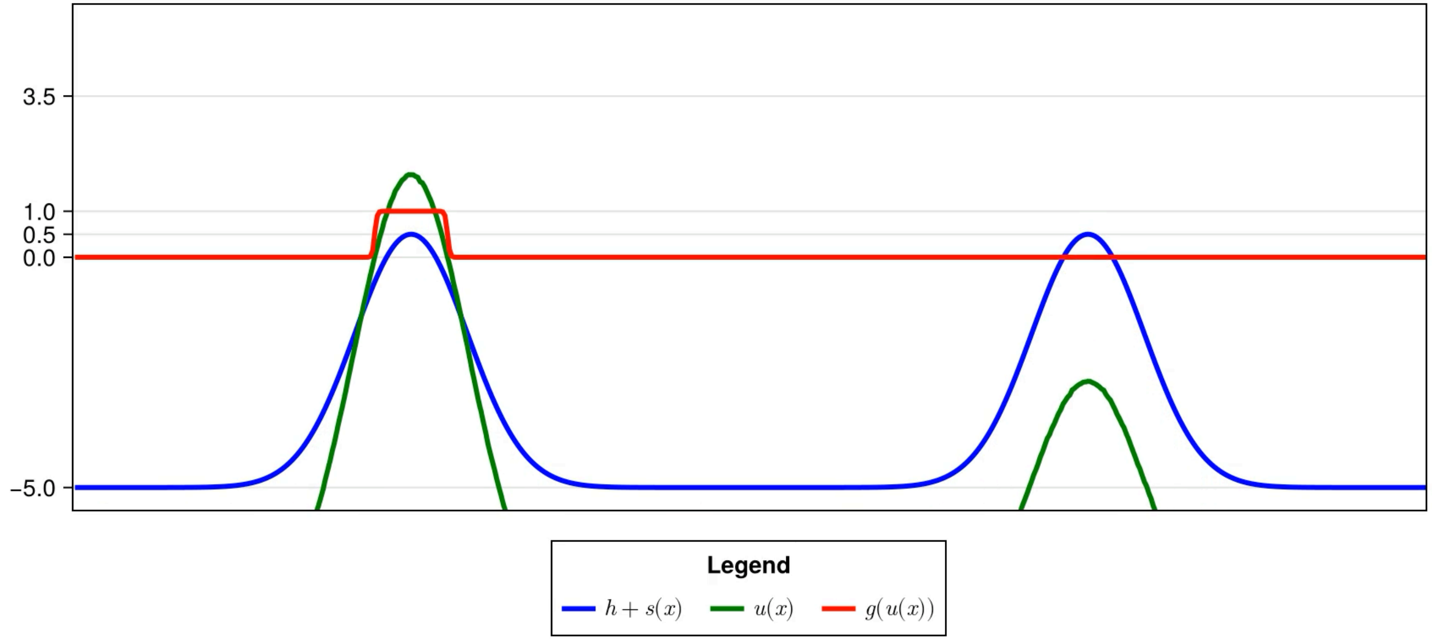
Instabilities

- detection instability
- reverse detection instability
- sustained activation
- selection
- selection instability
- boost driven detection/selection
- events and sequences

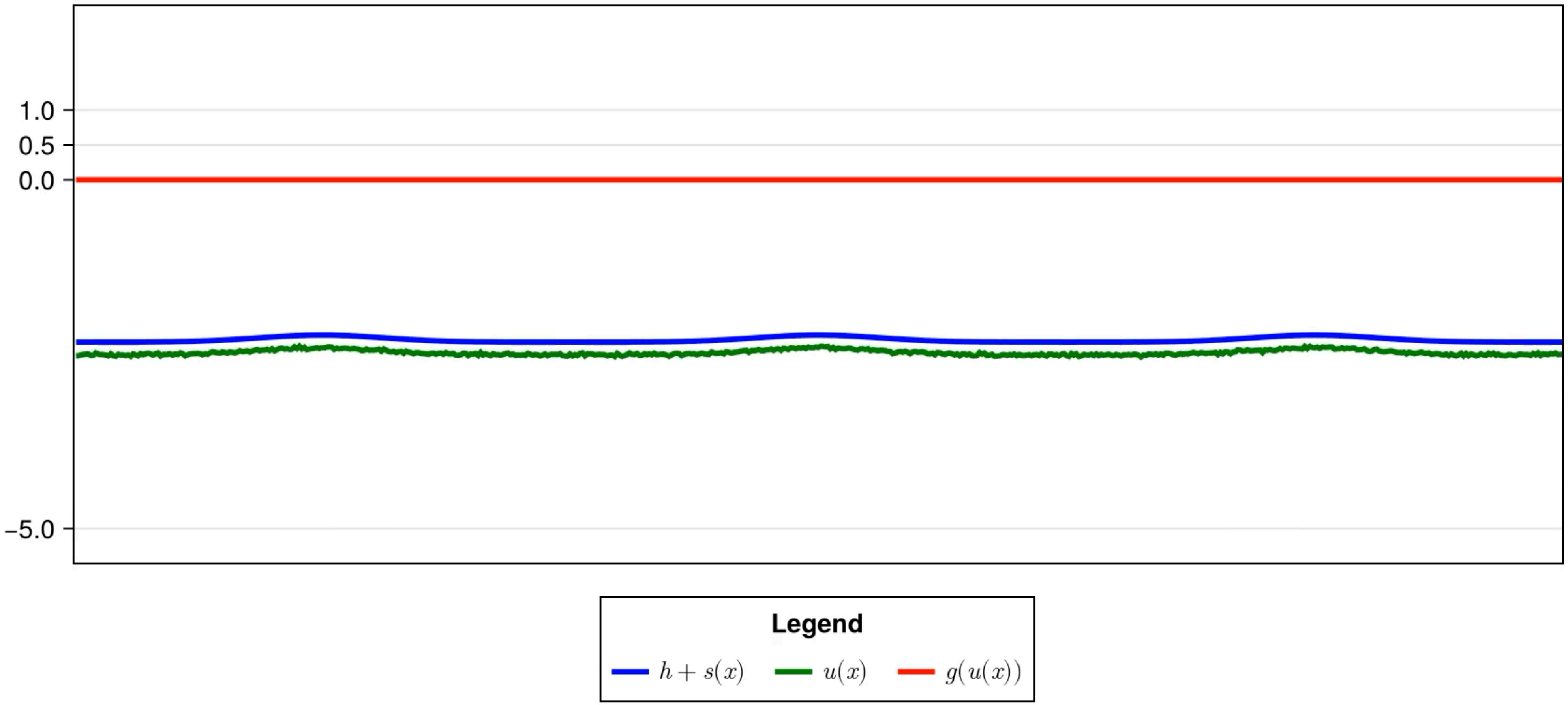
- sustained activation
- ~working memory

- selection
- selection instability

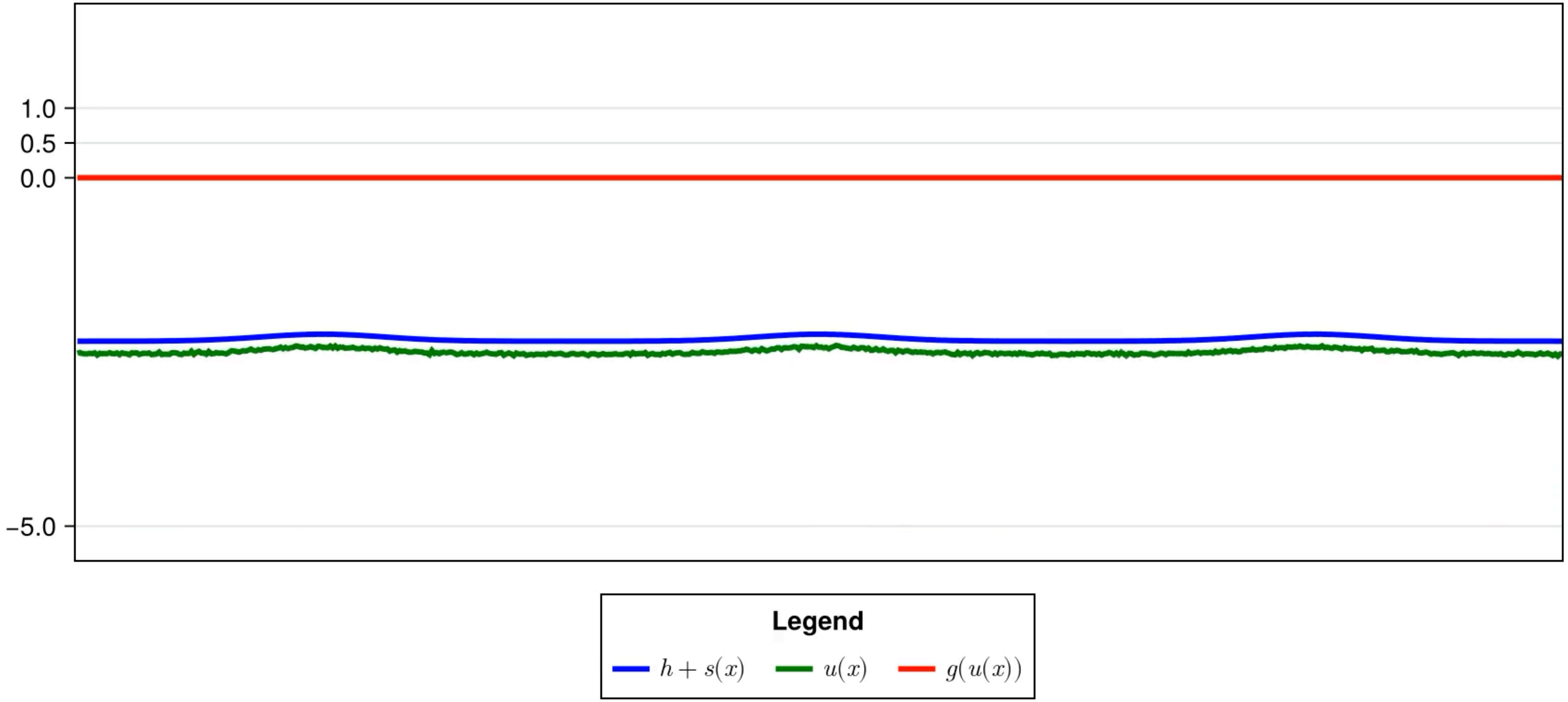




- detection and selection induced by homogeneous boost
- => amplify small inhomogeneities



- detection and selection induced by homogeneous boost
- => peak forms that amplifies small inhomogeneities



- the detection instability creates events at discrete moments in time
- even in response to time-continuous input
- => the basis of sequence generation

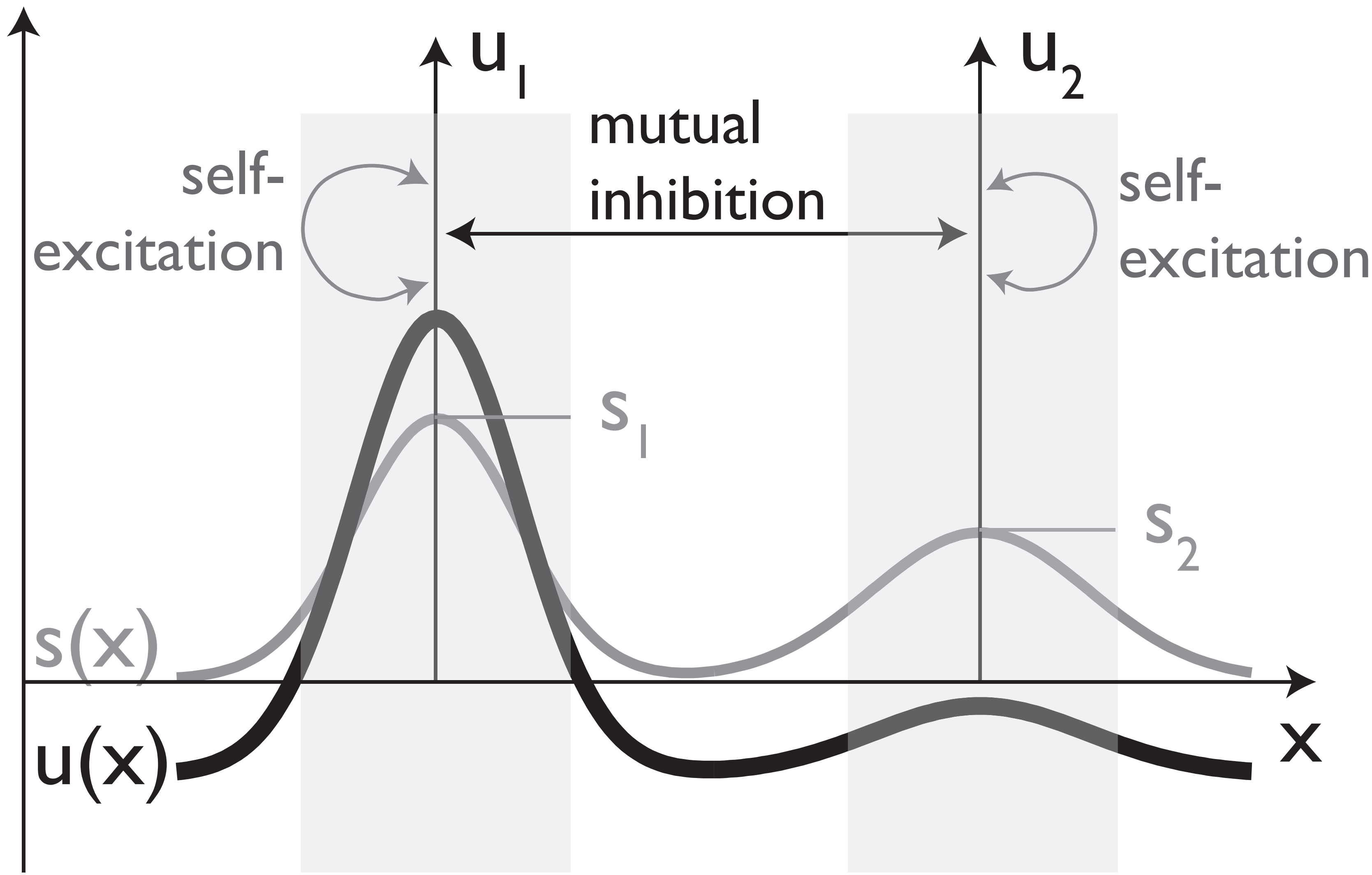
Roadmap

Instabilities

- detection instability
- reverse detection instability
- sustained activation
- selection
- selection instability
- boost driven detection/selection
- events and sequences

Tutorial

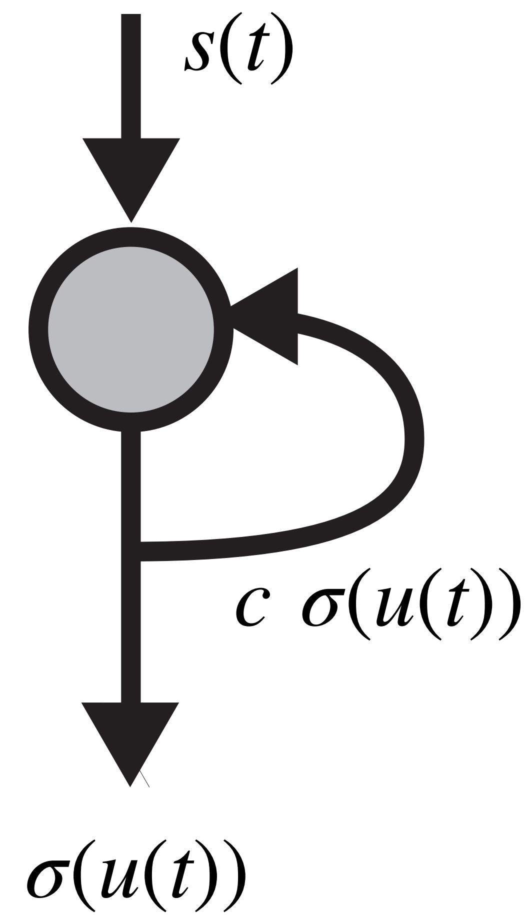
Analysis for discrete activation variables



Tutorial

Excitatory interaction = self-excitation

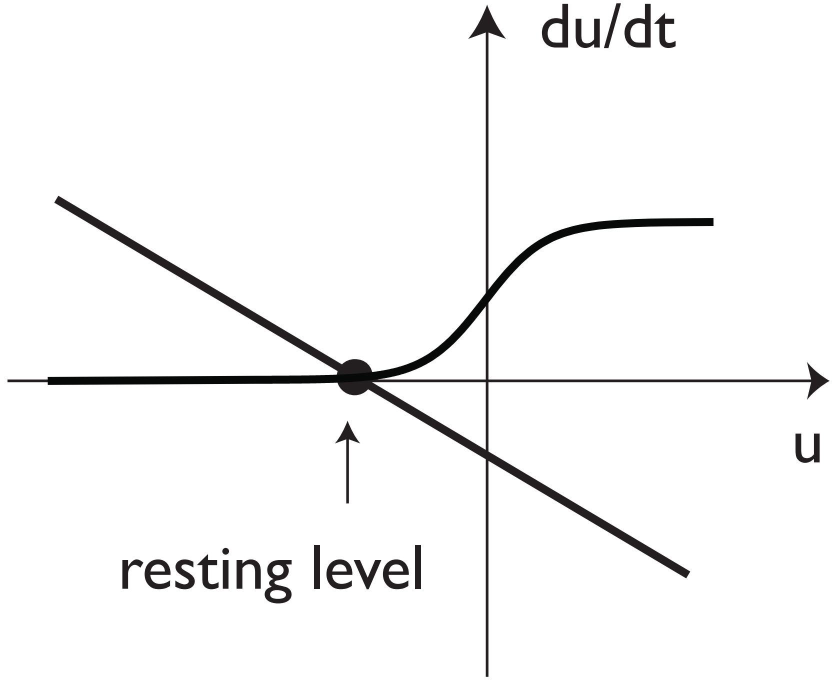
- a minimally recurrent network
- illustrates that "time" is conceptually necessary to understand these:
  - some inputs are outputs from the same neuron/population …
  - => not possible to frame as input-ouput systems
  - solution: time: past outputs are current inputs



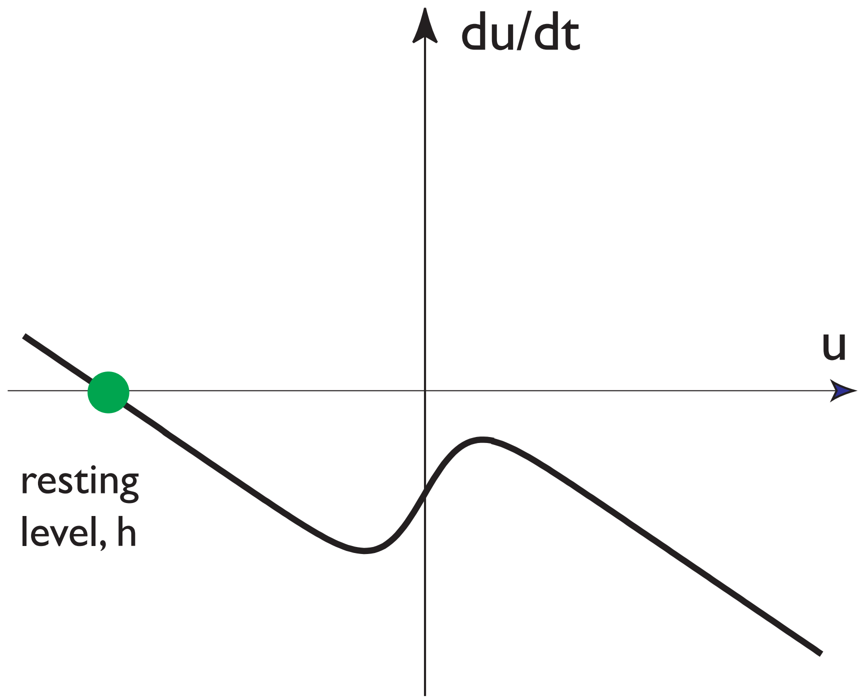
$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Tutorial

Neuronal dynamics with self-excitation



- nonlinear dynamics!

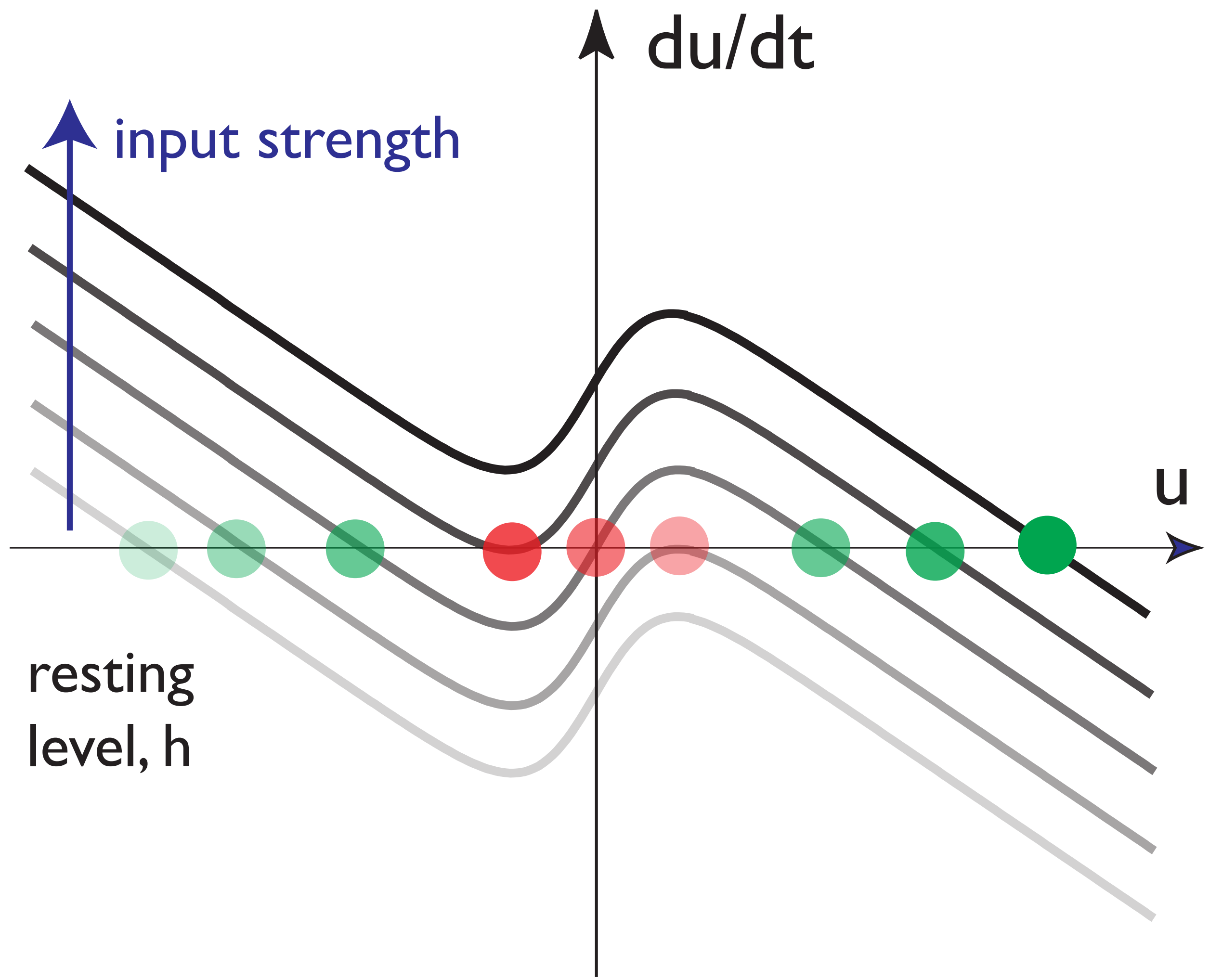


$$\tau\dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Tutorial

Neuronal dynamics with self-excitation

- varying input
- => number of attractors changes



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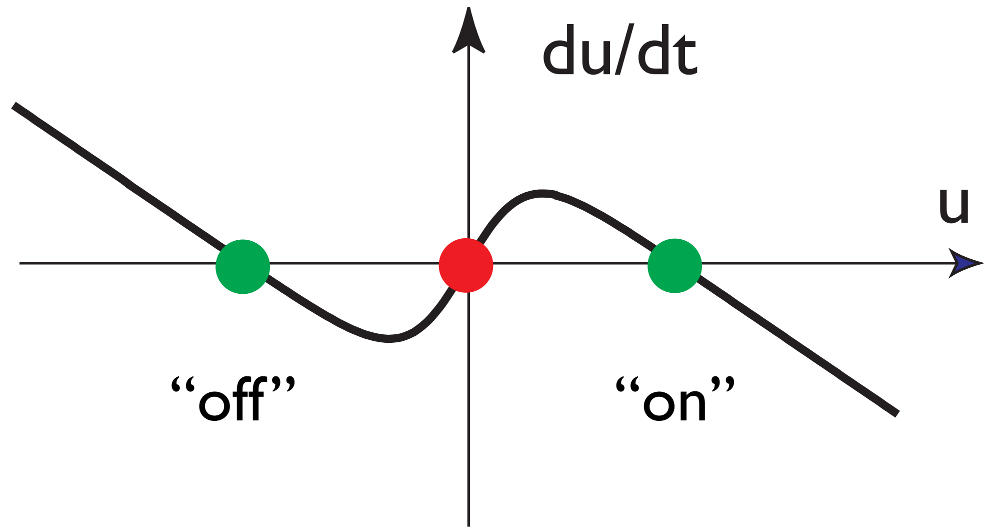
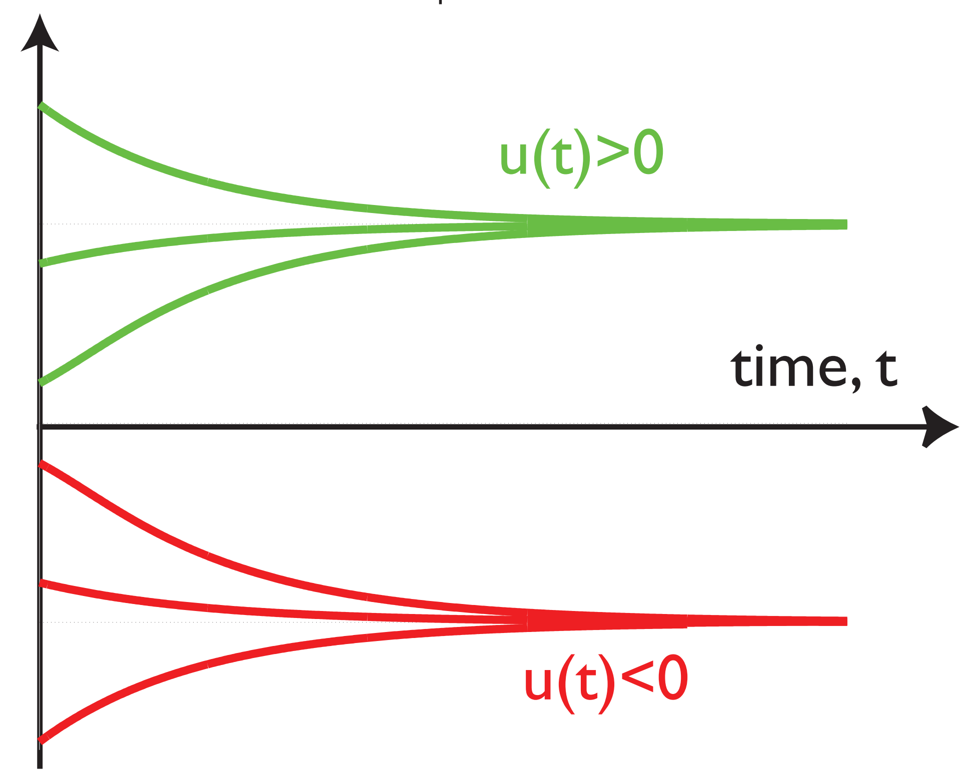
Tutorial

Neuronal dynamics with self-excitation



- at intermediate input levels: bistable dynamics
- "on" vs "off" state

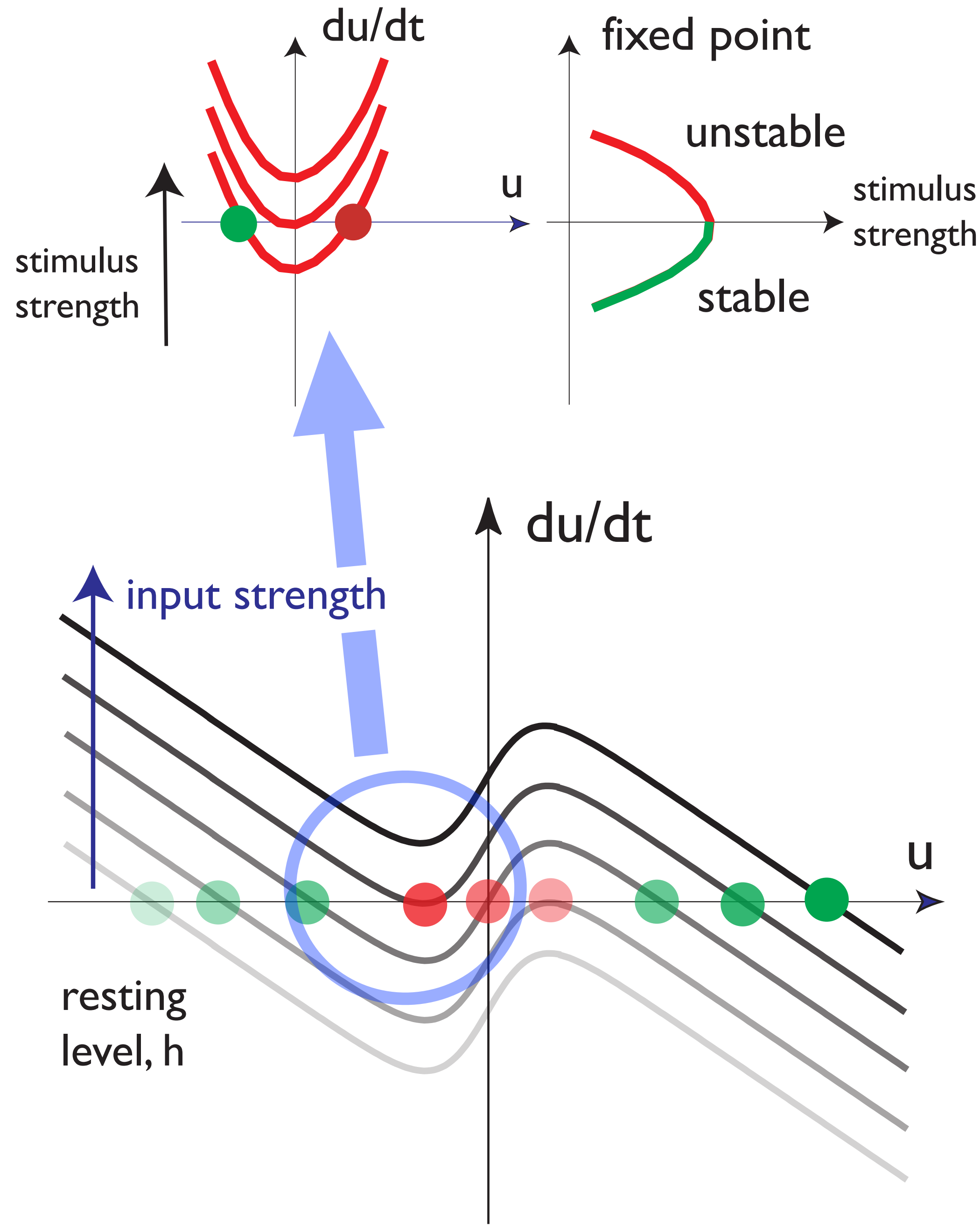


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Tutorial

Neuronal dynamics with self-excitation

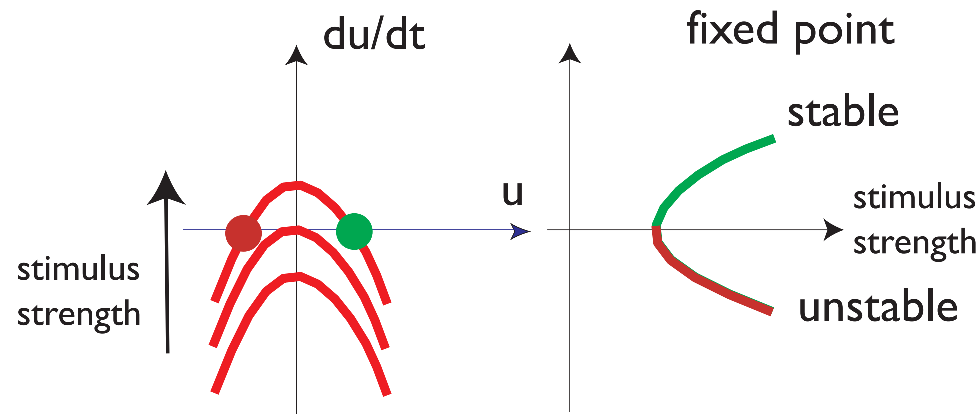
- increasing input strength => detection instability



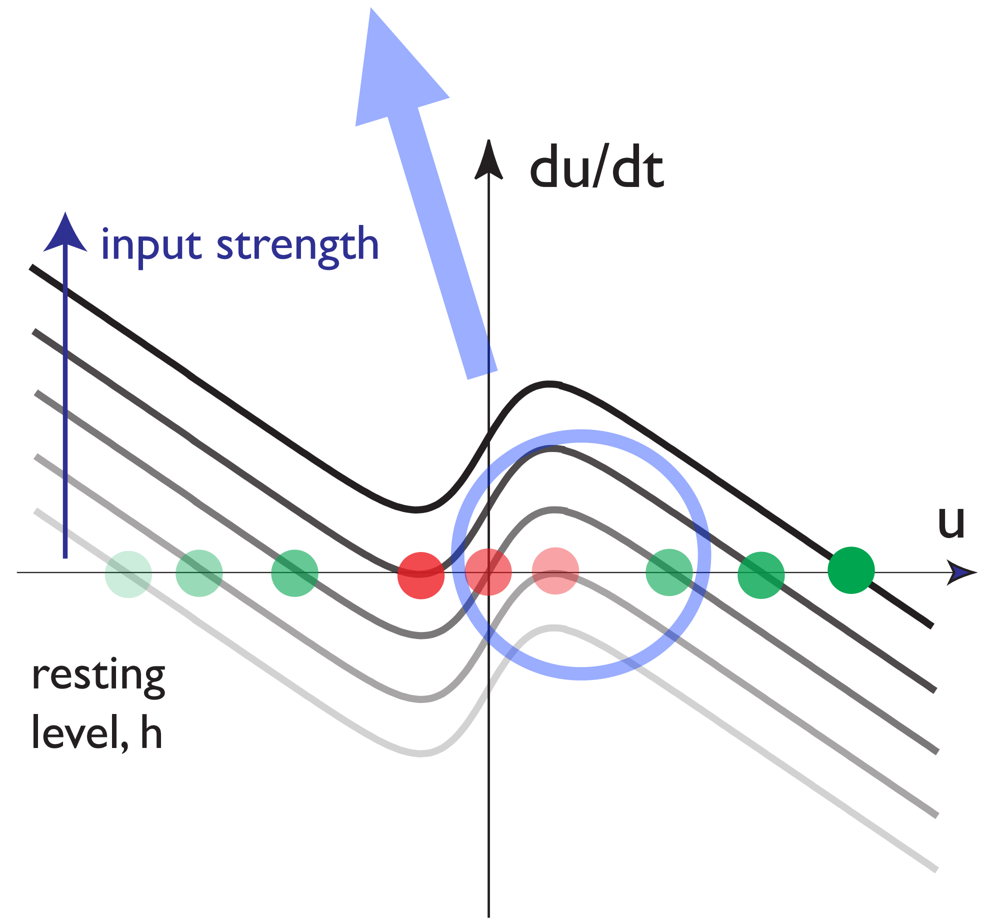
$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Tutorial

Neuronal dynamics with self-excitation



- decreasing input strength => reverse detection instability

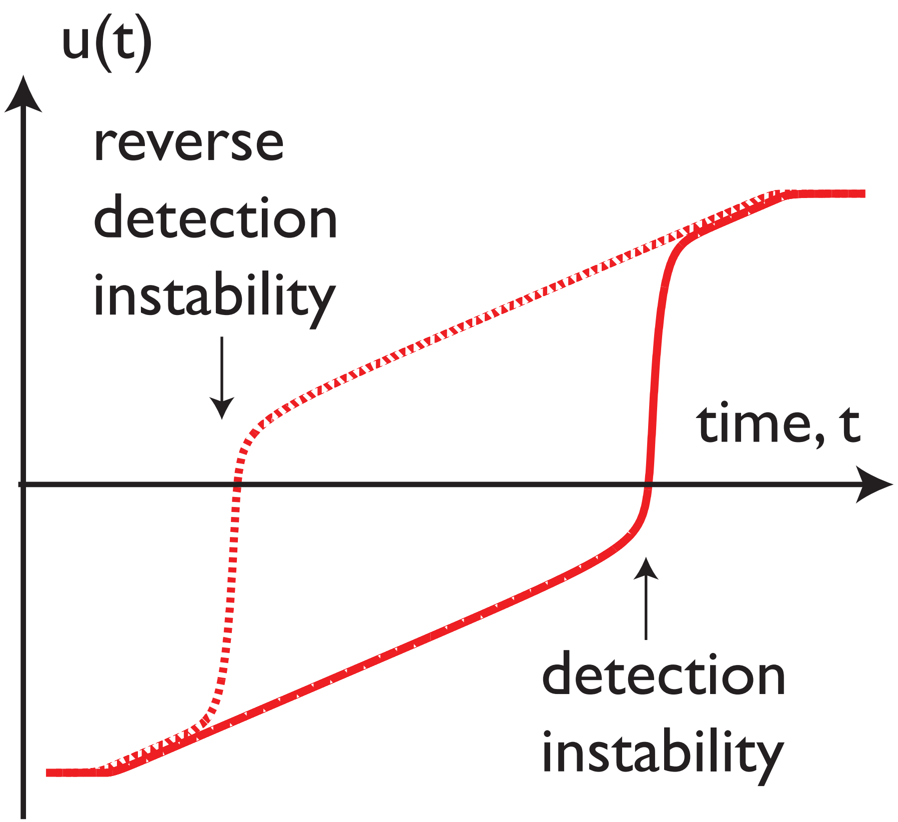


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Tutorial

Neuronal dynamics with self-excitation

- the detection and its reverse create events at discrete times from time-continuous changes

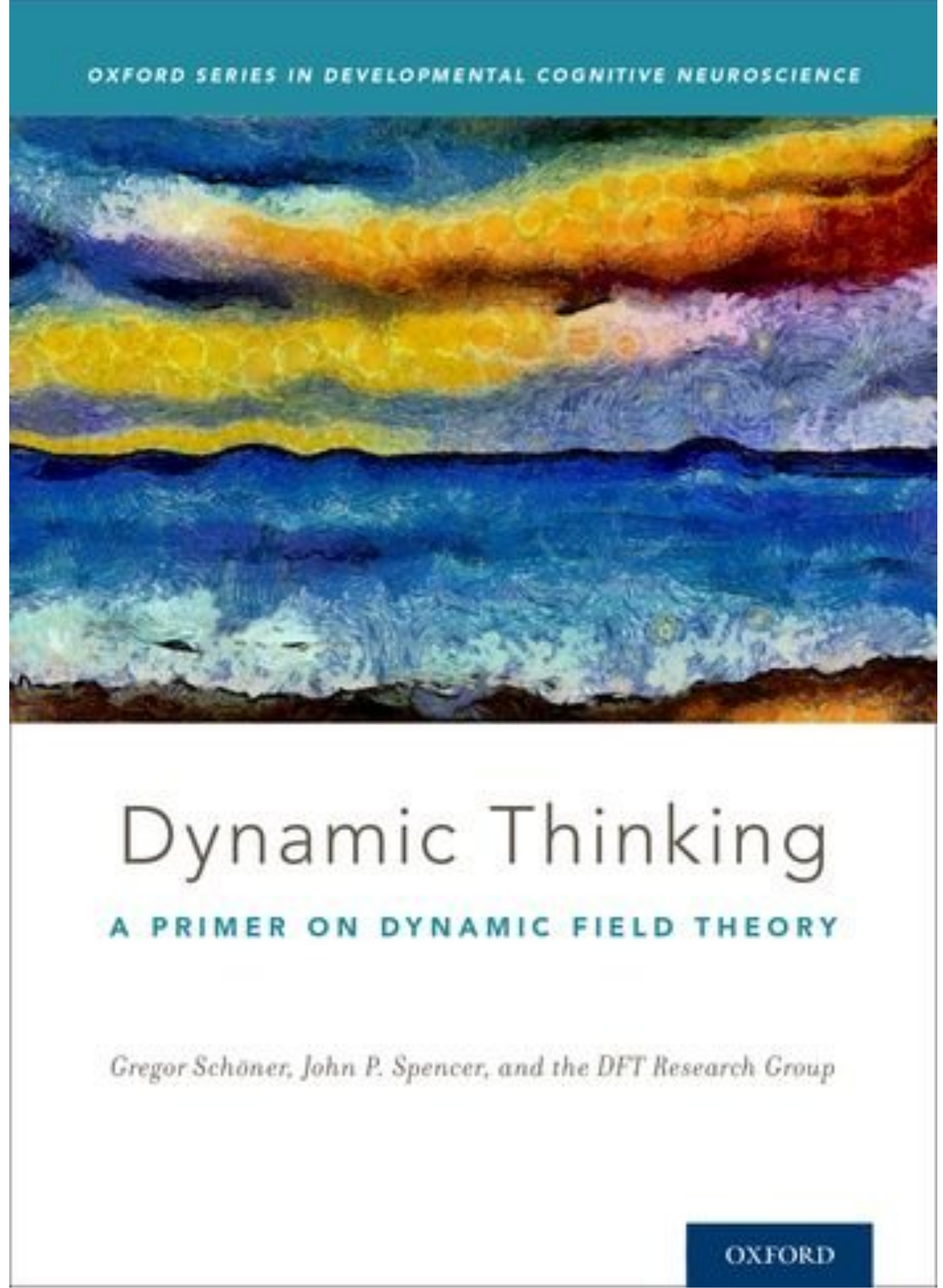


$$\tau \dot{u}(t) = -u(t) + h + s(t) + c\ \sigma(u(t))$$

Tutorial

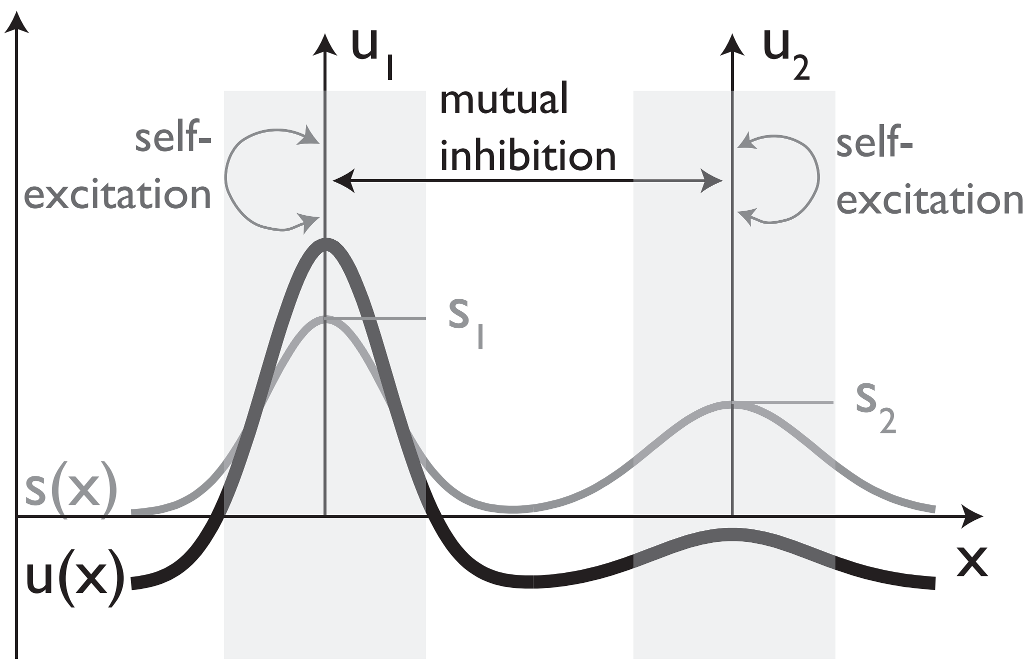
simulating discrete activation variables with self-excitation

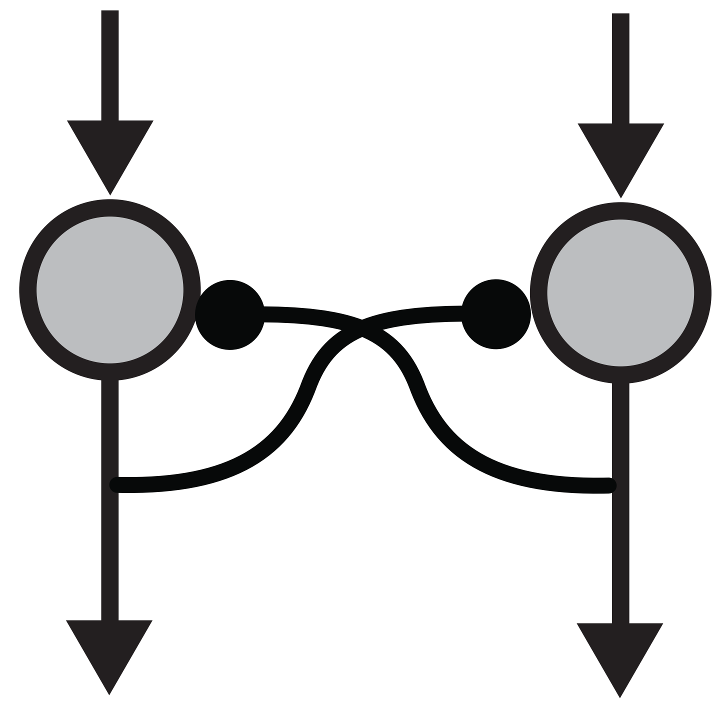
- dynamicfieldtheory.org



Tutorial

Inhibitory interaction: inhibitory recurrent connectivity





coupling/interaction

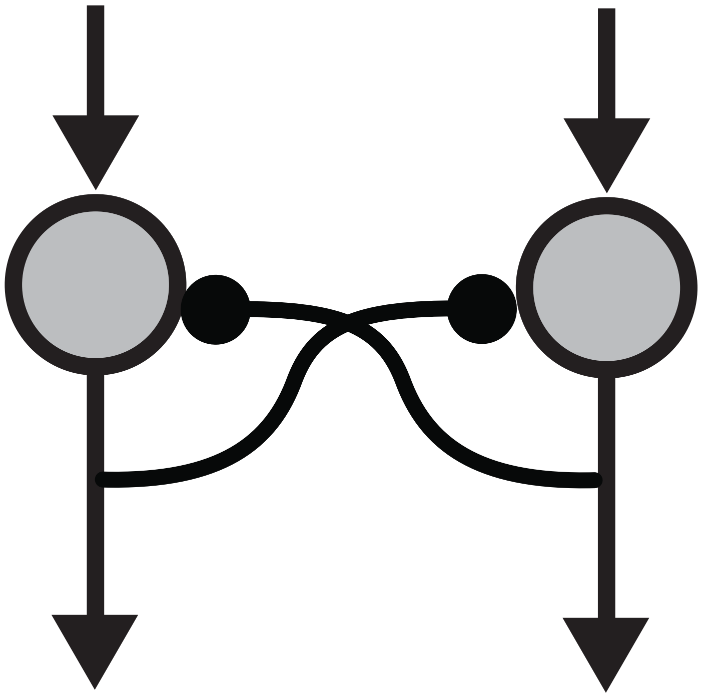
$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Tutorial

Inhibitory coupling

- two possible attractor stats
  - $u_2 > 0$ and $u_1 < 0$
  - $u_2 < 0$ and $u_1 > 0$
- => competition/selection

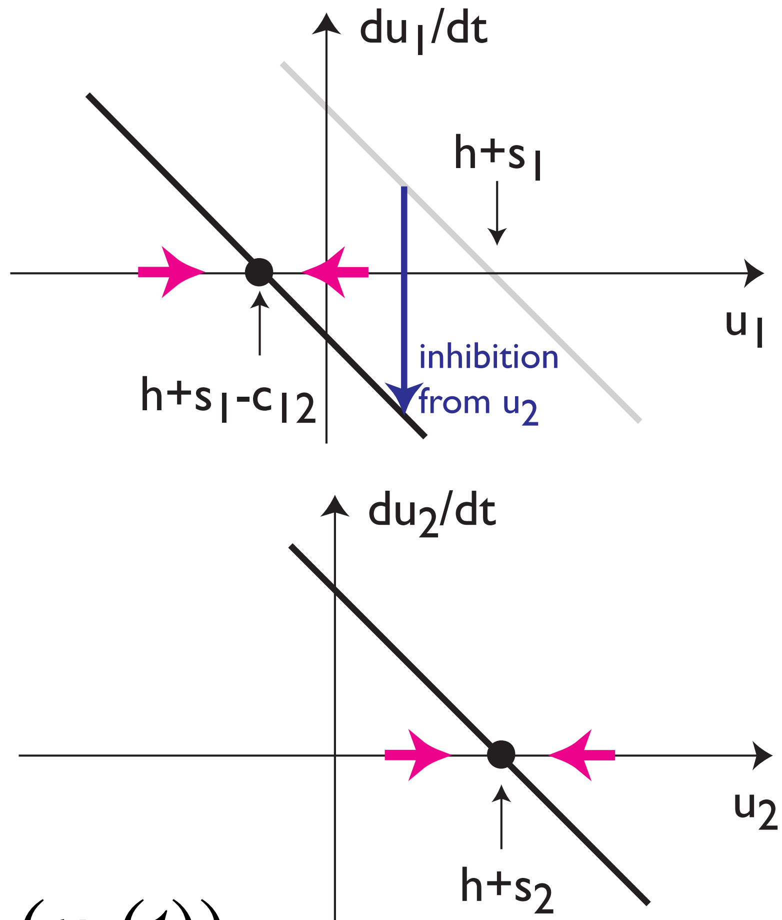


$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$
$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Tutorial

Inhibitory coupling

- to visualize, assume that u_2 has been activated by input to a positive level
- => it inhibits u_1



$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Tutorial

Inhibitory coupling

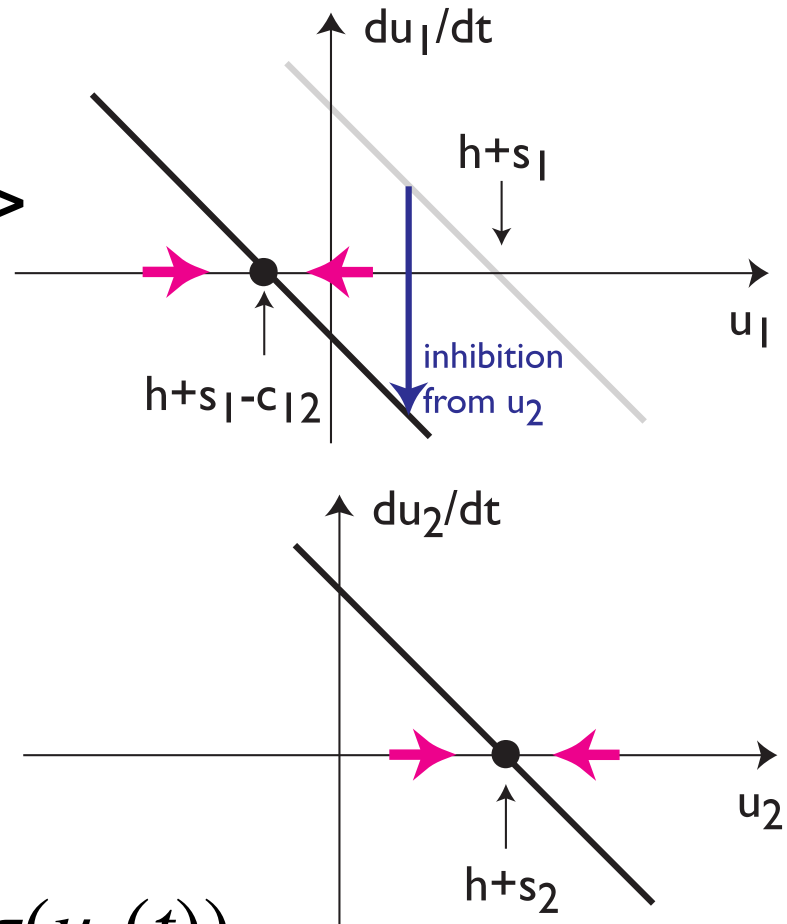
- symmetry: same logic if u_1 was initially activated it would prevent u_2 from activating
- => bistable selection of either u_1 or u_2

$$\tau \dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

$$\tau \dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Inhibitory coupling

- asymmetric case: e.g. more input to u_2 (better "match") => faster increase => u_2 selected
- => input advantage => time advantage => competitive advantage



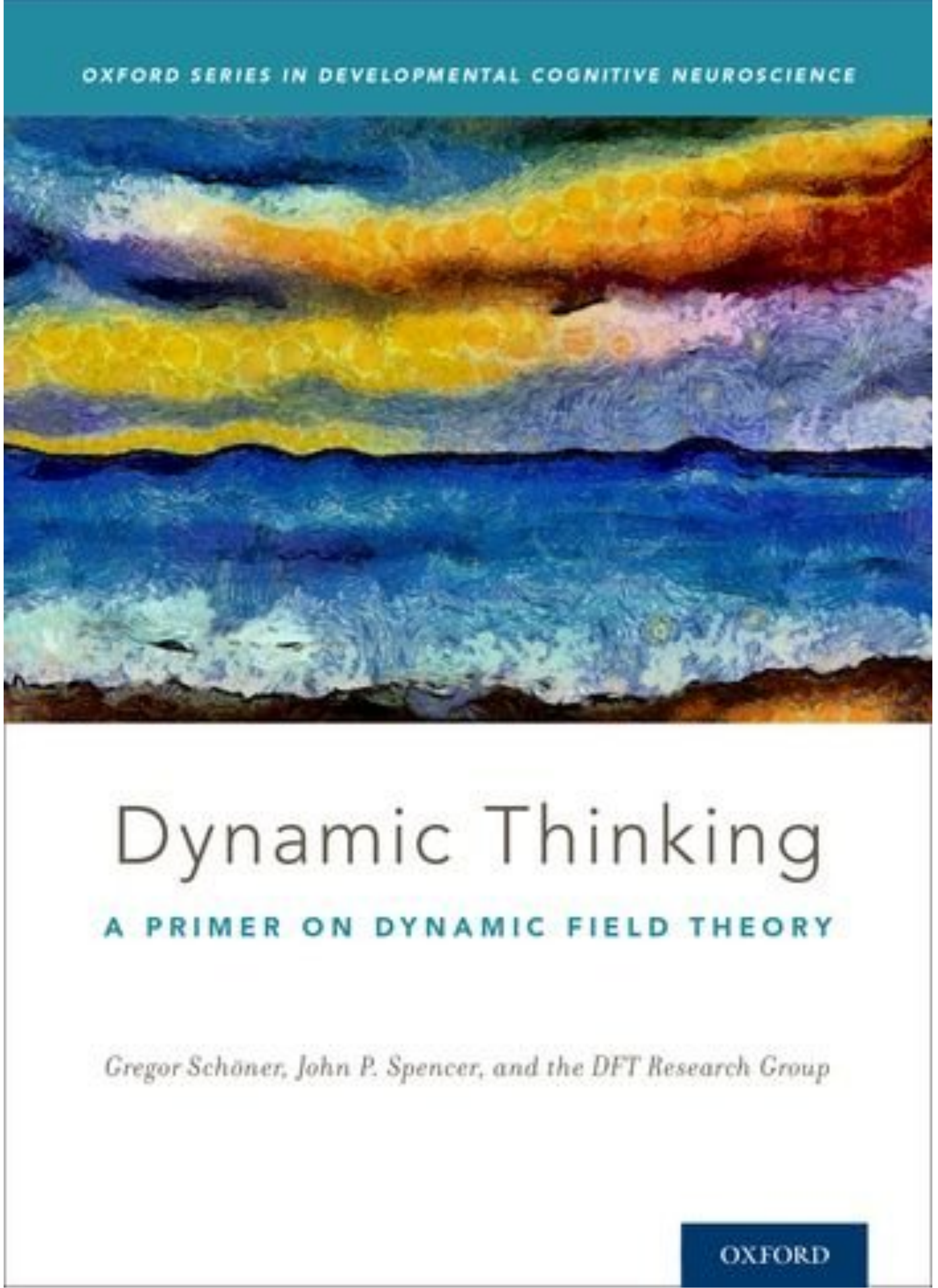
$$\tau\dot{u}_1(t) = -u_1(t) + h + s_1(t) - c_{12}\sigma(u_2(t))$$

$$\tau\dot{u}_2(t) = -u_2(t) + h + s_2(t) - c_{21}\sigma(u_1(t))$$

Tutorial

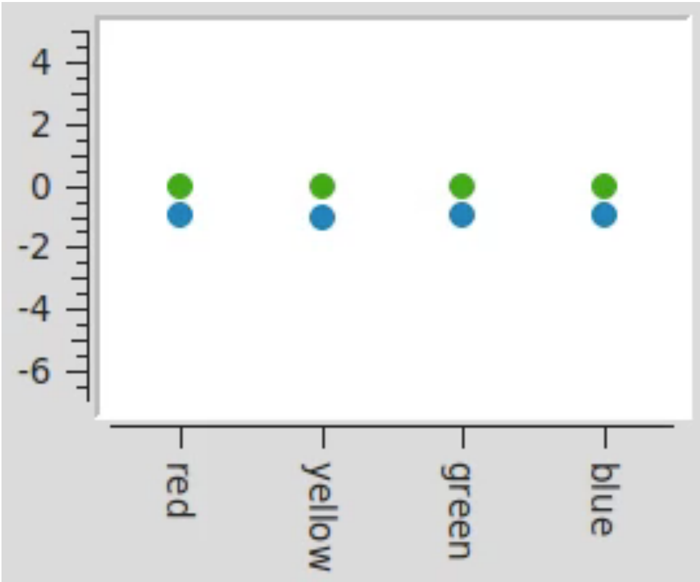
simulating inhibitorily coupled activation variables

- dynamicfieldtheory.org



Neural dynamic nodes

- sets of discrete activation variables as "nodes"
  - self-excitatory: "on" vs "off" states, detection instability, sustained activation
  - all nodes coupled inhibitorily: selection
  - => discretely sampled fields

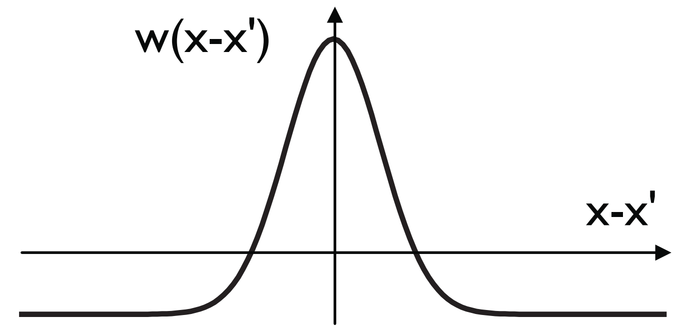


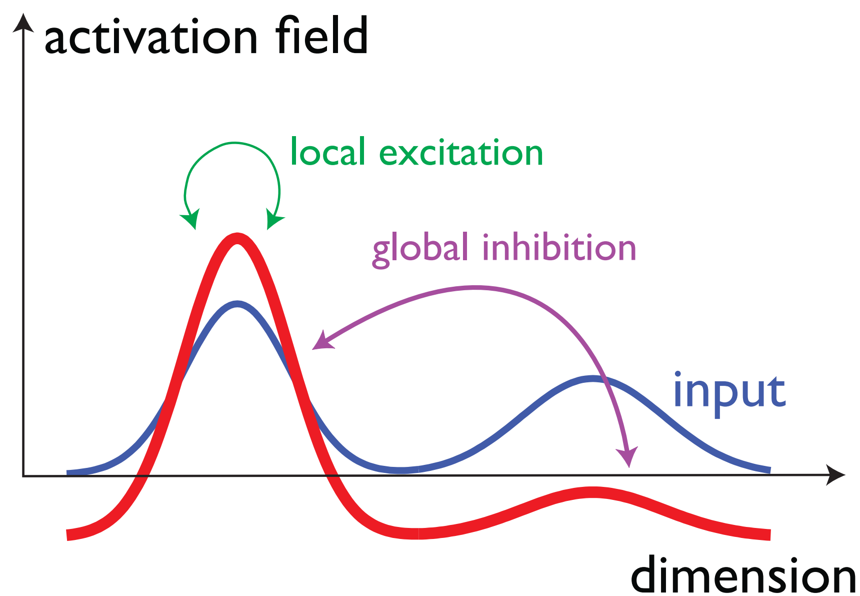
Supplement

Mathematical formalization

- kernel: local excitatory interaction/ global inhibitory interaction

$$w(x-x') = w_{\text{exc}} e^{-\frac{(x-x')^2}{2\sigma^2}} - w_{\text{inh}}$$





$$\tau \dot{u}(x,t) = -u(x,t) + h + s(x,t) + \int dx' \; w(x-x') \; \sigma(u(x'))$$

Supplement Mathematical formalization

Amari equation

$$\tau \dot{u}(x,t) = -u(x,t) + h + S(x,t) + \int w(x-x')\sigma(u(x',t))\ dx'$$

where

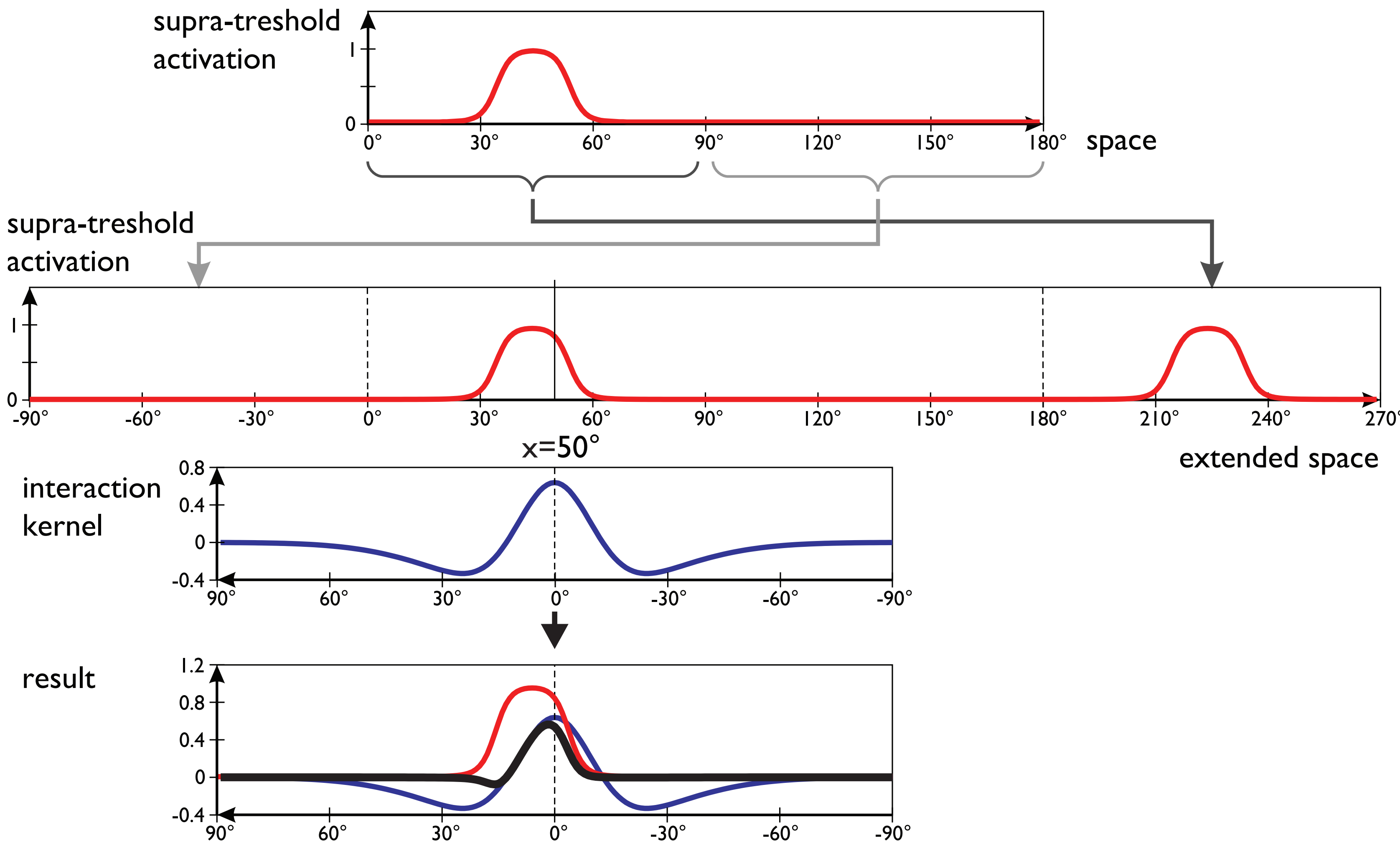
- time scale is τ
- resting level is $h < 0$
- input is $S(x,t)$
- interaction kernel is

$$w(x-x') = w_i + w_e \exp\left[-\frac{(x-x')^2}{2\sigma_i^2}\right]$$

- sigmoidal nonlinearity is

$$\sigma(u) = \frac{1}{1 + \exp[-\beta(u-u_0)]}$$

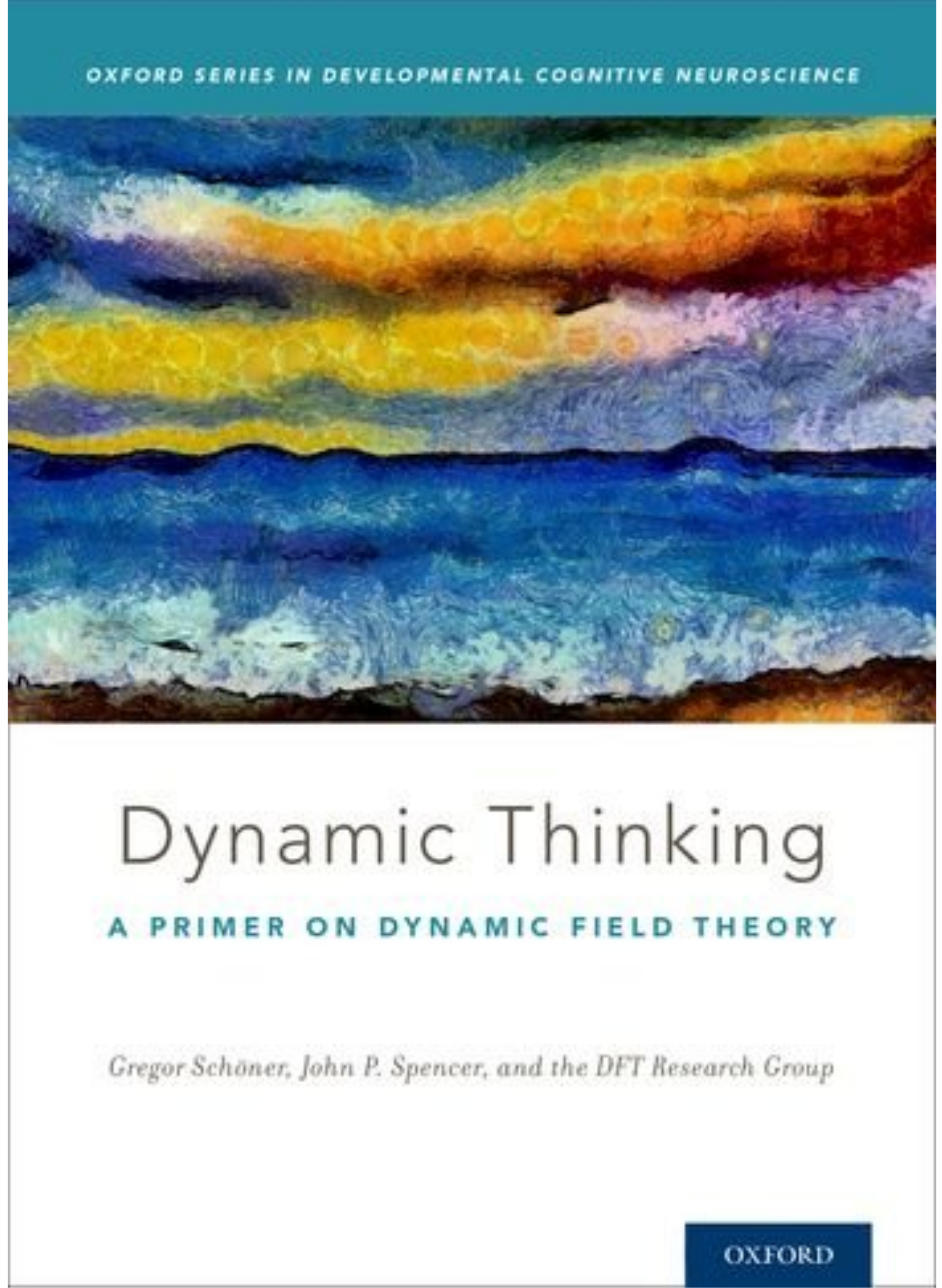
Interaction: convolution



Tutorial

simulating the instabilities of the field dynamics

- dynamicfieldtheory.org



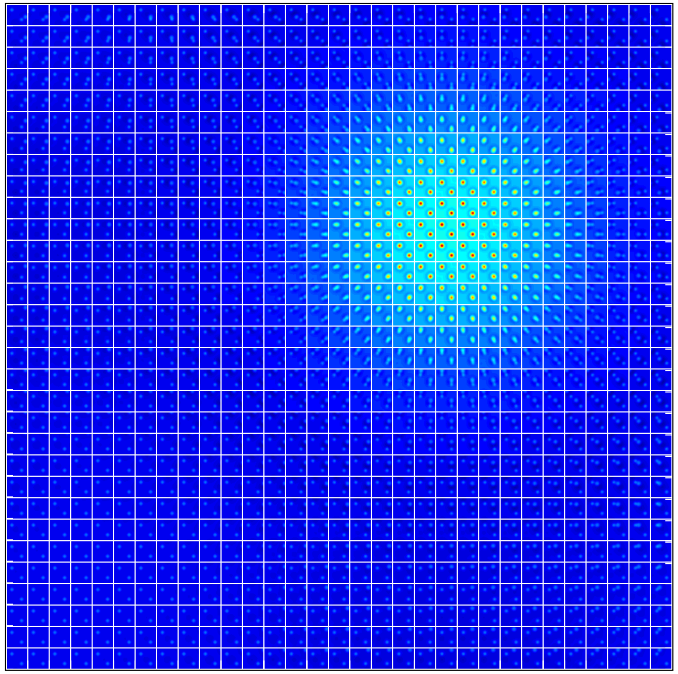
Dynamic regimes

- which attractors and instabilities arise as input patterns are varied
- examples
  - “perceptual regime”: mono-stable sub-threshold => bistable sub-threshold/peak => mono-table peak..
  - “working memory regime” bistable sub-threshold/peak => mono-table peak.. without mono-stable sub-threshold
  - single (“selective”) vs. multi-peak regime

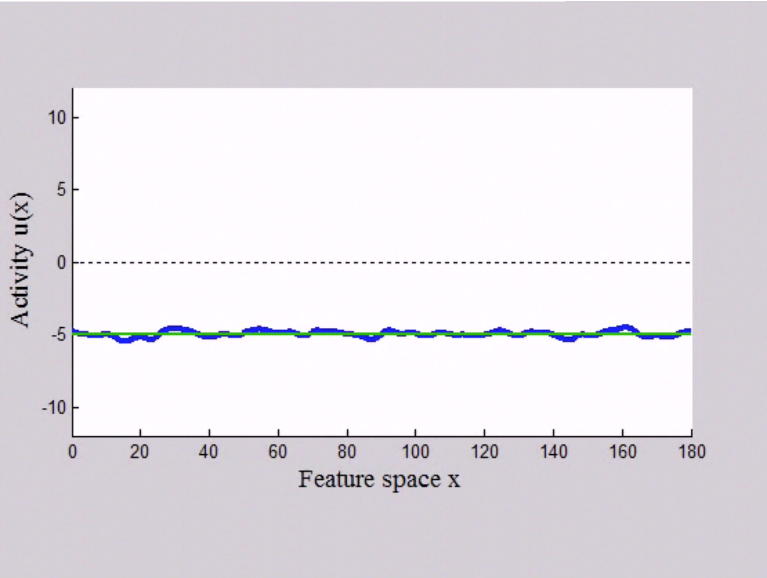
Field dynamics in different dimensions

- 1, 2, 3, 4… dimensions: peaks/ blobs as attractors

4-dimensional

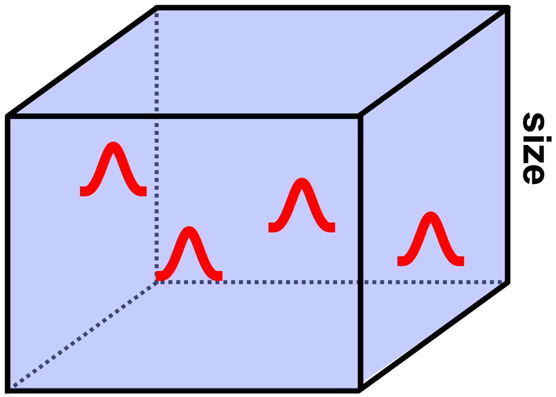


1-dimensional



2-dimensional

3-dimensional



Foundations 1: Space and time

- Space
- Time:
  - Neural dynamics
  - Interaction
- Instabilities
- Tutorial: discrete activation variables
- Supplement: mathematical formalization
- Tutorial: simulating fields