DFT Tutorial

ICANN 2025

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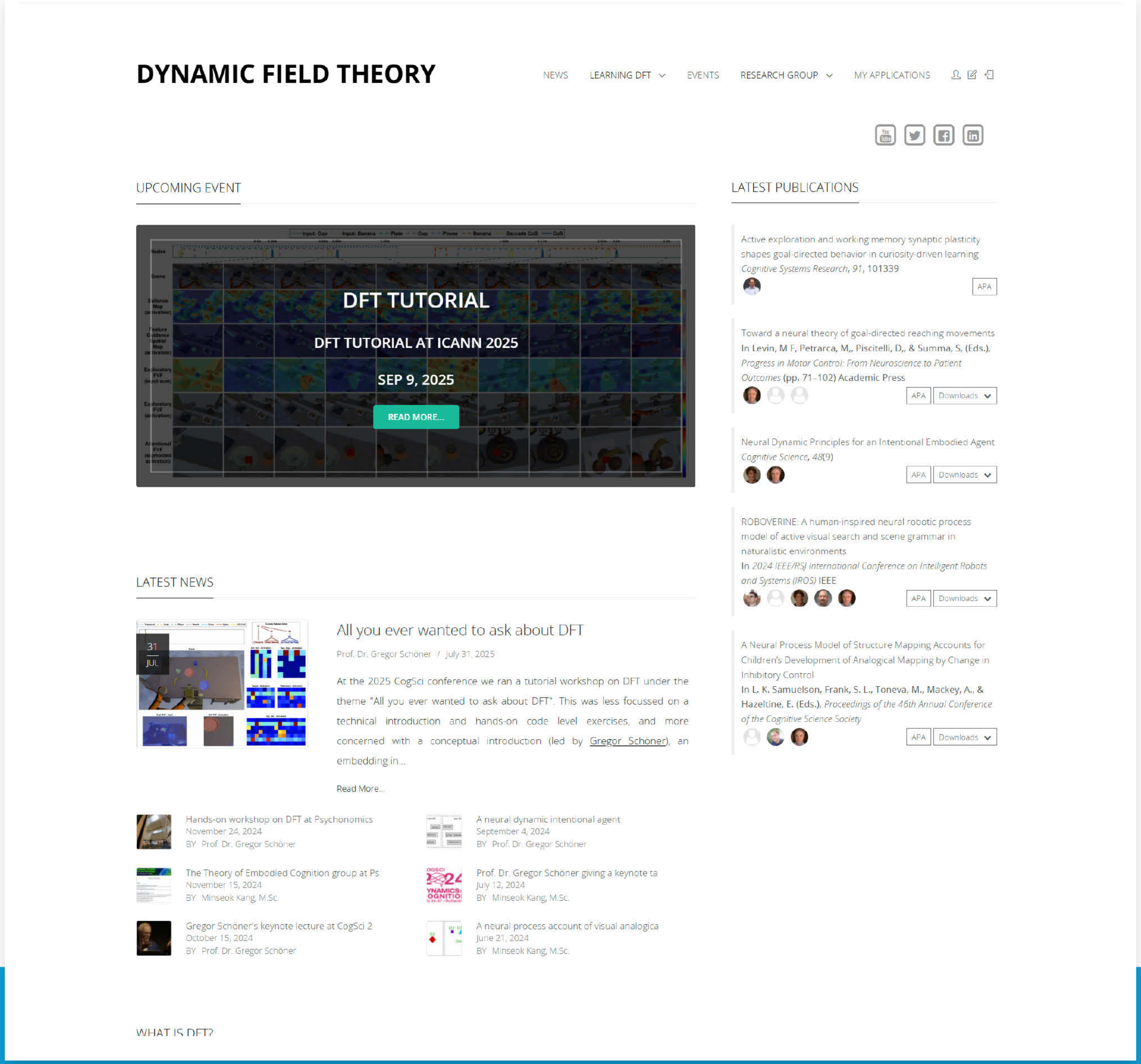
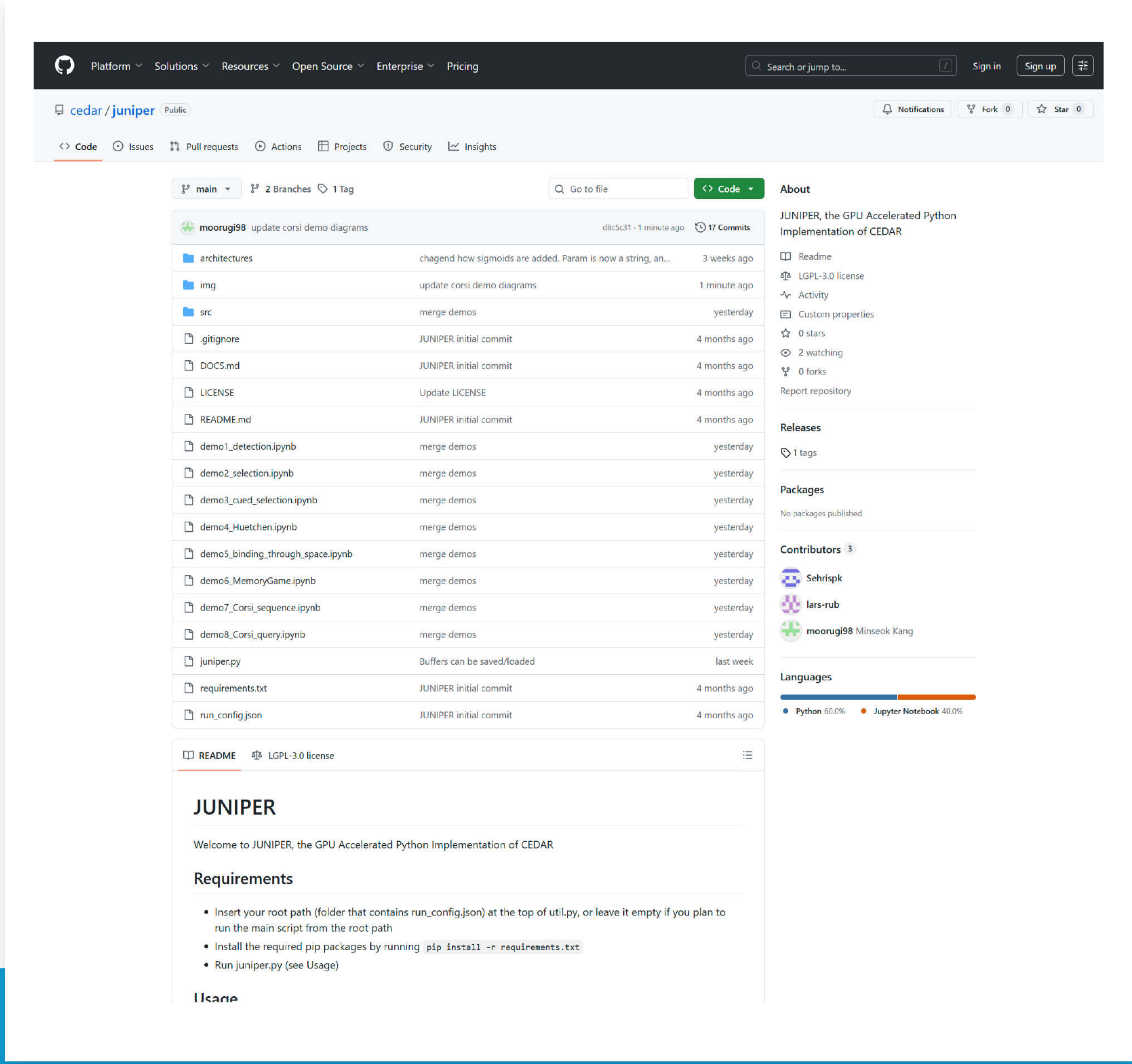
dynamicfieldtheory.org

- Tutorial documents (end of the week)
- DFT resources
  - Publications
  - Videos
  - Software
  - Book
  - News



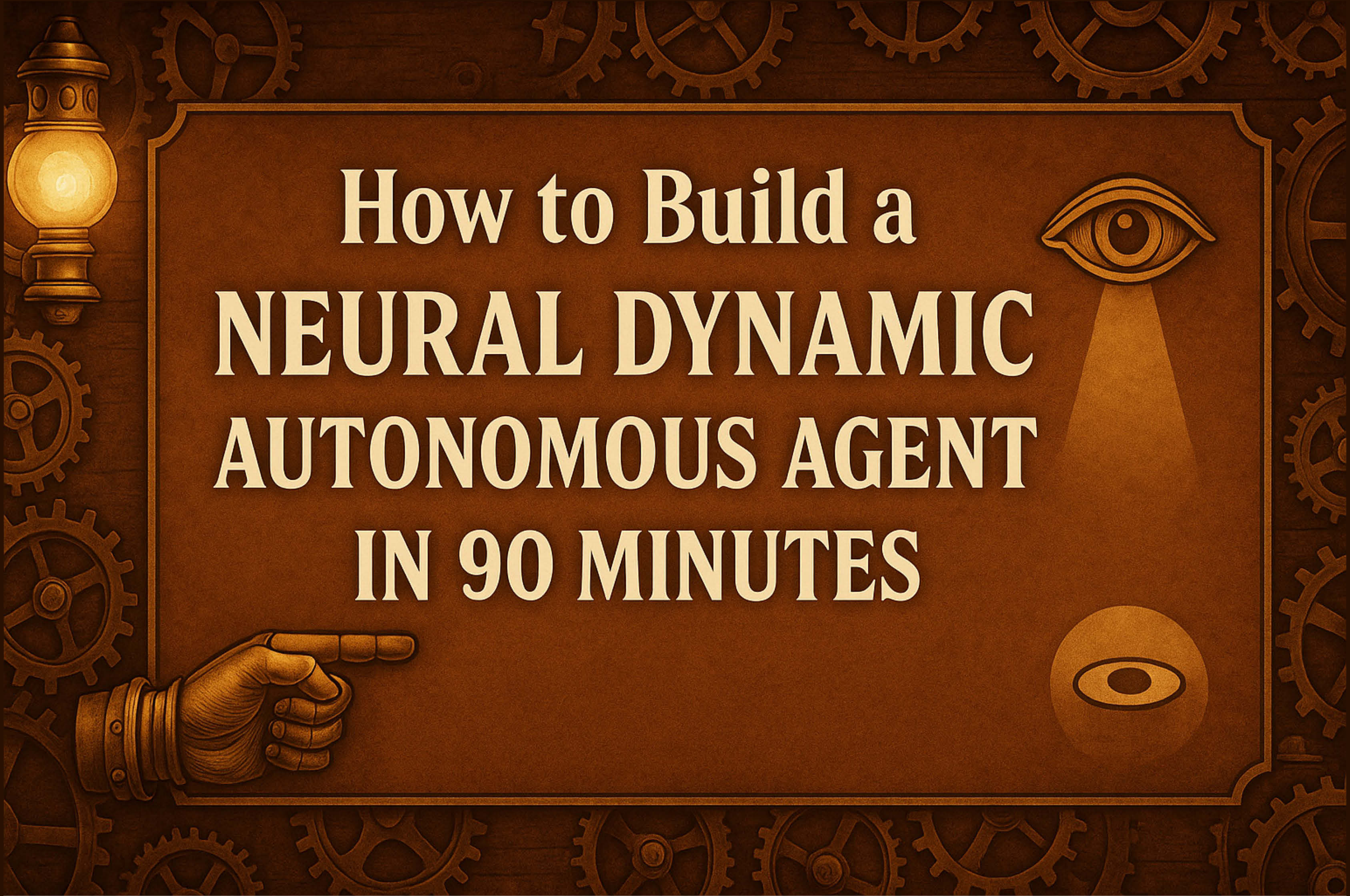
github.com/cedar/juniper

- GPU accelerated version of CEDAR
  - GitHub contains notebooks with demos used here
- Requirements:
  - jax[cuda12]
  - matplotlib

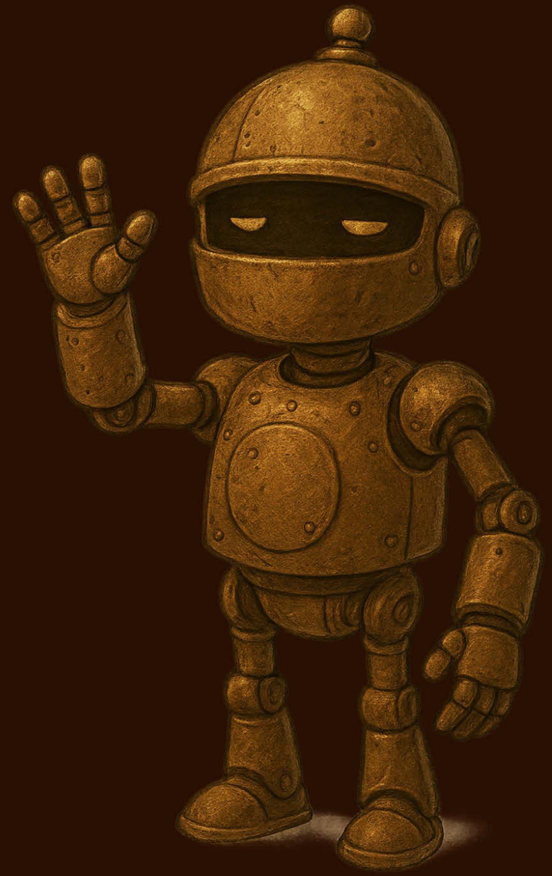


Schedule

- Building a Neural Dynamic Agent (90 min)
- Break and Networking (30 min)
- Hybrid Models with DFT (15 min)
- Latest DFT Applications (25 min)
- Open Discussion and Q&A (20 min)


How to Build a
NEURAL DYNAMIC
AUTONOMOUS AGENT
IN 90 MINUTES

This is Marvin:



“Oh God I’m so depressed”

And his new friend **Billy**:

“I want to play a game”


ESCAPE
ROOM


ESCAPE ROOM
1
8
2
4
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3
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Dynamic Field Theory (DFT)

- Dynamic Field Theory is a mathematical framework that aims to explain how
  - time-continuous evolution of neural population activation leads to cognition
  - discrete events emerge from instabilities in the underlying dynamics

G. Schöner and J. Spencer. Dynamic thinking: A primer on dynamic field theory. Oxford University Press, 2016.

Foundations

- **Cognition** is best understood at the level of **neural populations**
- Sensory information is transient
  - cognition needs **stable representations**
  - this stability must arise from recurrent connectivity within the neural population
- Neural **representations** needed for cognition are **low-dimensional**
  - they are easier to stabilize
  - enable invariance
- **Instabilities** in the dynamics enable **sequential transitions** between stable states
  - needed for cognition
- **Complex cognitive** functions emerge from interconnected **networks** of neural dynamics

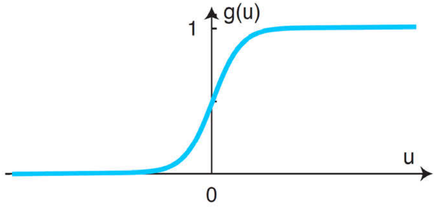
Schöner, G. (2019). The Dynamics of Neural Populations Capture the Laws of the Mind. Topics in Cognitive Science

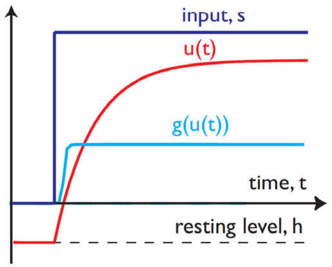
Dynamic neural field (DNF)

$$\underbrace{\tau}_{\text{time scale}}\ \overbrace{\dot{u}}^{\text{rate of change}}(\underbrace{x}_{\text{feature value}}, \overbrace{t}^{\text{time}}) = \overbrace{-u(x,t)}^{\text{stabilization factor}} + \overbrace{h}^{\text{resting level}} + \overbrace{s(x,t)}^{\text{external input}} + \overbrace{w_\xi \cdot \xi(x,t)}^{\text{neural noise}}$$

$$+ \int \underbrace{\omega}_{\text{kernel}}(x - x') \cdot \underbrace{g}_{\text{sigmoidal threshold}}(u(x',t))\, dx'$$

$$g(u) = \frac{1}{1 + \exp(-\beta u)}$$





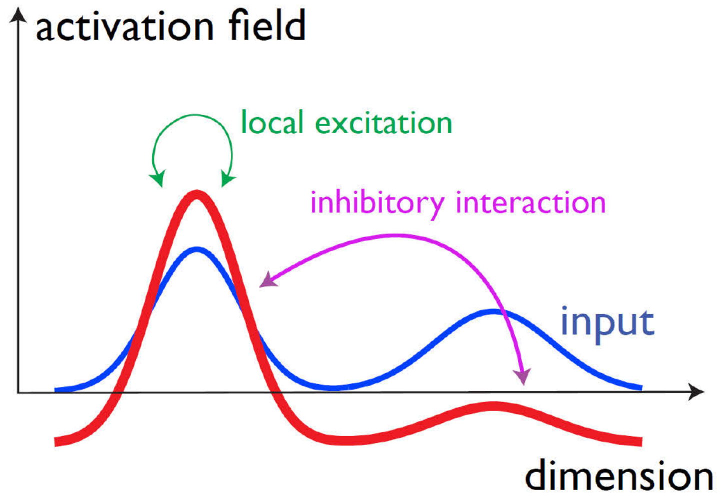
Underlying Neural Principles:

- Standard **connectionist** assumptions:
  - Continuous activation
  - Nonlinear transfer functions
  - Synaptic connectivity
- Plus:
  - Recurrent dynamics create **stability**
  - Continuous feature **spaces** allow **attention** and **binding**
  - Continuous **time** allows **autonomy**
- Dynamical systems theory is central
  - Stable states = **attractors**
  - Behavioral **robustness** emerges from attractor stability

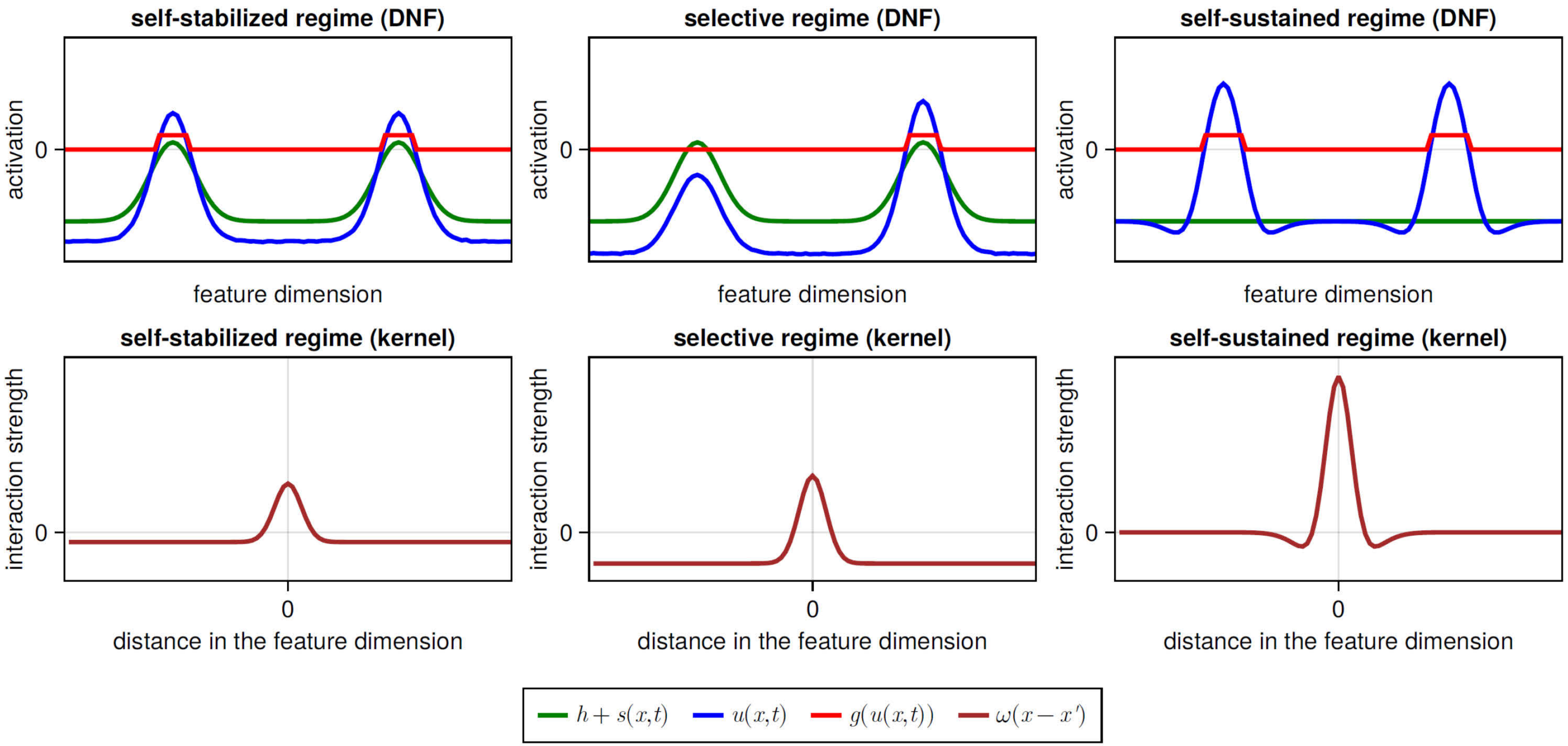
Neural interaction kernel

$$\omega(\Delta x) = \overbrace{\phi(\Delta x, a_{lex}, 0, \sigma_{lex})}^{\text{lateral excitation}} \underbrace{- \phi(\Delta x, a_{lin}, 0, \sigma_{lin})}_{\text{lateral inhibition}} \overbrace{- \psi}^{\text{global inhibition}}$$

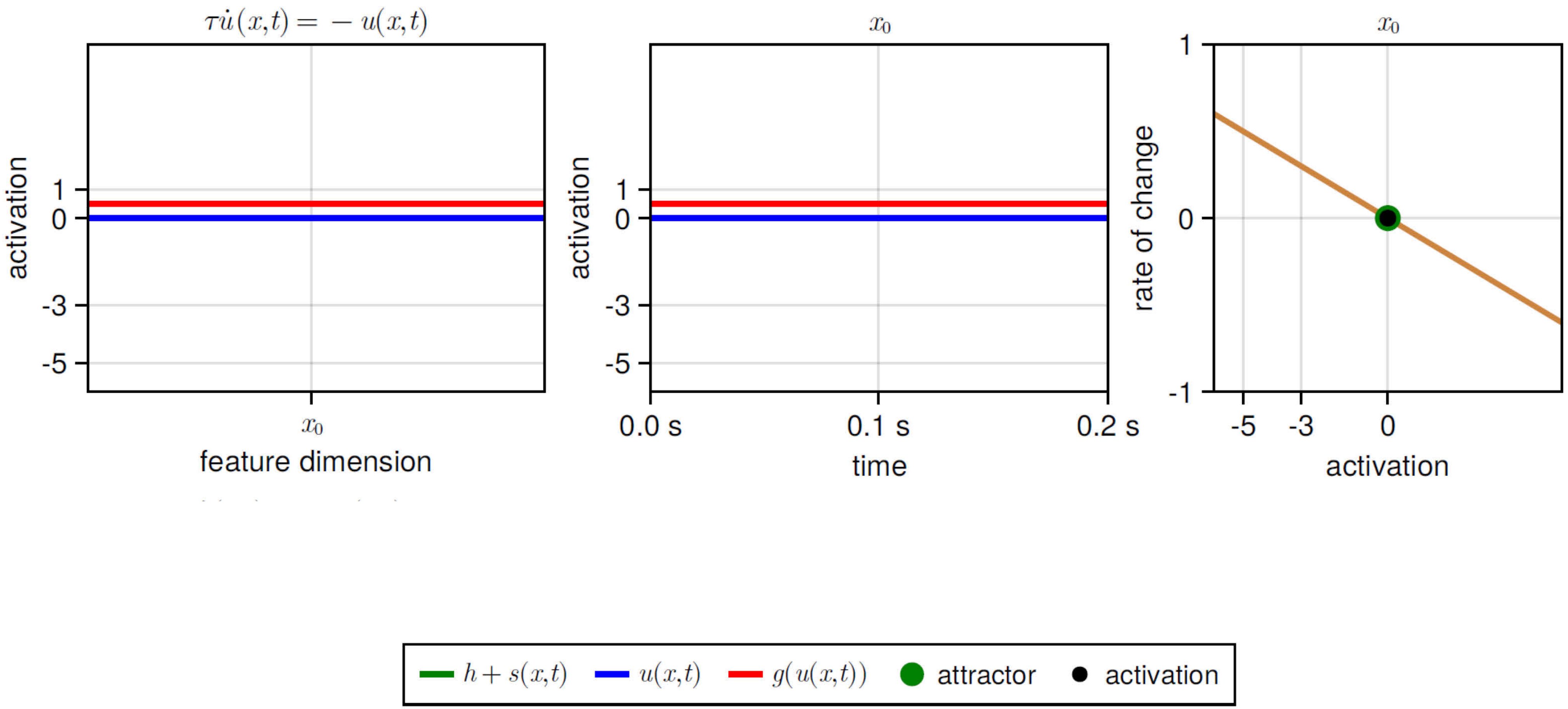
$$\phi(\Delta x, a, \mu, \sigma) = a \cdot \exp\left(-\frac{(\Delta x - \mu)^2}{2\sigma^2}\right)$$



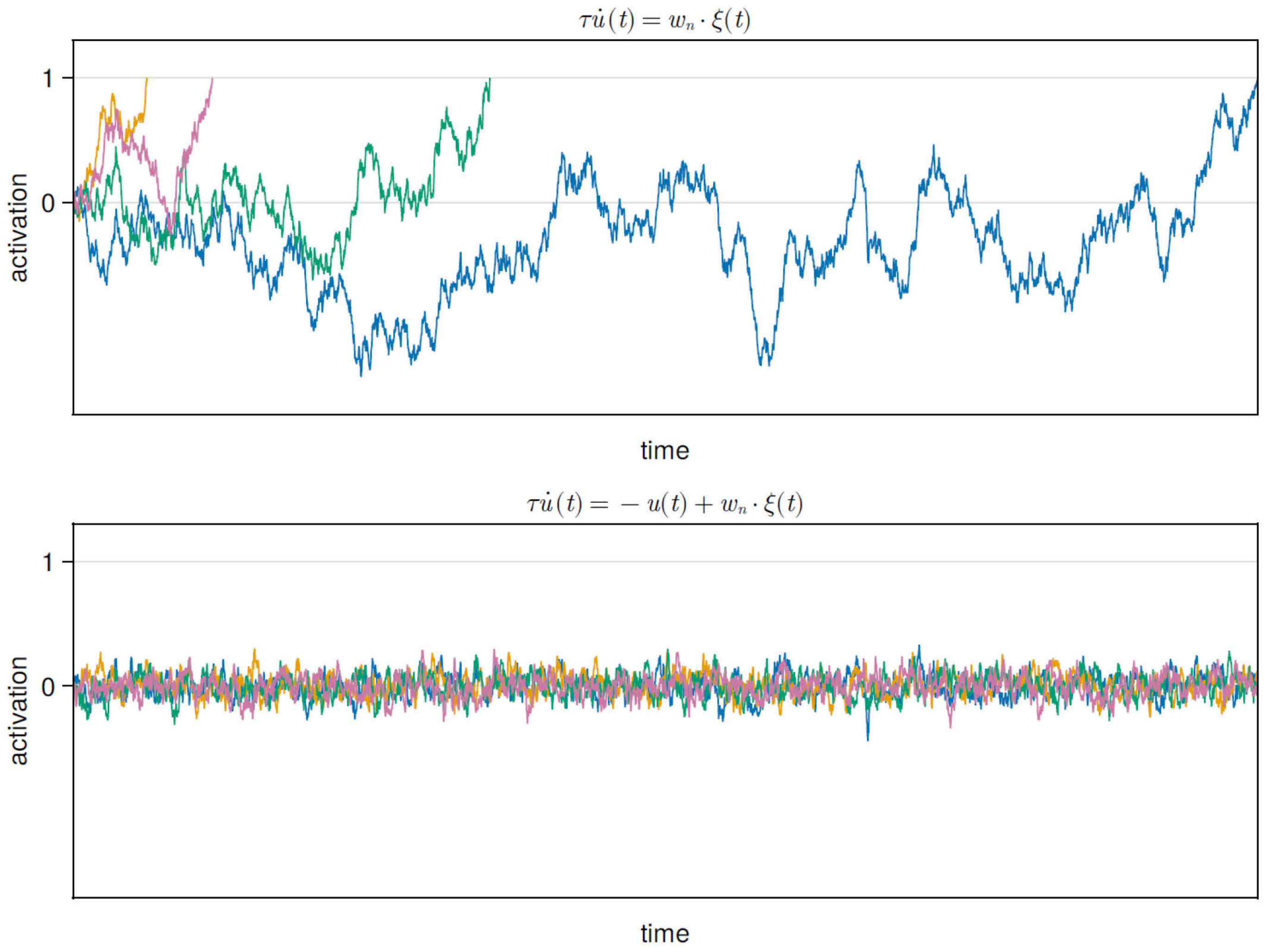
Neural dynamic regimes



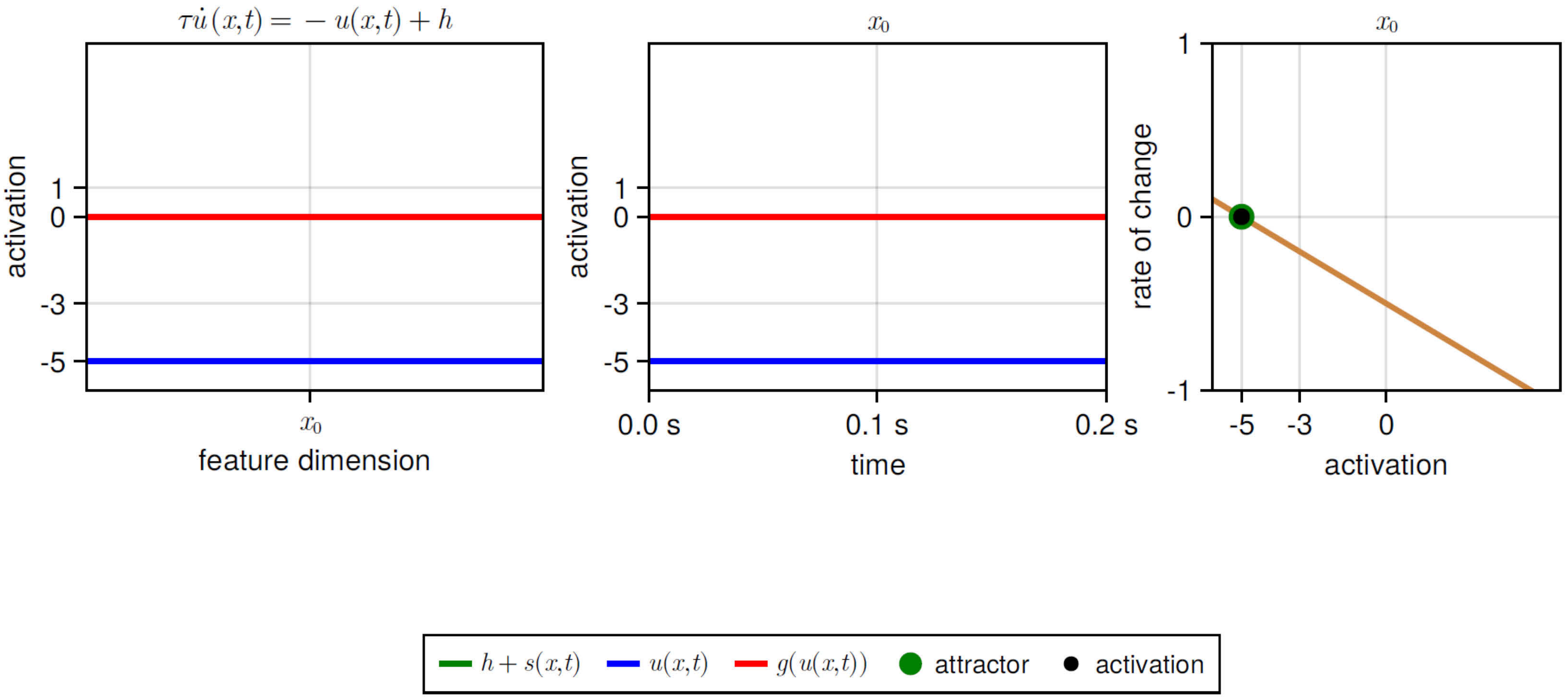
Attractor states and instabilities



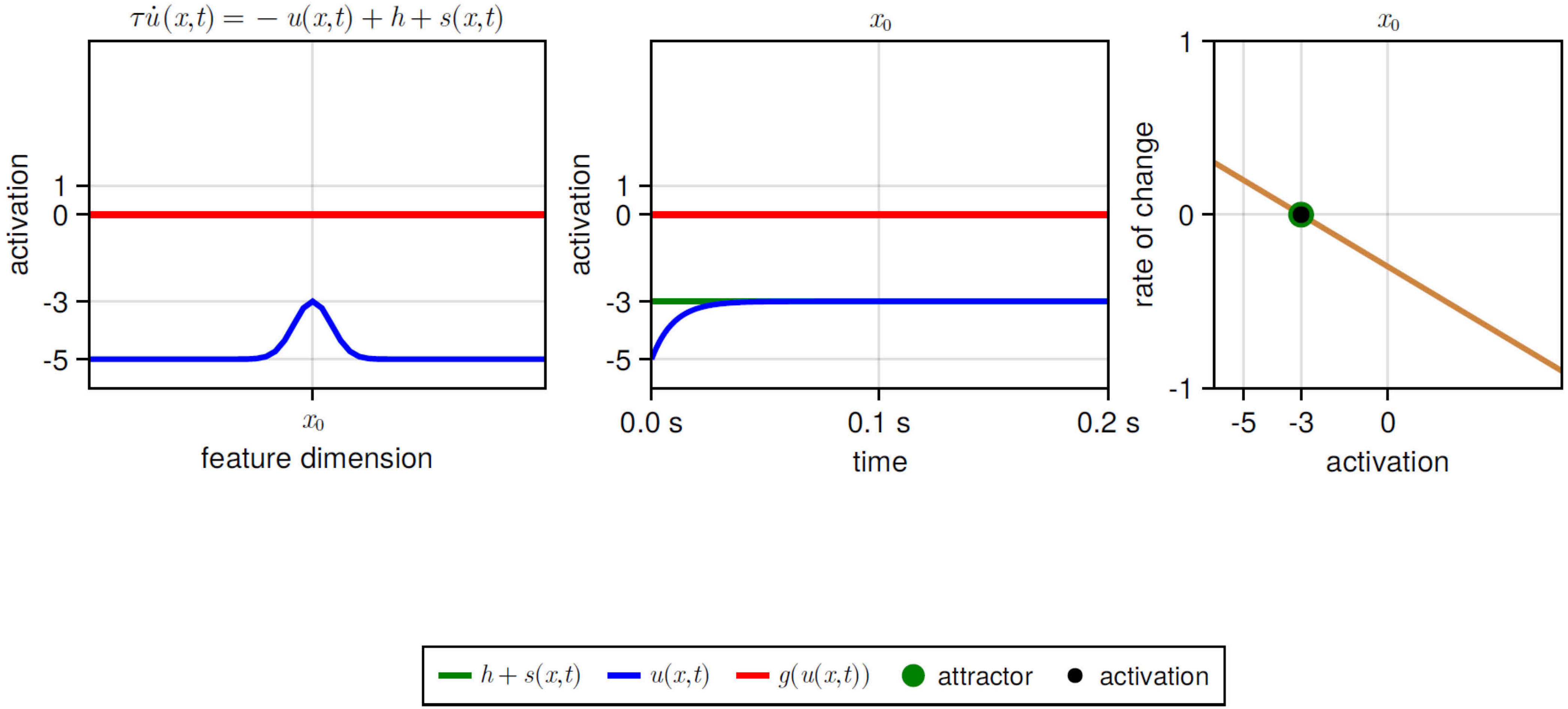
Attractor states and instabilities



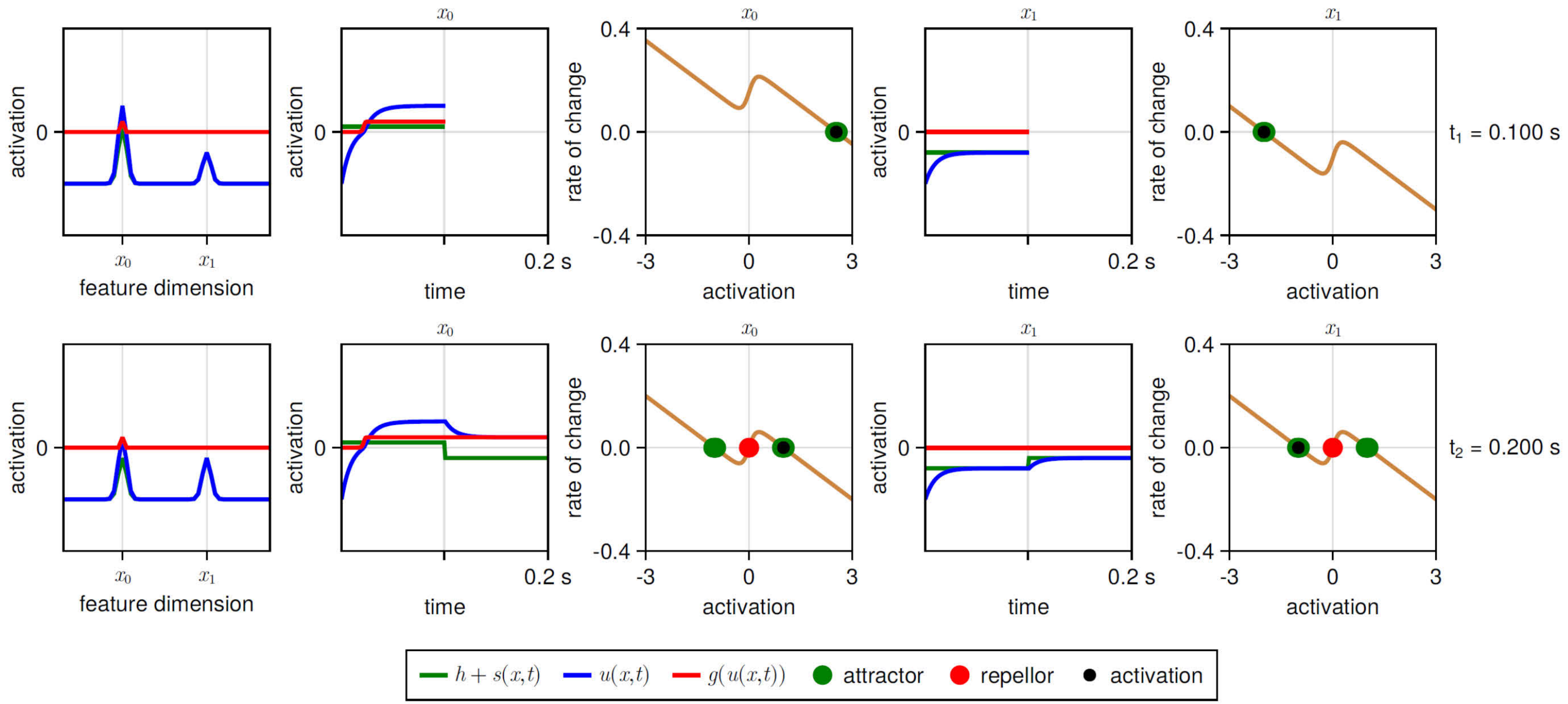
Attractor states and instabilities

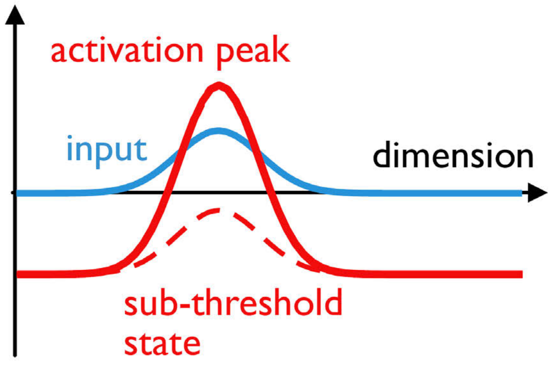


Attractor states and instabilities

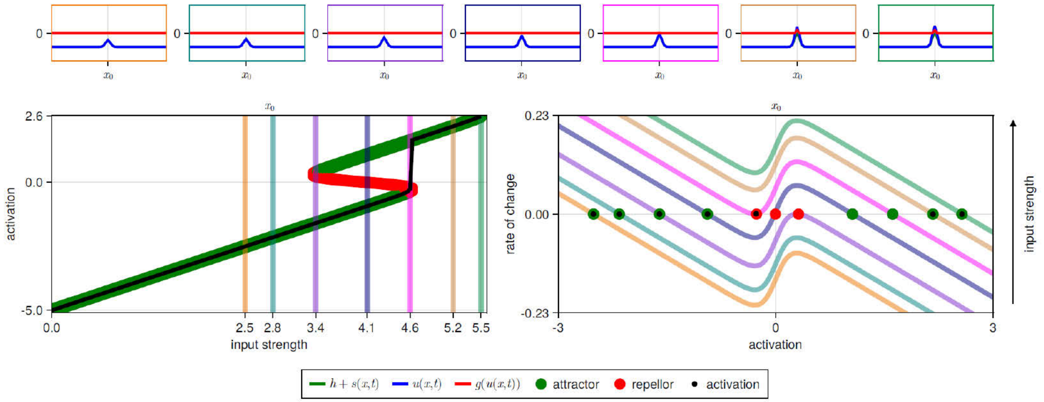


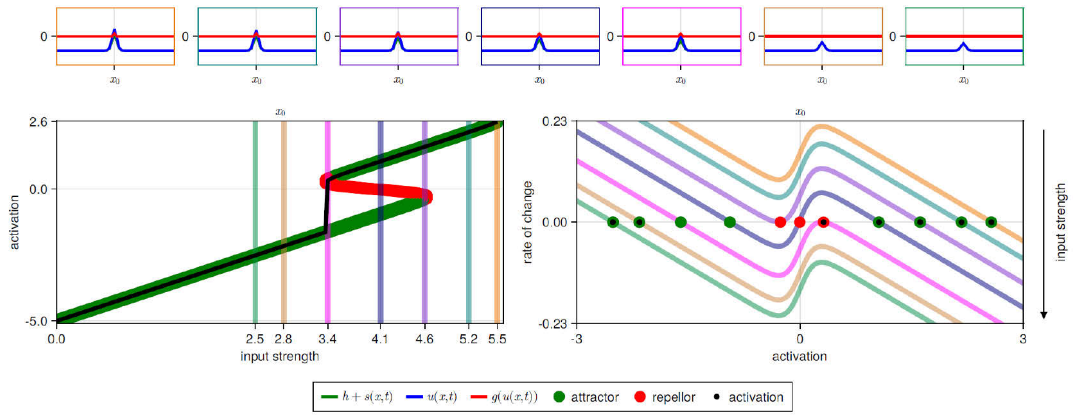
Sub- and suprathreshold attractor states





Detection and reverse detection






ESCAPE
ROOM
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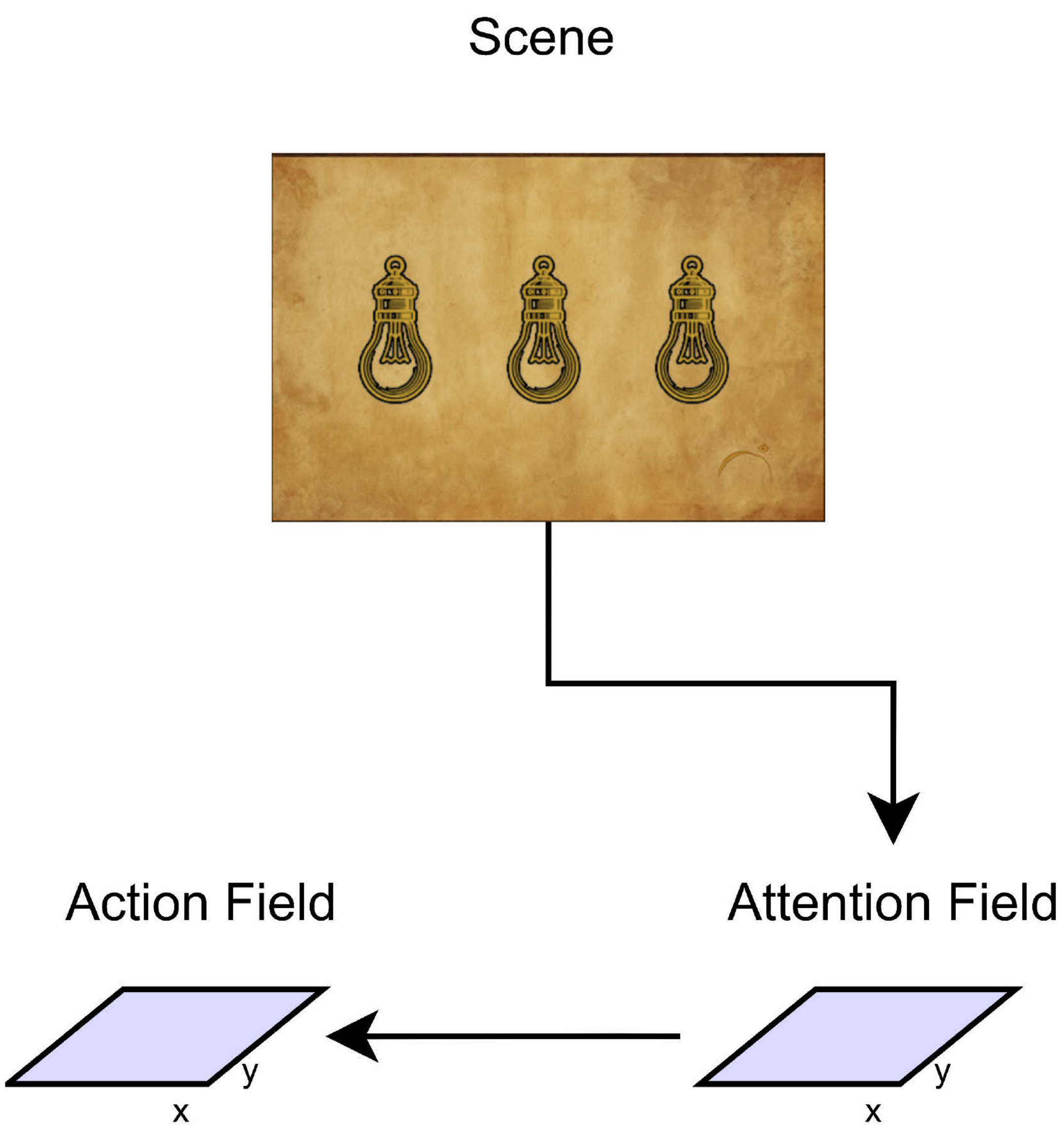
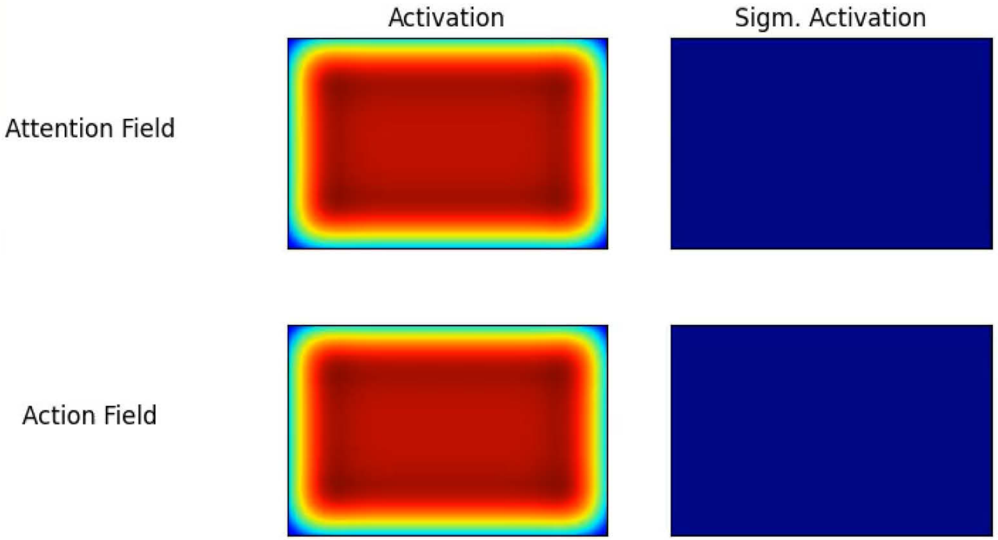
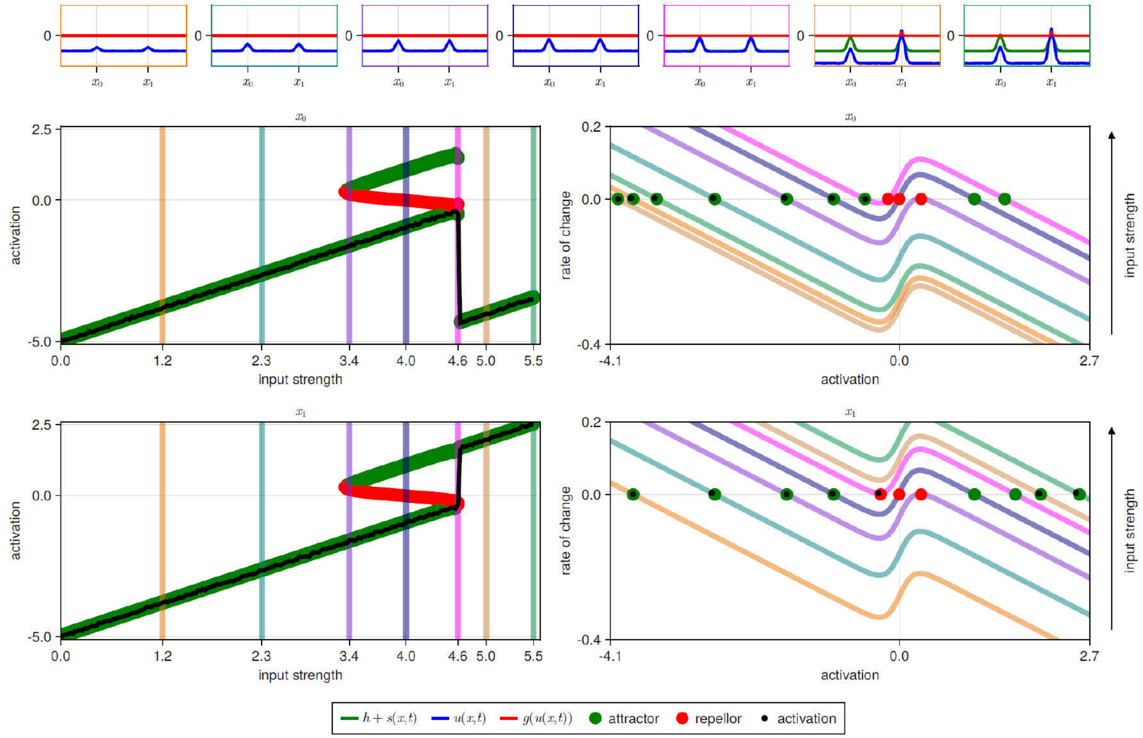

Scene
Action Field
Attention Field
y
x
y
x



https://github.com/cedar/juniper/blob/main/demo1_detection.ipynb

Selection



input strength
activation
rate of change
attractor
repellor
activation

Legend
$h+s(x)$
$u(x)$
$g(u(x))$


ESCAPE ROOM
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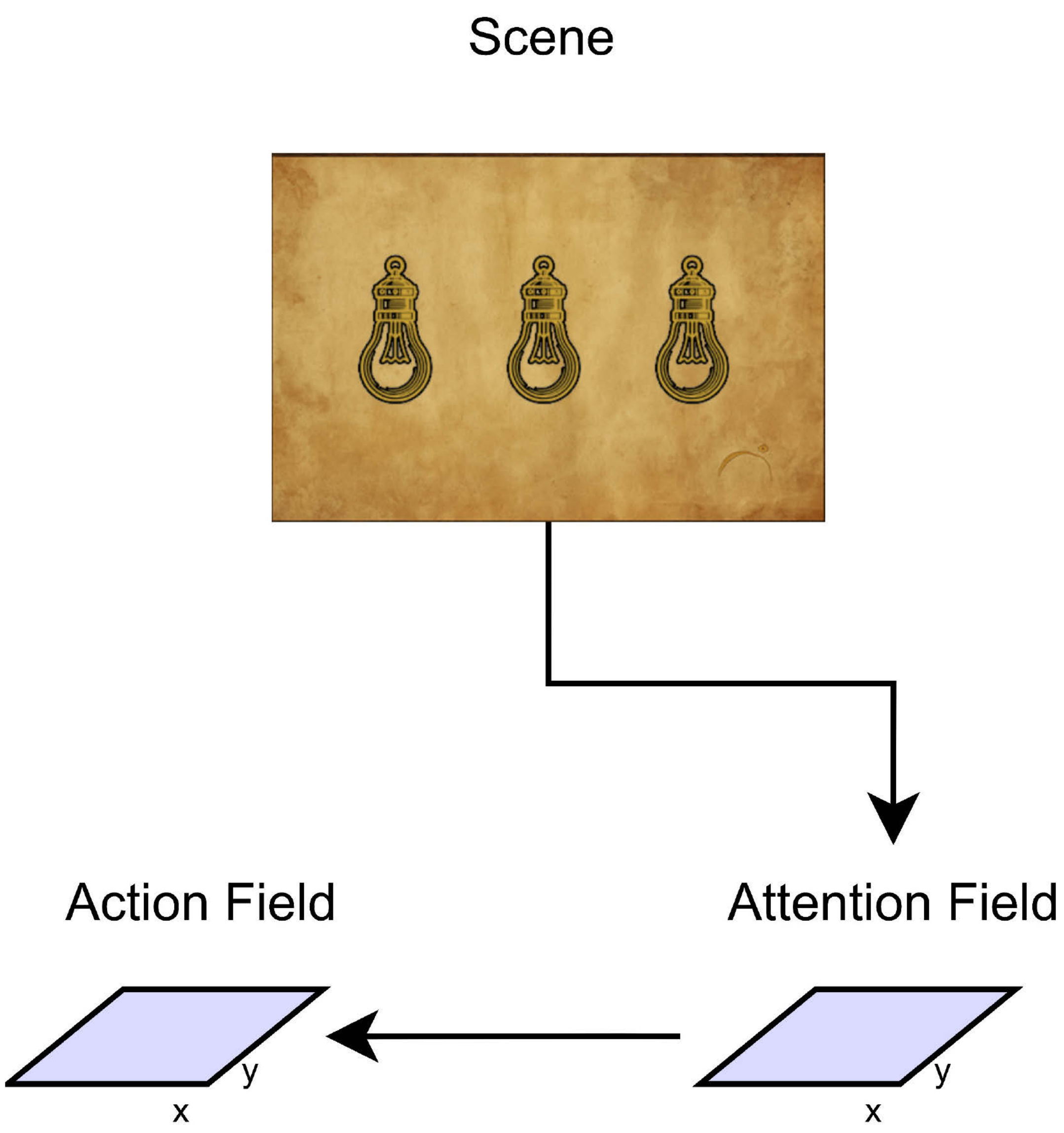
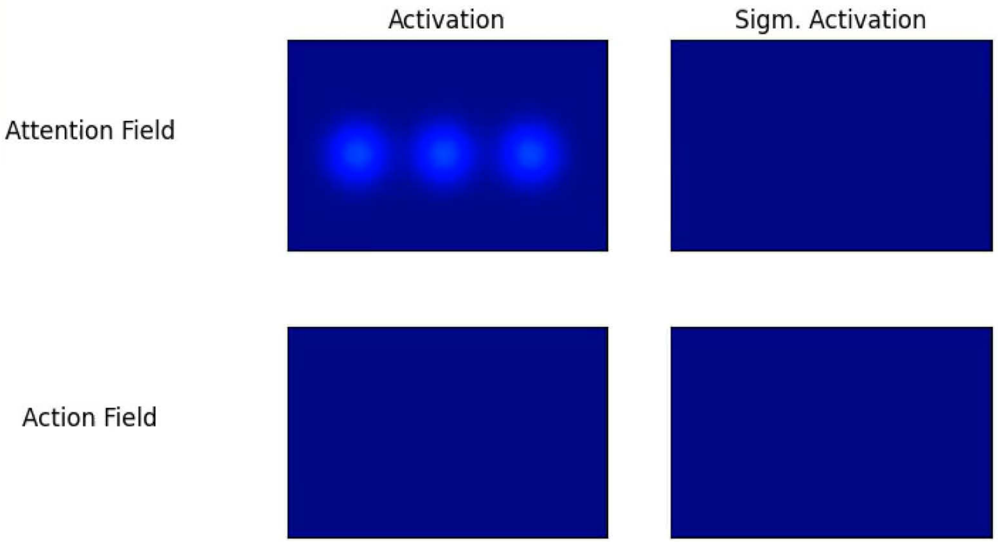
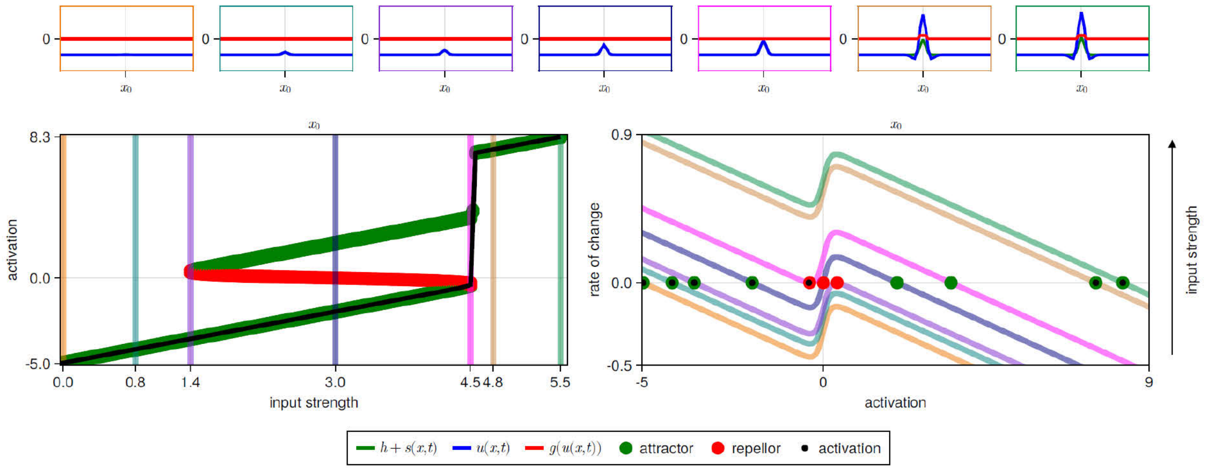

Scene
Action Field
Attention Field
y
x
y
x

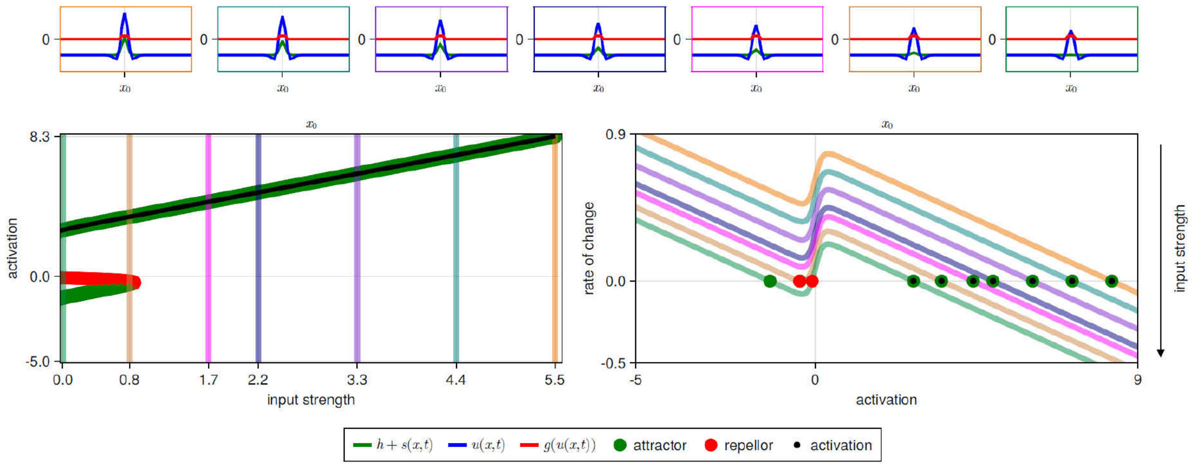


https://github.com/cedar/juniper/blob/main/demo2_selection.ipynb

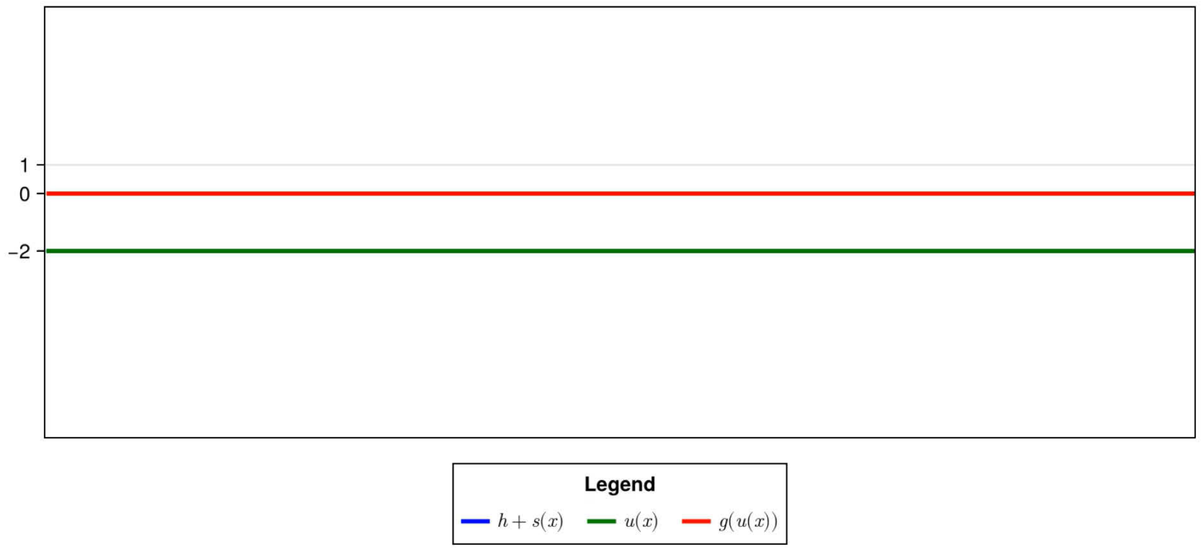
Memory



activation
input strength
rate of change
input strength
$h + s(x,t)$ $u(x,t)$ $g(u(x,t))$ attractor repellor activation



activation
input strength
rate of change
input strength
$h + s(x,t)$ $u(x,t)$ $g(u(x,t))$ attractor repellor activation



Legend
$h + s(x)$ $u(x)$ $g(u(x))$


ESCAPE ROOM
8
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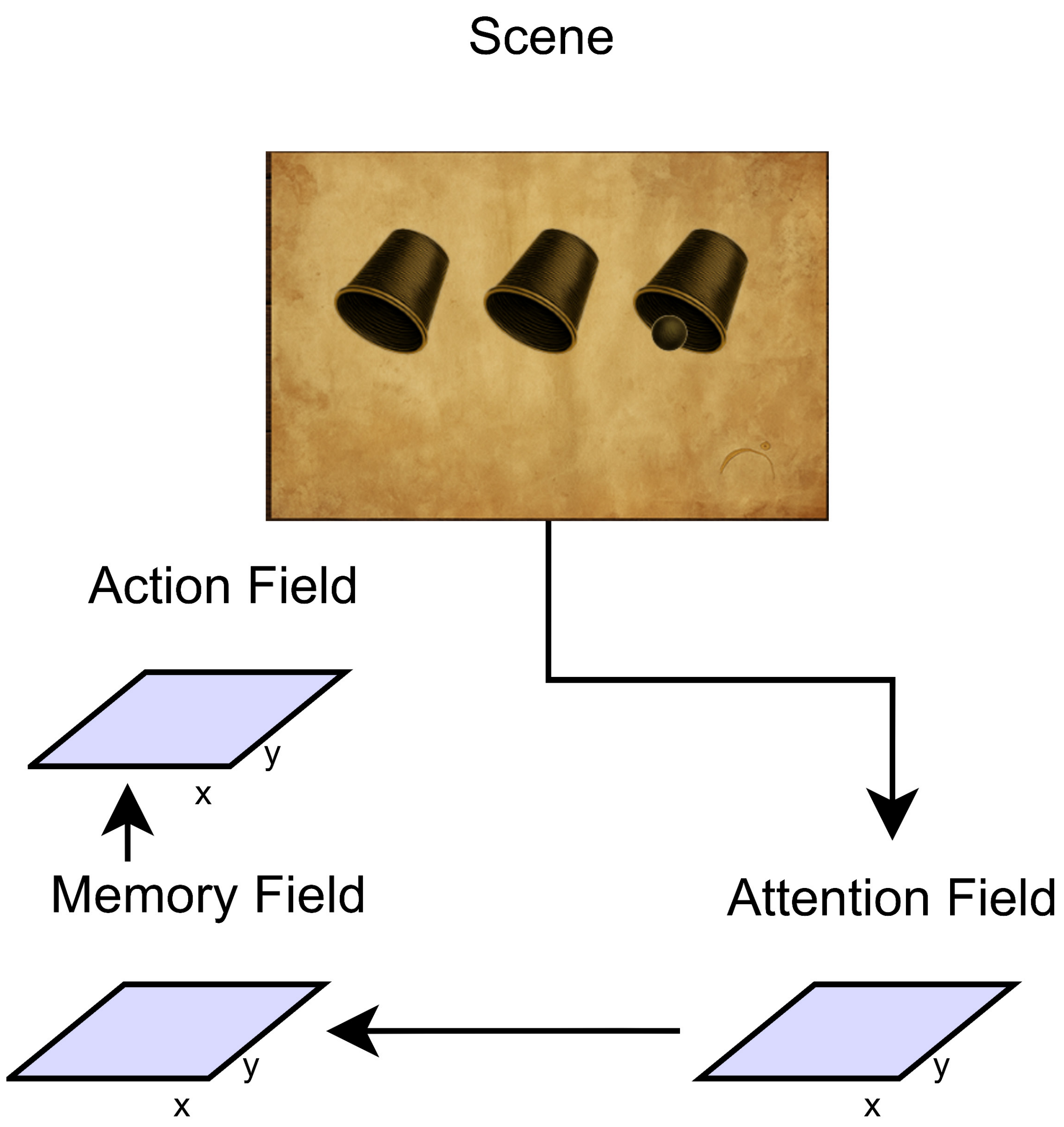
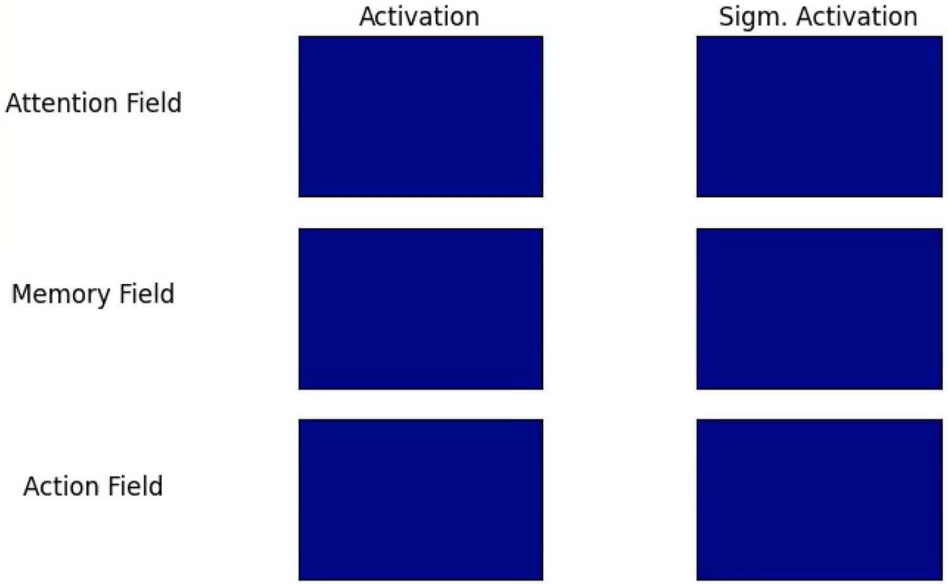
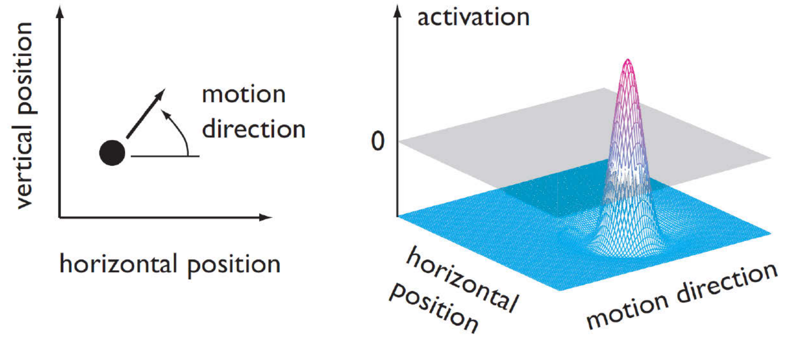

Scene
Action Field
y
x
Memory Field
Attention Field
y
x
y
x

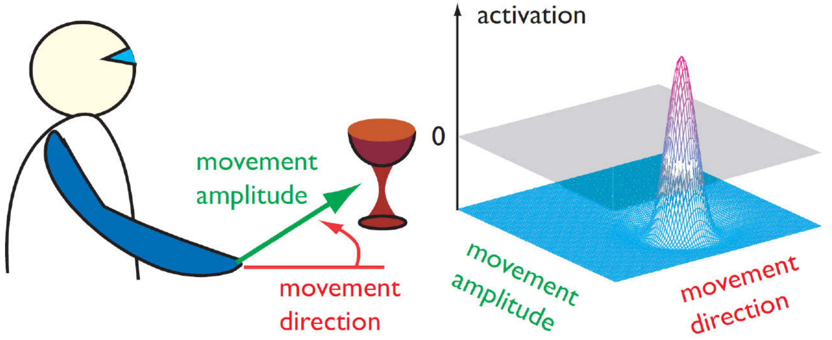


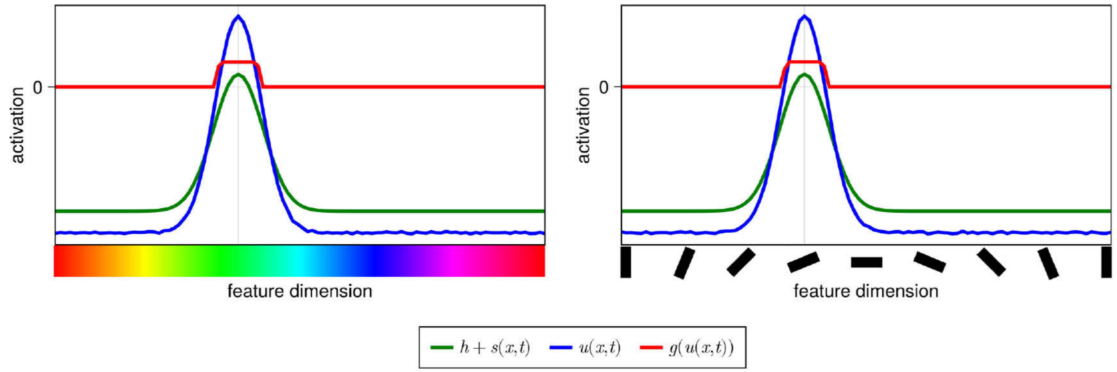
https://github.com/cedar/juniper/blob/main/demo4_Huetchen.ipynb

Continuous Feature Spaces

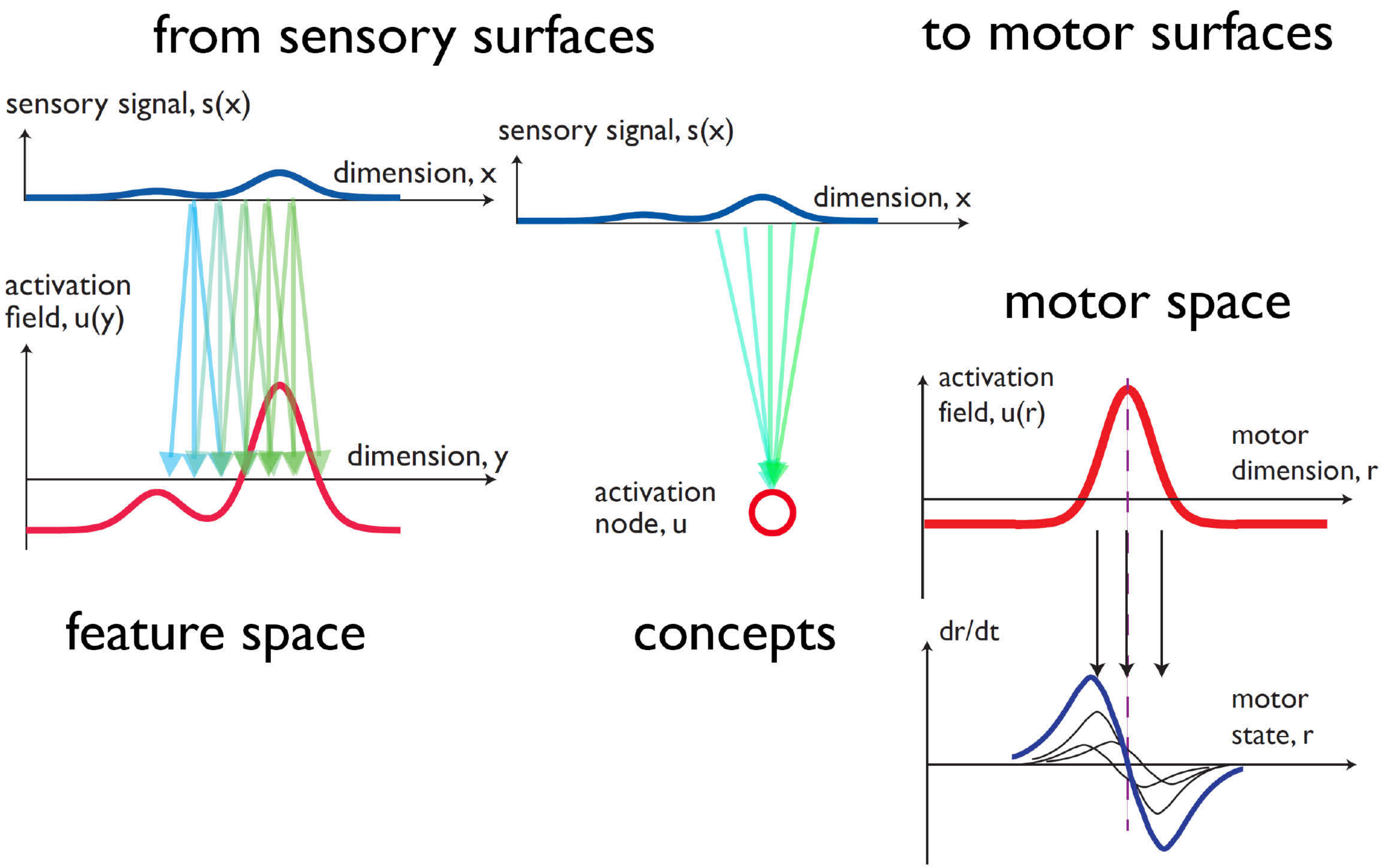
- Fields are defined over low-dimensional continuous spaces
- Facilitates:
  - Attention
  - Decision making
  - Binding
  - Generalization
  - Grounding
  - Compositional representation
  - …







Feature space

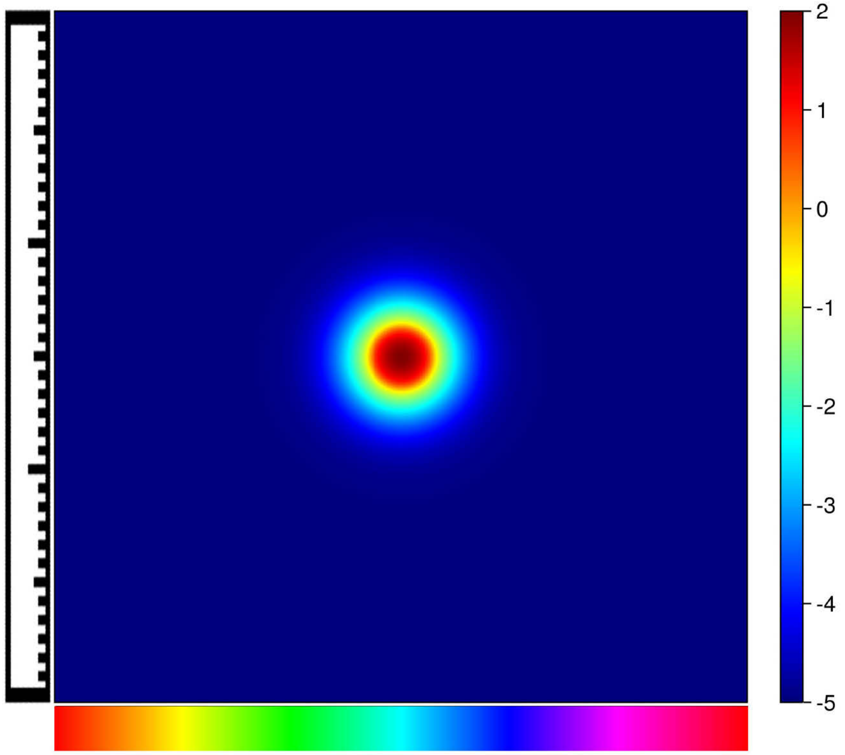


Dimensionality

$$\tau \dot{u}(\chi, t) = -u(\chi, t) + h + s(\chi, t) + w_\xi \cdot \xi(\chi, t) + \int \cdots \int \omega(\chi - \chi') \cdot g(u(\chi', t)) \; dx'_1 \ldots dx'_n$$

$$\omega(\Delta\chi) = \phi(\Delta\chi, a_{lex}, \vec{0}, \sigma_{\vec{lex}}) - \phi(\Delta\chi, a_{lin}, \vec{0}, \sigma_{\vec{lin}}) - \psi$$

$$\phi(\Delta\chi, a, \vec{\mu}, \vec{\sigma}) = a \cdot \exp\left(- \sum_{i=1}^{|\Delta\chi|} \frac{(\Delta x_i - \mu_i)^2}{2\sigma_i^2} \right)$$



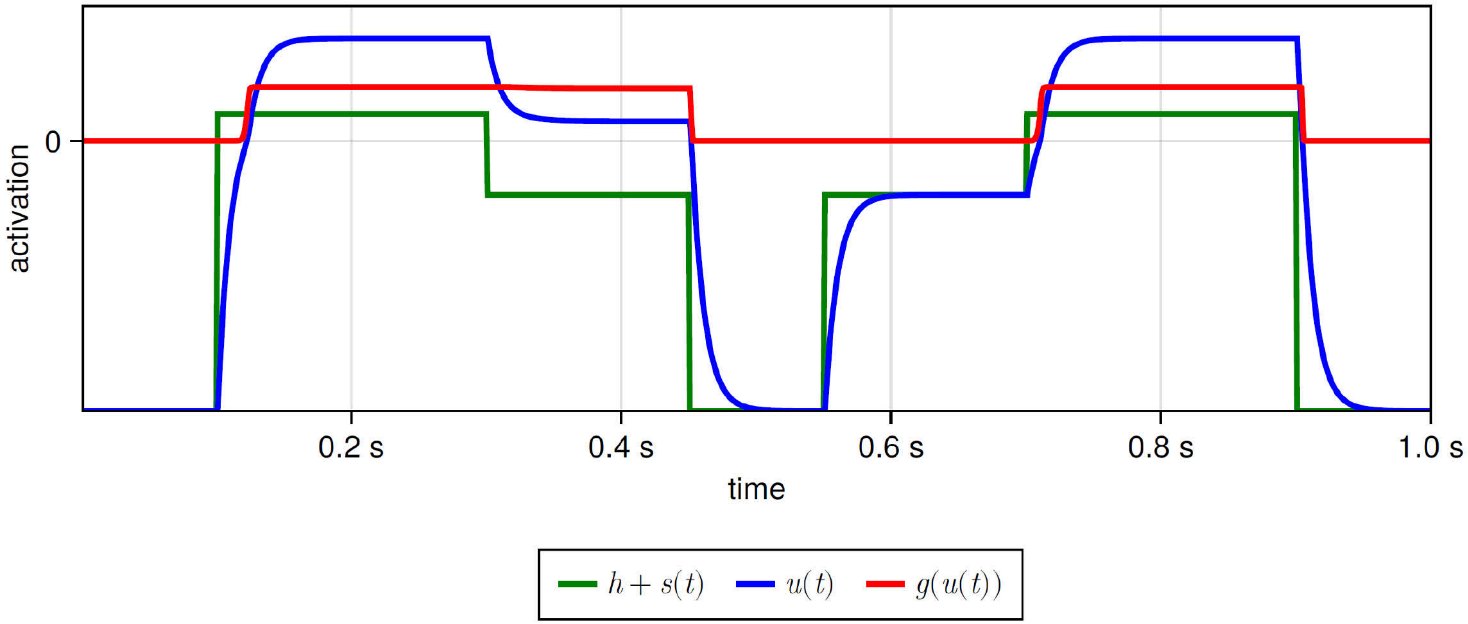
Dynamic neural node (DNN)

stabilization factor

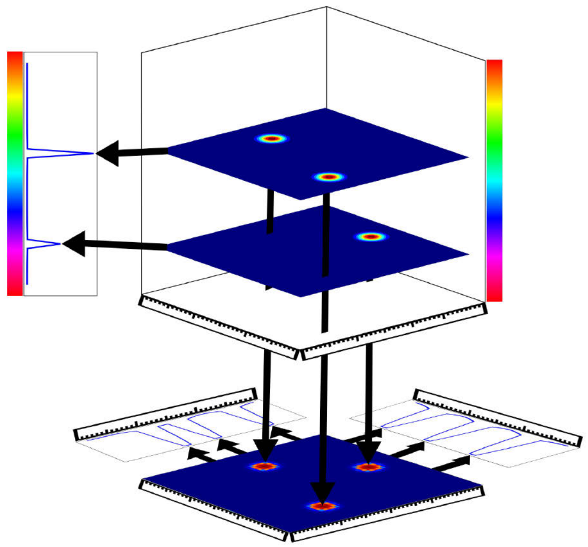
time scale time external input neural noise

$$\tau \, \dot{u}(t) = -u(t) + h + s(t) + w_\xi \cdot \xi(t) + w_e \cdot g(u(t))$$

rate of change resting level self excitation strength sigmoidal threshold

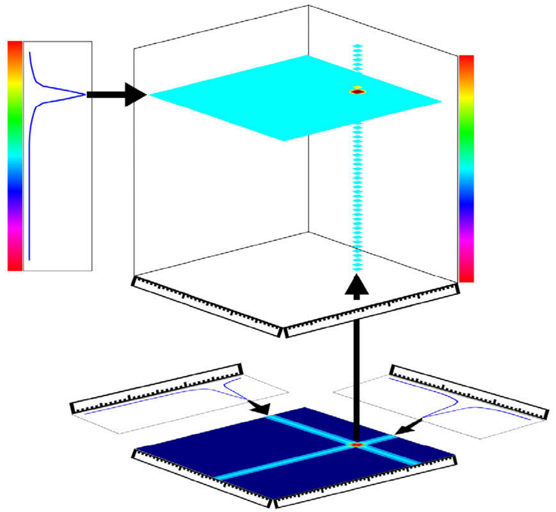


Connections between DNFs



$$\tau_T \dot{u}_T(\chi^T, t) = -u_T(\chi^T, t) + h_T + w_{T,\xi} \cdot \xi_T(\chi^T, t)$$
$$+ \int \cdots \int \omega_T(\chi^T - \chi^{T'}) \cdot g(u_T(\chi^{T'}, t)) \; dx_1^{T'} \dots dx_n^{T'}$$
$$+ w_{S,T} \left(\int \cdots \int w_{S,T}^{\gamma} \cdot \gamma_{S,T}(\chi^S, \chi^T, t) \cdot \kappa_{S,T}(\chi^S - \chi^\kappa) \right.$$
$$\left. \cdot \; g(u_S(\chi^\kappa, t)) \; dx_1^\kappa \dots dx_m^\kappa \; dx_1^S \dots dx_m^S \right)$$

$$\gamma_{S,T}^{\text{fix}}(\chi^S, \chi^T, t) = \begin{cases} 1, & \text{if } x_i^S = x_i^T, \; \forall i \in \mathbb{N}_{>0}, \; i \leq \min\{|\chi^S|, |\chi^T|\} \\ 0, & \text{otherwise} \end{cases}$$



$$\kappa_{S,T}(\Delta\chi) = \phi(\Delta\chi, a_{ex}, \vec{0}, \vec{\sigma_{ex}}) - \phi(\Delta\chi, a_{in}, \vec{0}, \vec{\sigma_{in}})$$

Hebbian connections between DNFs

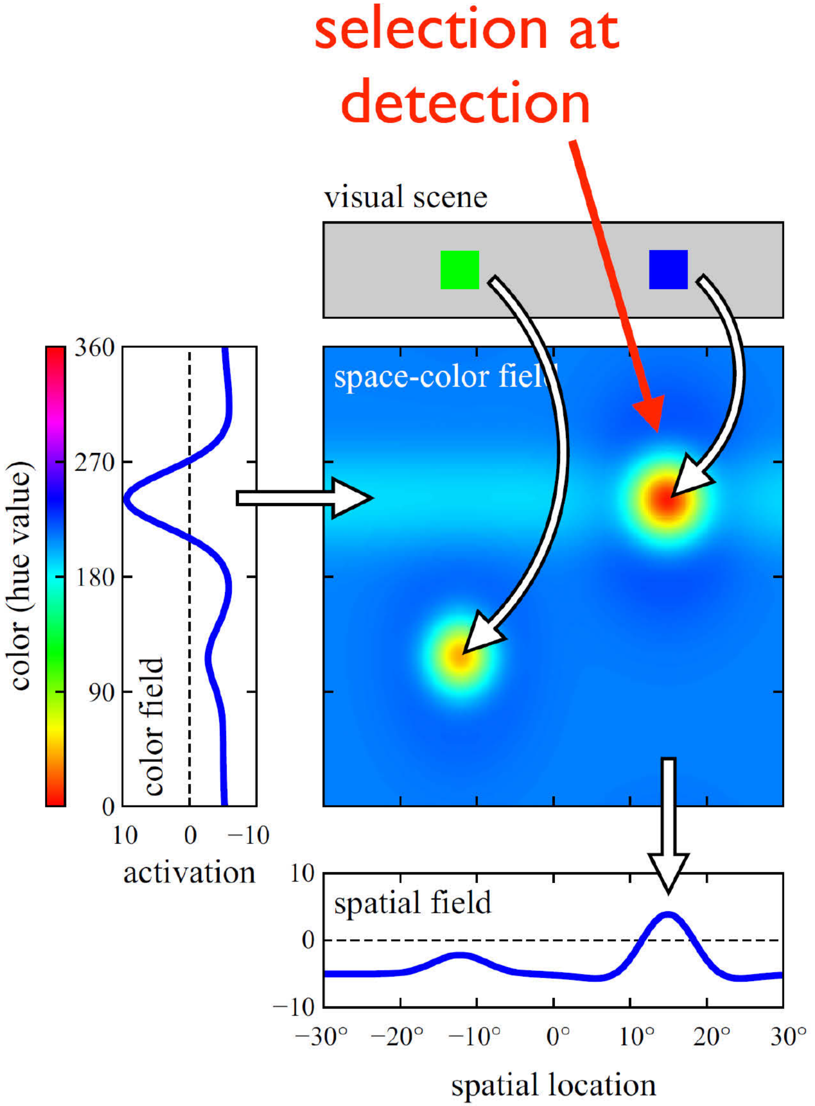
$$\begin{aligned}\tau_T \dot{u}_T(\chi^T, t) = &- u_T(\chi^T, t) + h_T + w_{T,\xi} \cdot \xi_T(\chi^T, t) \\ &+ \int \cdots \int \omega_T(\chi^T - \chi^{T'}) \cdot g(u_T(\chi^{T'}, t)) \; dx_1^{T'} \ldots dx_n^{T'} \\ &+ w_{S,T} \Bigg(\int \cdots \int w^{\gamma}_{S,T} \cdot \gamma_{S,T}(\chi^S, \chi^T, t) \cdot \kappa_{S,T}(\chi^S - \chi^\kappa) \\ &\qquad \cdot g(u_S(\chi^\kappa, t)) \; dx_1^\kappa \ldots dx_m^\kappa \; dx_1^S \ldots dx_m^S \Bigg)\end{aligned}$$

$$\gamma^{\mathrm{heb}}_{S,T}(\chi^S, \chi^T, t) = w^{\gamma *}_{S,T}(\chi^S, \chi^T, t)$$

$$\tau \dot{w}^{\gamma *}_{S,T}(\chi^S, \chi^T, t) = \eta \cdot g(u_{\mathrm{learn}}(t)) \cdot g(u_S(\chi^S, t)) \cdot (g(u_T(\chi^T, t)) - w^{\gamma *}_{S,T}(\chi^S, \chi^T, t))$$

$$\kappa_{S,T}(\Delta\chi) = \phi(\Delta\chi, a_{ex}, \vec{0}, \vec{\sigma_{ex}}) - \phi(\Delta\chi, a_{in}, \vec{0}, \vec{\sigma_{in}})$$

Cued selection




ESCAPE
ROOM
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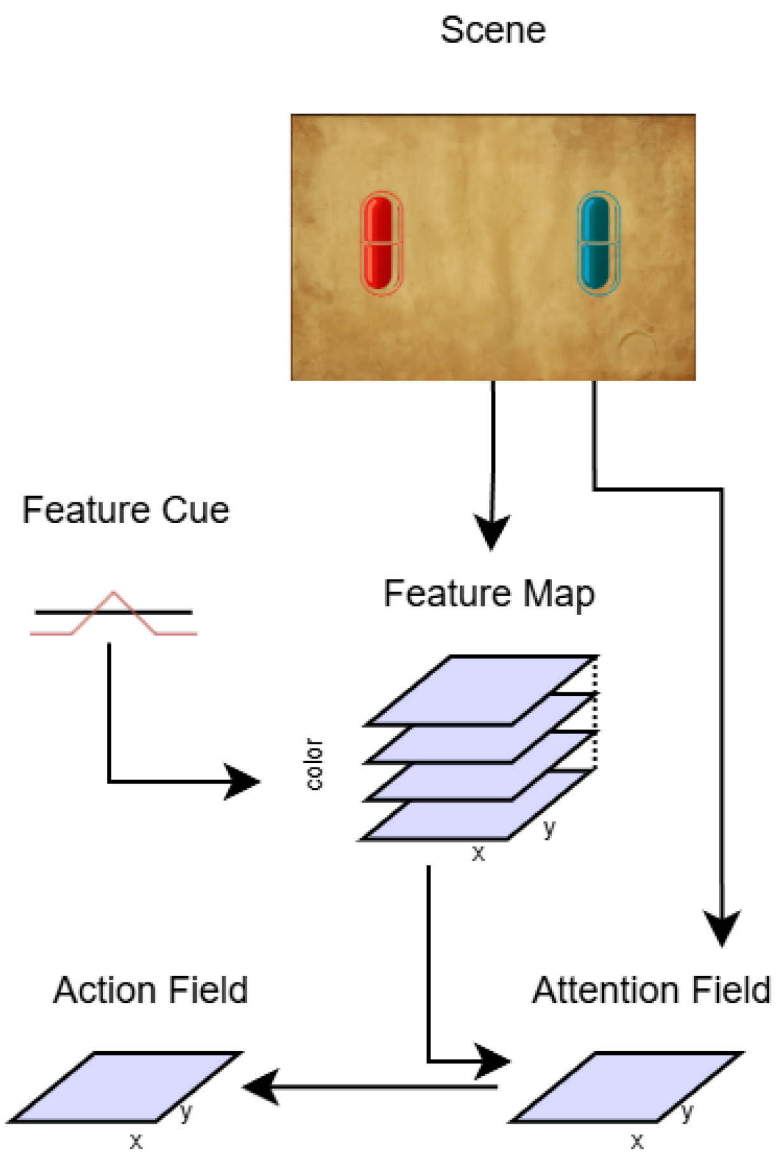
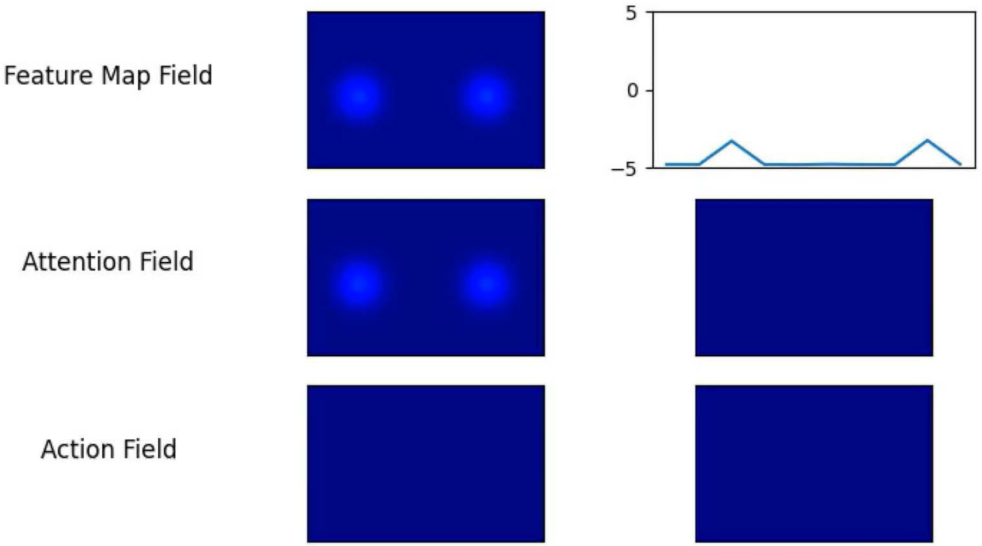
RED

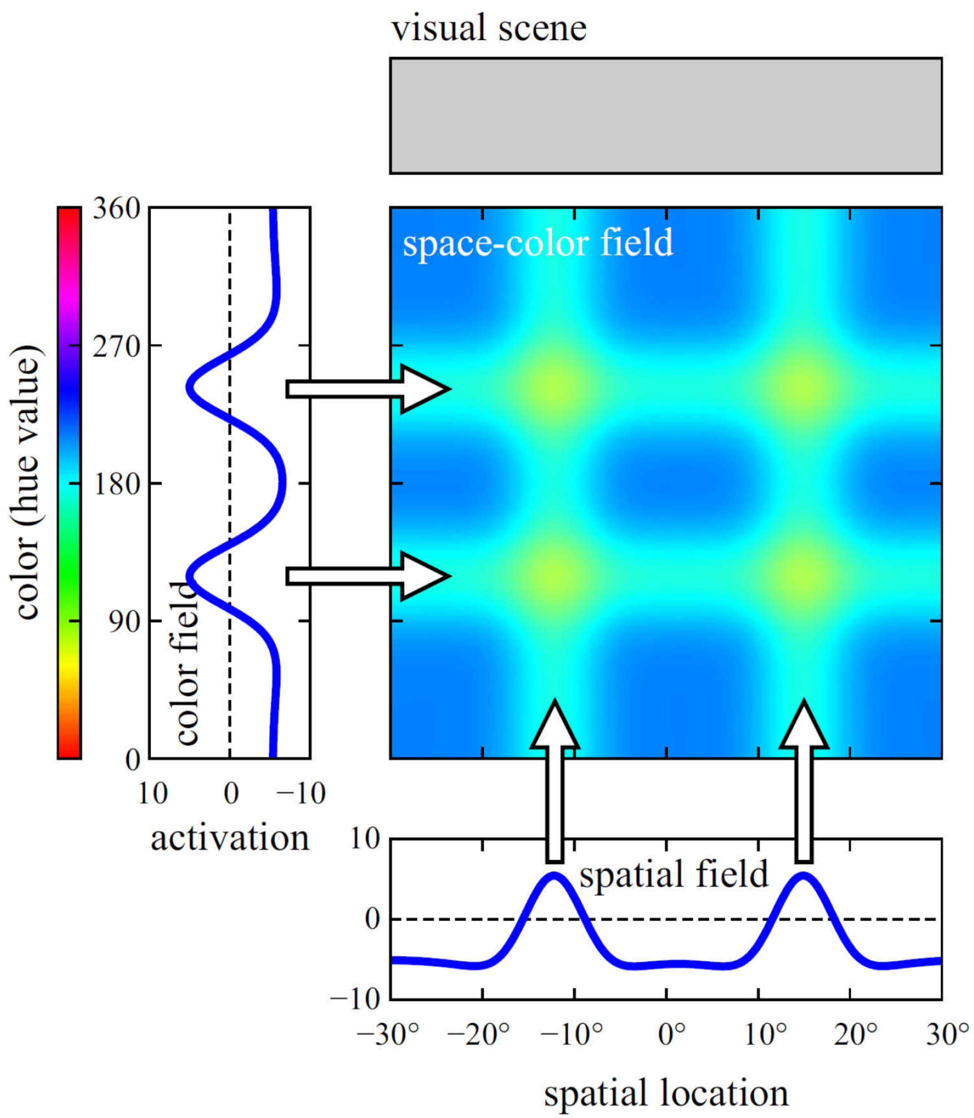


Scene
Feature Cue
Feature Map
color
y
x
Action Field
Attention Field
y
x
y
x

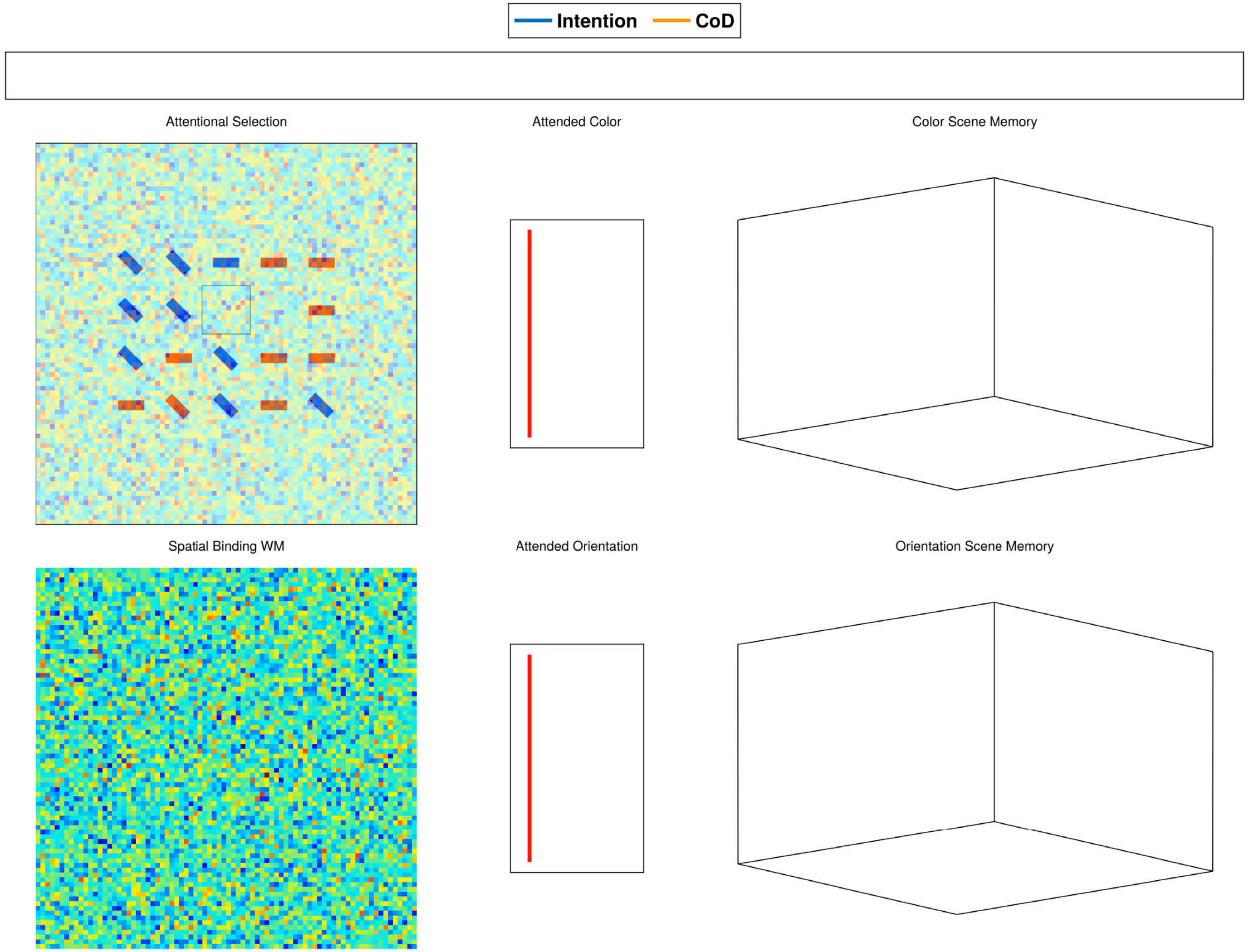


https://github.com/cedar/juniper/blob/main/demo3_cued_selection.ipynb

The binding problem



Binding through space



Grieben et al. (2020). Scene memory and spatial inhibition in visual search, Attention, Perception & Psychophysics


ESCAPE
ROOM
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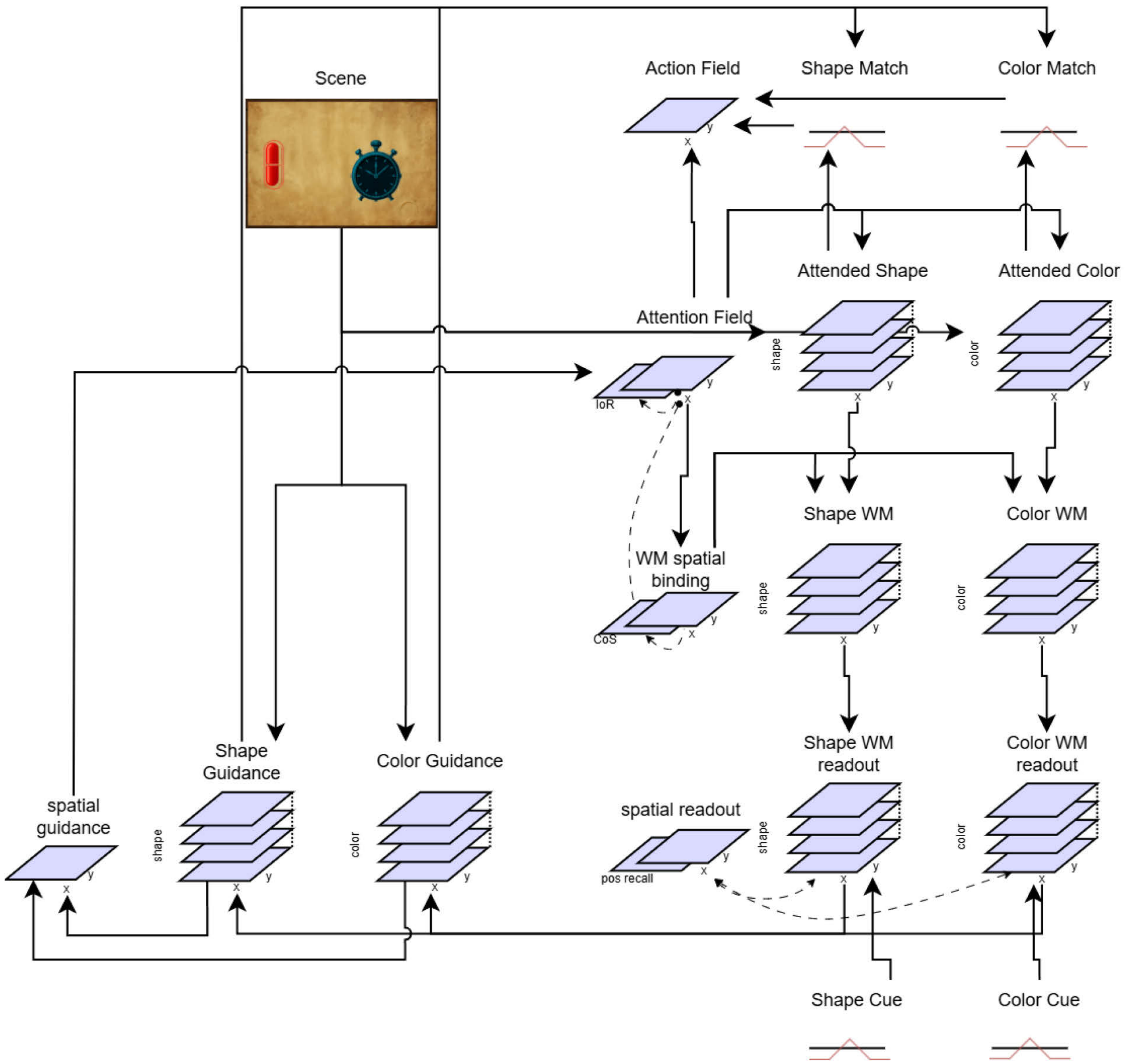
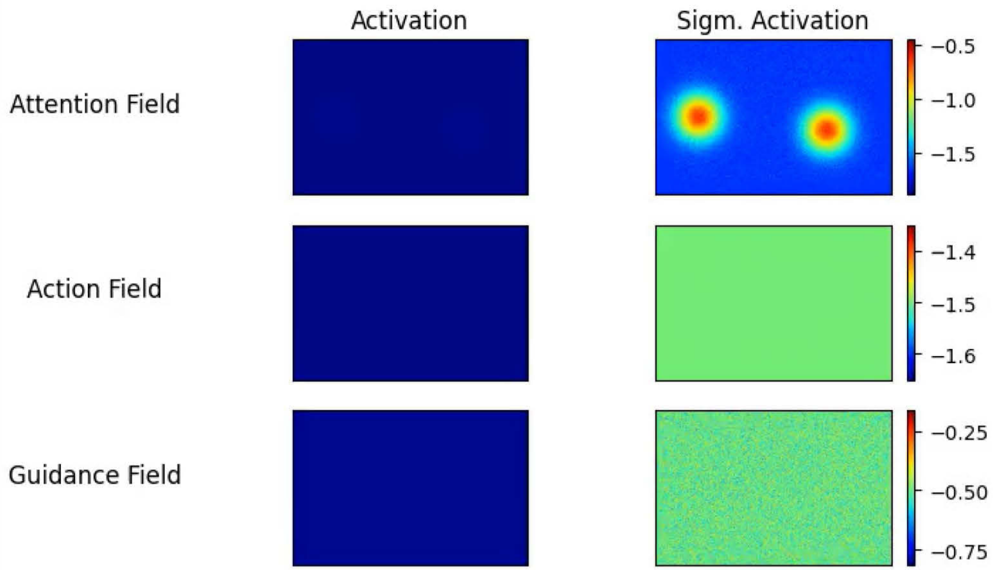
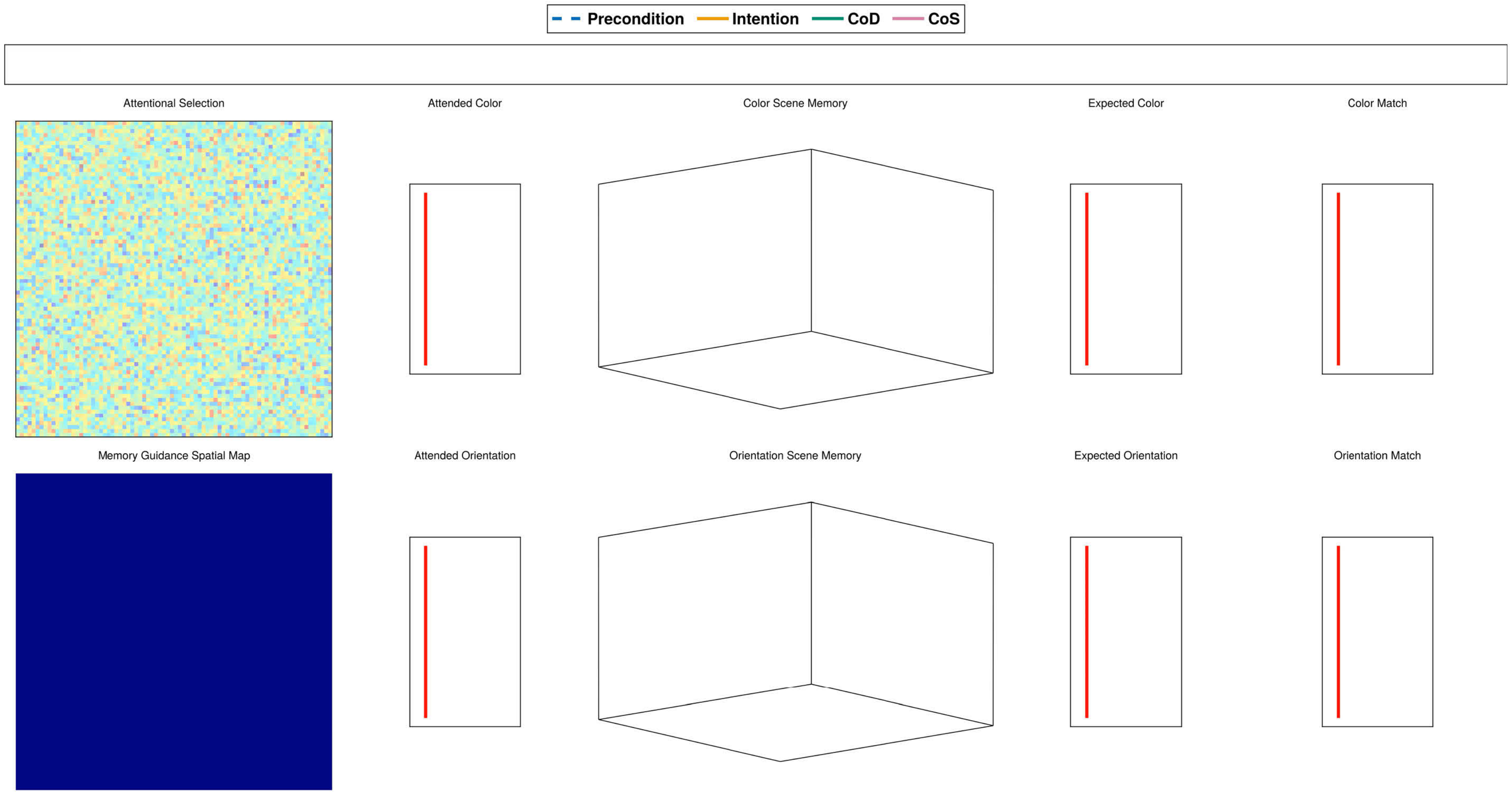

Scene
Action Field
Shape Match
Color Match
Attended Shape
Attended Color
Attention Field
shape
color
IoR
Shape WM
Color WM
WM spatial binding
CoS
Shape WM readout
Color WM readout
spatial readout
pos recall
Shape Guidance
Color Guidance
spatial guidance
Shape Cue
Color Cue



https://github.com/cedar/juniper/blob/main/demo5_binding_through_space.ipynb

Visual exploration / scene memory guidance



Grieben et al. (2020). Scene memory and spatial inhibition in visual search, Attention, Perception & Psychophysics


ESCAPE
ROOM
8
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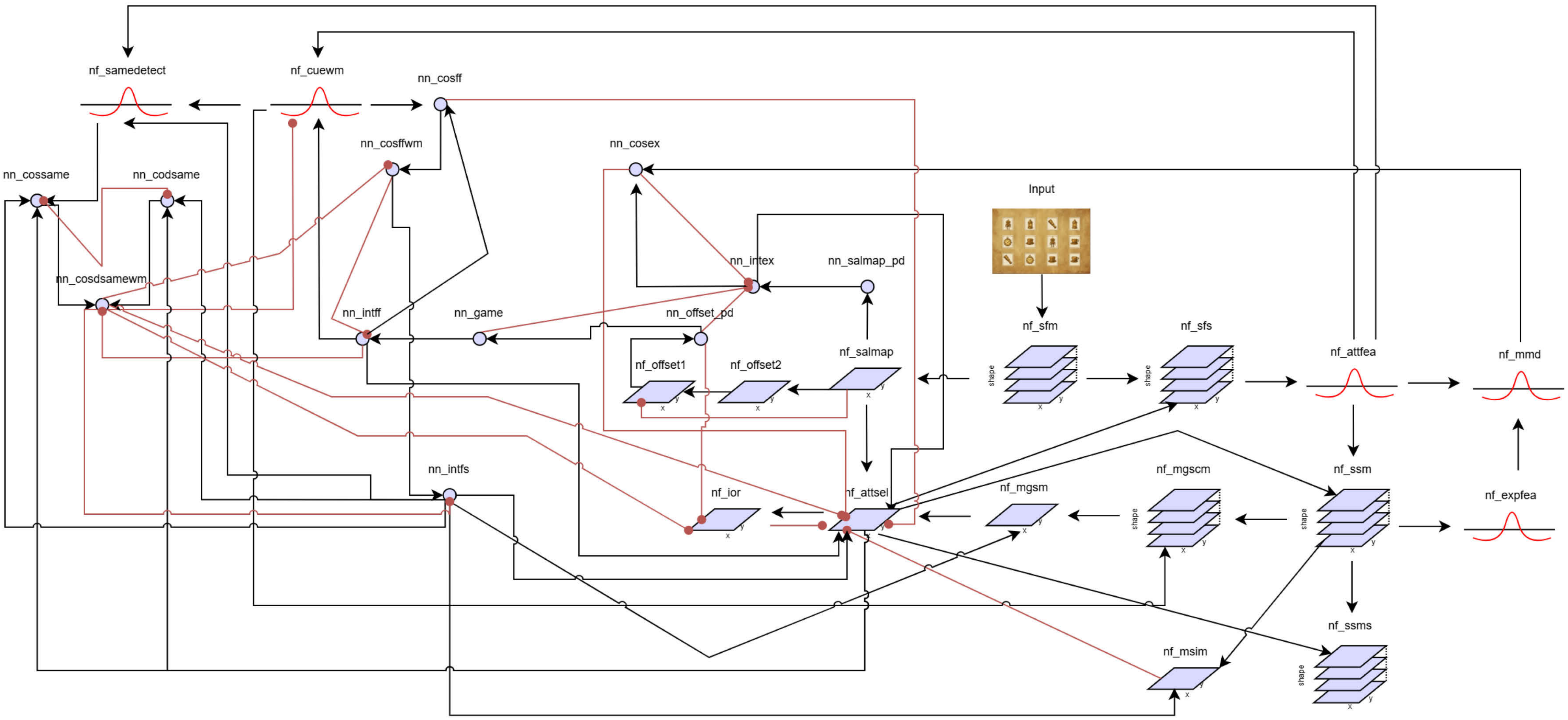
nf_samedetect
nf_cuewm
nn_cosff
nn_cosffwm
nn_cosex
nn_cossame
nn_codsame
Input
nn_intex
nn_salmap_pd
nn_cosdsamewm
nn_intff
nn_game
nn_offset_pd
nf_sfm
nf_sfs
nf_salmap
nf_attfea
nf_mmd
nf_offset1
nf_offset2
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nf_mgsm
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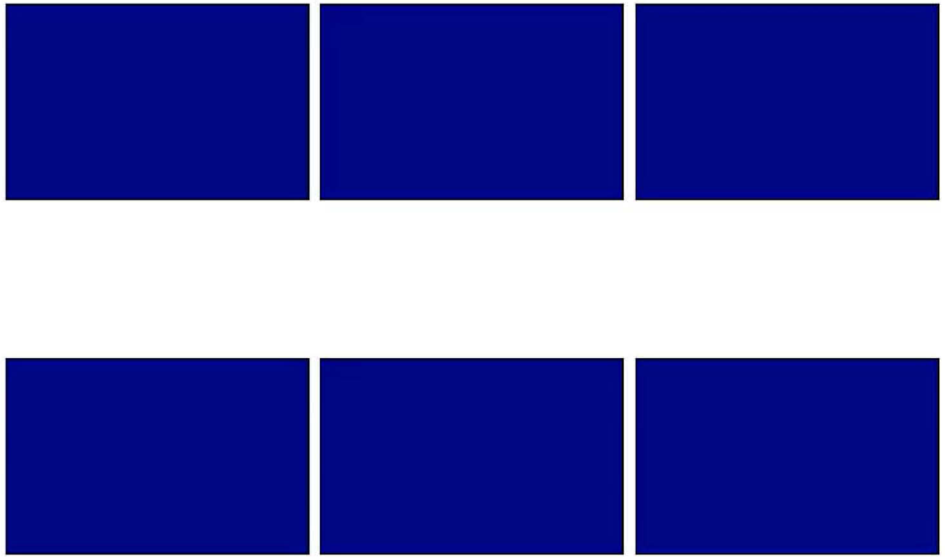
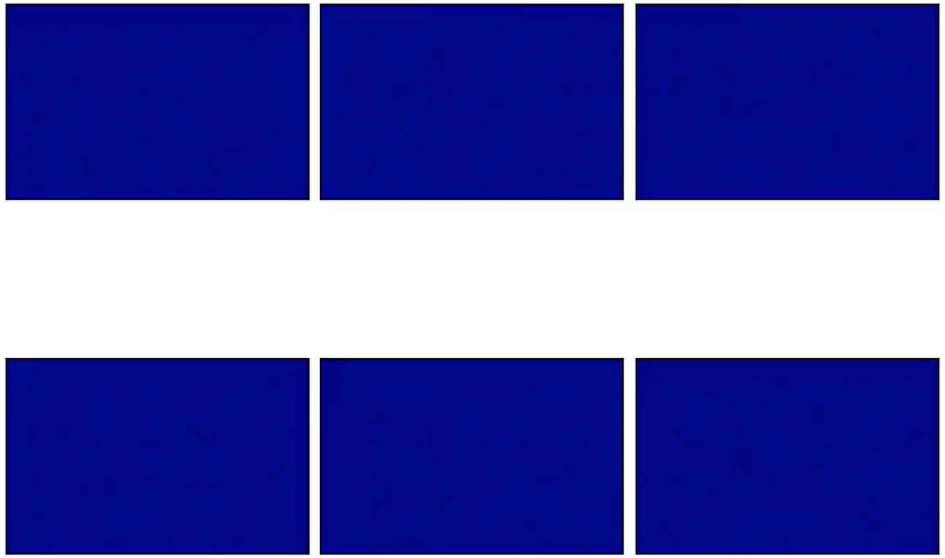
nf_ssms
nf_msim

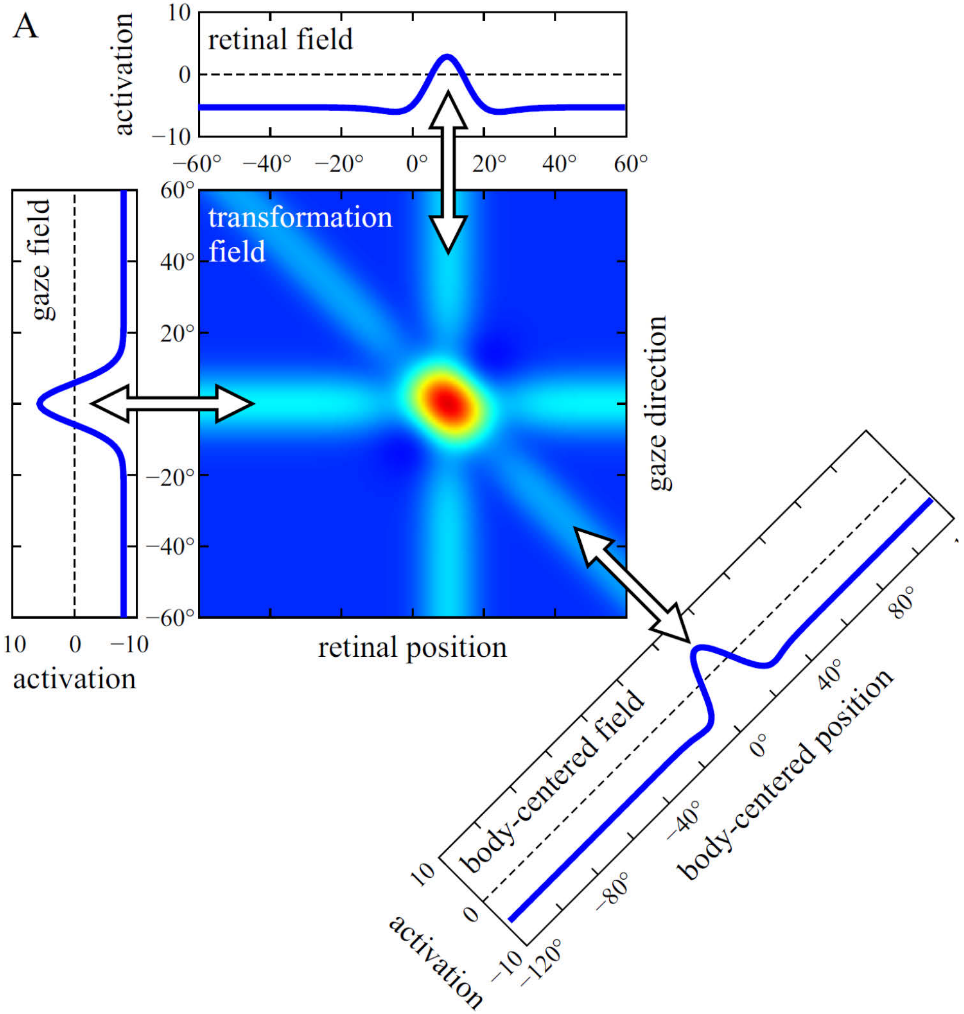


https://github.com/cedar/juniper/blob/main/demo6_MemoryGame.ipynb

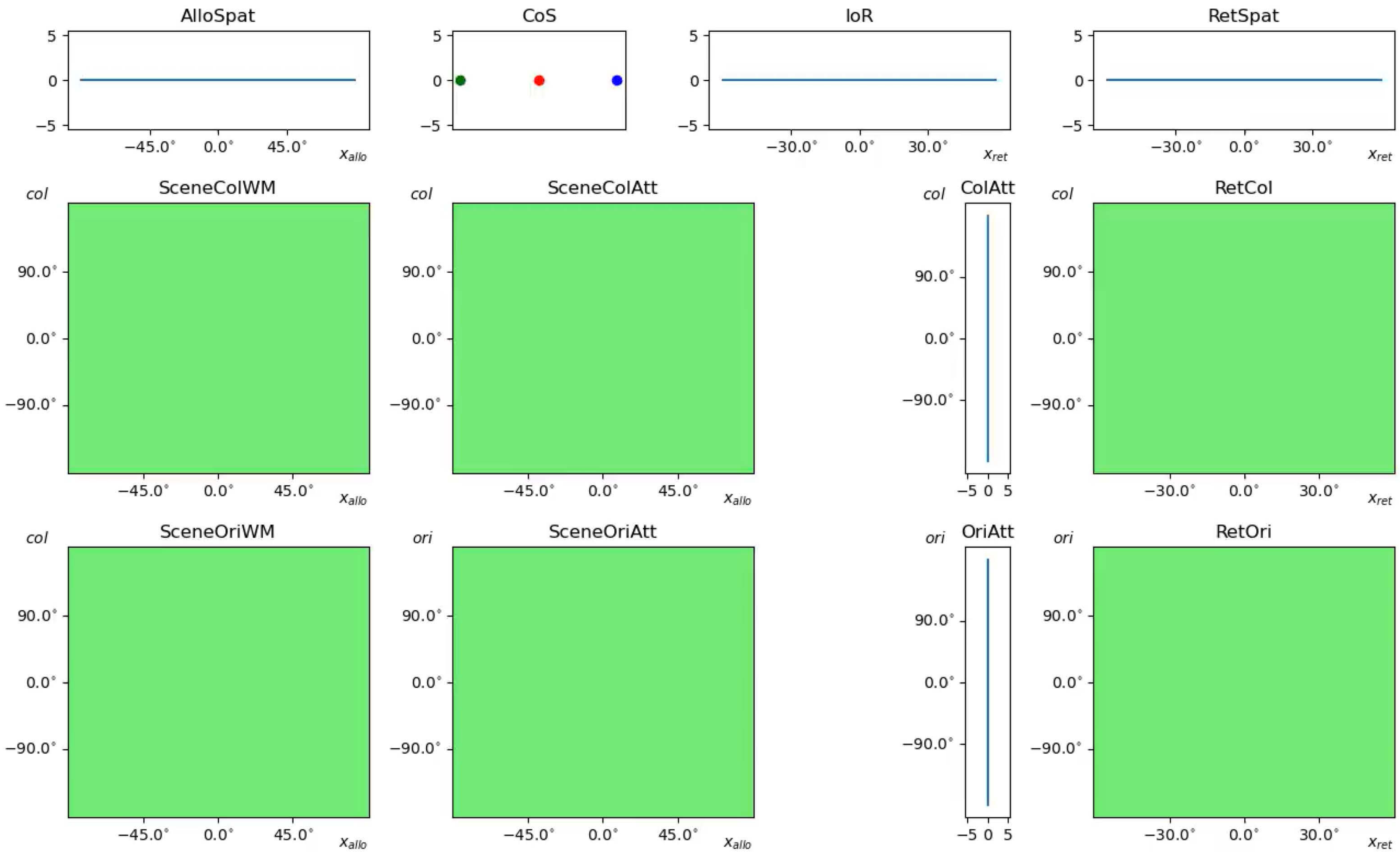


https://github.com/cedar/juniper/blob/main/demo6_MemoryGame.ipynb

Coordinate transformation



Schneegans et al. (2014). Dynamic interactions between visual working memory and saccade target selection. Journal of vision



AlloSpat
CoS
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SceneColWM
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